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JULY, 1949

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ACOUSTICAL SOCIETY OF AMERICA

FOUNDED 1929

THE ACOUSTICAL SOCIETY OF AMERICA was founded in 1929 to increase and diffuse the knowledge of acoustics and promote its practical applications. Any person or corporation interested in acoustics shall be eligible for membership in this society.

Annual Dues

Fellows and members, $8.00; Associates, $6.00; Sustaining members, $50

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The Journal of the Acoustical Society of America

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THE JOURNAL of the Acoustical Society of America

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Sustaining Members

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Any person or corporation contributing Fifty Dollars ($50.00) or more annually may be elected a Sustaining Member of the Acoustical Society and will receive one subscription to the Journal for the year and a yearly membership certificate suitable for framing. Application for membership may be made to Secretary Wallace Waterfall.

THE JOURNAL
OF THE
ACOUSTICAL SOCIETY OF AMERICA

Volume 21 Number 4

JULY • 1949

The Twentieth Anniversary Meeting

"Research is the effort of the mind to comprehend relationships which no one has previously known, and in its finest exemplifications it is practical as well as theoretical; trending always toward worthwhile relationships, demanding common sense as well as uncommon ability."—Harold De Forest Arnold.

Inscription in the foyer of Arnold Auditorium at Bell Telephone Laboratories, Murray Hill, New Jersey.

"The Anniversary meeting fulfilled all expectations in being successful and interesting. There was a record-breaking attendance, which furnished opportunities for renewal of friendships among members, with many discussions of problems among various groups." Thus wrote Editor Watson in our July 1939 issue in describing the Tenth Anniversary meeting held in the Hotel Pennsylvania in New York, but the statement is equally applicable to the Twentieth Anniversary meeting held in the Hotel Statler in New York. Just as this is the same hotel under a different name, the Acoustical Society is the same virile organization under the leadership of different officers, except Wallace Waterfall, who has served with distinction as Secretary for the full twenty-year period. The registration for our Twentieth Anniversary meeting totaled 417 as compared with 290 at the Tenth Anniversary meeting. Our membership now totals about 1400 as compared with 700 ten years ago.

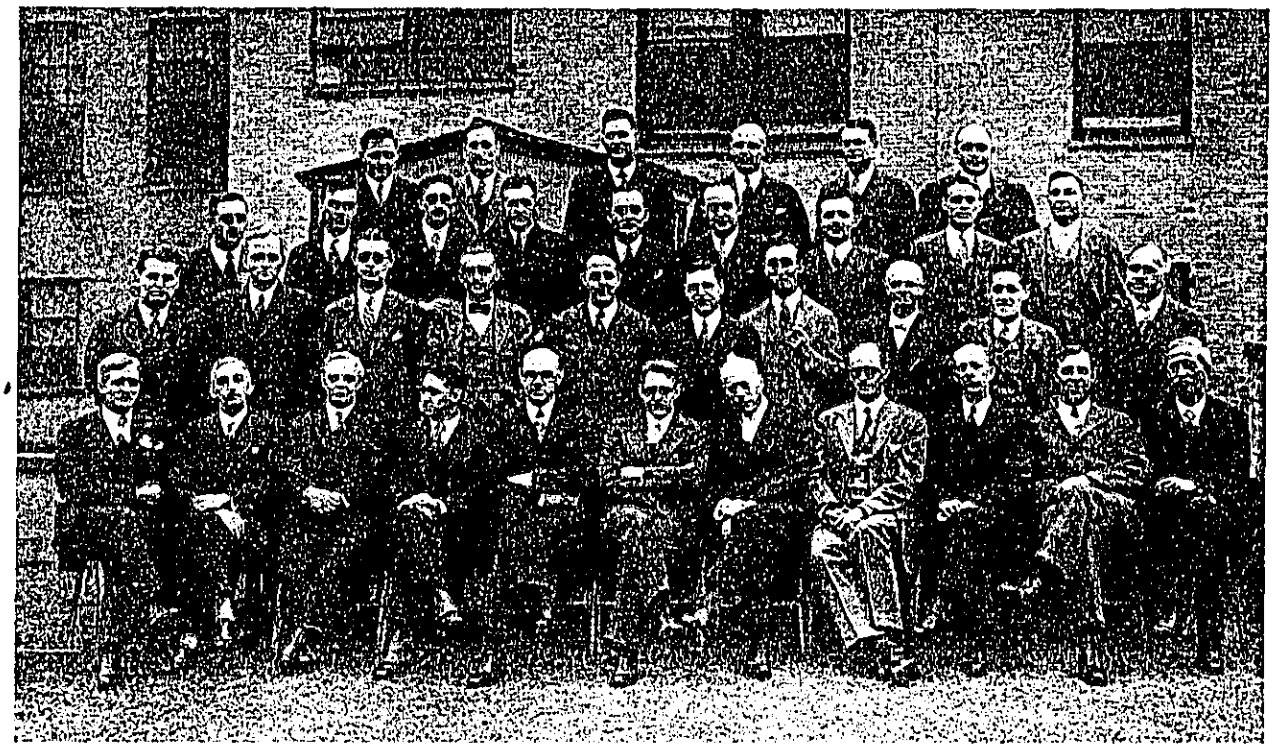
The theme of the meeting was Acoustics and Man, the papers being classified according to function in the following categories: Acoustics in Communication; Acoustics in the Arts; Acoustics in Comfort and Safety; and Acoustics in Research. Invited papers on these subjects were presented in addition to the contributed papers.

A founders' luncheon was attended by a considerable fraction of those far-seeing members who twenty years ago assembled on the roof of the Bell Telephone Laboratories at 463 West Street for a photograph after having completed the plans for the organization of the Acoustical Society of America. This group was photographed again at the recent luncheon, 20 years older, 20 db wiser (see the following two pages). These founders could be identified at the recent meeting by their white carnations and justifiable air of pride.

Since the Society got its start at the Bell Telephone Laboratories, it was appropriate that Friday should be spent in a tour of the beautiful new Bell Telephone Laboratories at Murray Hill, New Jersey. Half of the day was spent in visiting various portions of the laboratory devoted to acoustical research, while the other half consisted in a series of demonstration lectures in the Arnold Auditorium. These demonstrations, listed in the program at the end of this issue, were executed with that beauty and flourish which is seldom seen except at the Bell Laboratories. For instance, we all know that sound will travel around curves inside of a tube, but Winston E. Kock showed sound following around a curve along the outside of a rod covered with disks about the size a penny and about a half-inch apart.

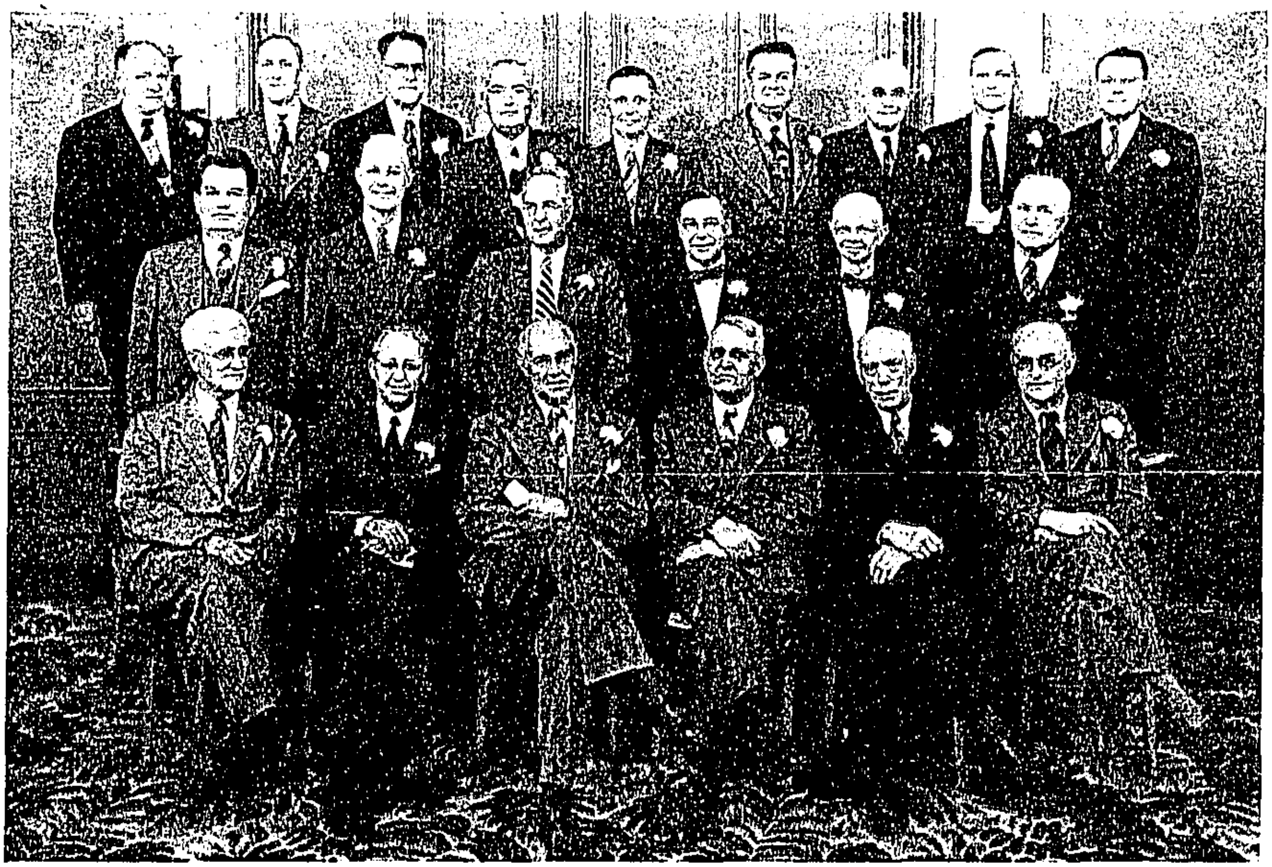
Following the dinner, Dr. Harvey Fletcher, first president of the Society, was presented with a Certificate of Honorary Membership following remarks by his one-time student, Dr. Vern O. Knudsen (see page 293). This was followed by divertissement conducted by the society's humorist and court jester, Pat Norris, who outlined the many advances in acoustics made during the twenty years that he has been a member, starting with the open window unit of sound absorption and ending with the vastly improved modern sound recording systems which now require three turntable speeds instead of the one which formerly sufficed.

To Program Chairman Harold Burris-Meyer and his unusually large Program Committee, we express our appreciation for arranging this memorable occasion. May we speculate on the status of acoustical science on the occasions of our one hundredth or one thousandth anniversaries?



Part of the group of organizers of the Acoustical Society, Bell Telephone Laboratories, December, 1928

Bottom row, left to right: F. A. Saunders, R. V. Parsons, D. C. Miller, W. Waterfall, V. O. Knudsen, H. Fletcher, C. F. Stoddard, J. P. Maxfield, F. R. Watson, F. K. Richtmyer, G. R. Anderson.
Second row from bottom, left to right: H. A. Erf, H. C. Harrison, J. B. Kelly, R. L. Wegel, H. A. Frederick, N. R. French, C. W. Hewlett, A. T. Jones, I. Wolff, J. B. Taylor.
Third row from bottom, left to right: L. J. Sivian, E. L. Norton, W. A. MacNair, R. F. Mallina, L. Green, Jr., R. H. Schroeter, H. W. Lamson, C. N. Hickman, D. G. Blattner.
Top row, left to right: W. P. Mason, J. C. Steinberg, V. L. Chrisler, E. J. Schroeter, E. C. Wente, W. C. Jones.



20 Years Older, 20 db Wiser

The same organizers at the Founders' Luncheon, Hotel Statler, May 5, 1949

Bottom row, left to right: F. A. Saunders, Wallace Waterfall, Vern O. Knudsen, Harvey Fletcher, C. F. Stoddard, J. P. Maxfield.
Middle row, left to right: H. A. Erf, J. B. Kelly, H. A. Frederick, N. R. French, A. T. Jones, J. B. Taylor.
Top row, left to right: W. P. Mason, J. C. Steinberg, V. L. Chrisler, Lonsdale Green, Jr., H. Lamson, E. C. Wente, W. C. Jones, D. G. Blattner, C. N. Hickman.

Dinner, Twentieth Anniversary Meeting, Hotel Statler, New York, May 6, 1949

Harvey Fletcher

Presentation of Certificate of Honorary Membership to Doctor Harvey Fletcher

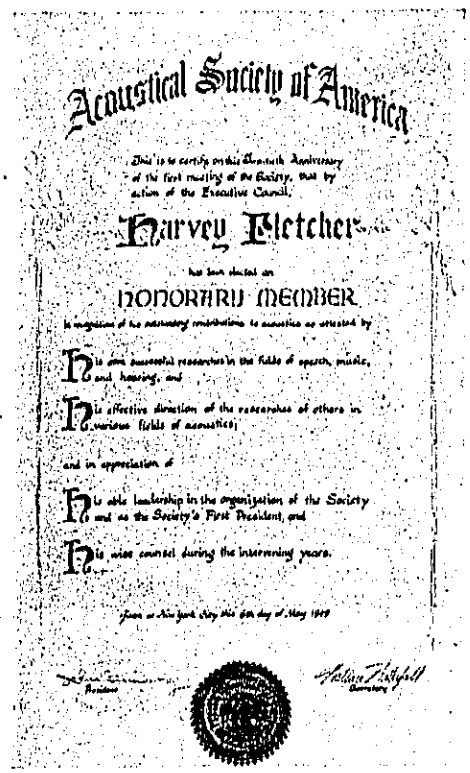
Remarks of Dr. Vern O. Knudsen on the Occasion of the Presentation of the Certificate of Honorary Membership to Dr. Fletcher at the Acoustical Society Dinner on May 6, 1949.

HARVEY FLETCHER, distinguished scientist and engineer, trail-blazing investigator of the nature of speech and hearing, was born of pioneer parents in Provo, Utah, September 11, 1884.

Dr. Fletcher acquired at an early age the art of fishing—and in no mean degree! The native trout in mountain streams can usually see, hear, or otherwise sense the unwary angler, but they don't sense Uncle Harvey until they are on the hook! Once, when returning to camp, he had not only his creel—a California orange crate—bulging with trout, but, hung from his shoulder, a coyote that had threatened violence, and over which Harvey, with the aid of his hunting knife, had won a decision.

Dr. Fletcher received his B.S. degree at the Brigham Young University in 1907, and his Ph.D. at the University of Chicago in 1911. Since then, honorary degrees have been conferred upon him by Columbia University, Case Institute of Technology, Kenyon College, Stevens Institute of Technology, and the University of Utah.

From 1911 until 1916 Dr. Fletcher was Professor of Physics at B.Y.U.; in fact, he was the staff of the Department—and for good measure he taught courses in Differential Equations, Vector Analysis, and Theology. All of the lower and division physics, and most of the upper division mathematics that Carl Eyring, Wayne Hales, and I—among many others—learned at B.Y.U., were taught by Harvey Fletcher. If we learned too little, it was not because Fletcher spread his teaching too wide and thin. He was a profound and proficient teacher in such diverse courses as Electron Physics, Kinetic Theory, Electricity and Magnetism, Spectroscopy, and Thermodynamics. In order to carry his teaching load of much more than 20 hours a week, supplemented with an active research program and with extra-curricular services to Boy Scouts and a variety of other civic and church organizations, it



became necessary for Dr. Fletcher to schedule his first class at 6:45 A.M.

In 1916, he joined the research staff that later became the Bell Telephone Laboratories. His contributions and leadership in acoustical research during the ensuing thirty-three years are so well and so highly appreciated by the members of this Society as to make trite any remarks of mine tonight about his researches in acoustics. Most of his published works, beginning with his doctoral dissertation on "A Verification of the Theory of Brownian Movements and a Direct Determination of the Value of NE for Gaseous Ionization,"[1] and continuing through an epochal series of scientific papers dealing with loudness, auditory masking, speech, music, and theories of hearing, bear the most distinguished of all hall marks—a dark band around the edges of the pages—a band that will become even more conspicuous as succeeding generations of scholars leave their finger marks on these honored pages.

Dr. Fletcher has received many scientific honors. I already have referred to the honorary degrees that have been conferred upon him. But that is not all. He is a member of the National Academy of Sciences, honorary member of the American Otological Society, and a recipient of the Louis Edward Levy medal. He was President of the Utah Academy of Science in 1915–16, of the American Society for the Hard of Hearing in 1929–30, and of the American Physical Society in 1945. But he is best known to this audience as the first President of the *Acoustical Society of America*—and in the minds and hearts of the members of this Society he is, I am confident, its first and most distinguished member.

[1] Phys. Rev. 33, 81–110 (1911).

Professor Wallace C. Sabine, 1868–1919, pioneer of the science of architectural acoustics.

Letter of Appreciation of the Work of Professor Wallace C. Sabine

16 June 1949

Mrs. Wallace C. Sabine
348 Marlborough Street
Boston, Massachusetts

Dear Mrs. Sabine:

The Acoustical Society of America, on the occasion of its Twentieth Anniversary, wishes to extend cordial greetings to you in honor of the great contributions of Wallace Clement Sabine to the science of acoustics.

The pioneering work of Professor Sabine laid the foundations for important scientific and engineering advances in the design of auditoriums for better hearing of speech and music. Out of his achievements have come results of great significance to several arts and professions. His name is recognized in architecture, music, communications and public health, as well as in many industrial fields. Mankind has benefitted from his teachings which led to the control of noise and the enhancement of musical sounds in rooms. The formation of our Society twenty years ago was inspired in large measure by widespread applications of the principles of acoustics which were first formulated by Professor Sabine.

This formal expression of our sentiment was read to the members of the Society at their Twentieth Anniversary banquet, 6 May 1949, and was endorsed by a unanimous rising vote. The memory of Professor Sabine will always live as a guide and inspiration to all who pursue the field of acoustics.

Very sincerely yours,
Richard H. Bolt

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 21, NUMBER 4 JULY, 1949

Vern O. Knudsen

Acoustics in Comfort and Safety

VERN O. KNUDSEN
University of California, Los Angeles, California
(Received May 11, 1949)

ACOUSTICS has long been a servant of comfort and safety. The mere mention of musical instruments, telephony, radio, hearing aids, noise abatement, sound-insulation, room acoustics, and psycho-acoustics reminds us of the diverse and important contributions acoustics has made to human comfort. Similarly, a reference to the whistle, the siren, and the fog-horn will remind us of advances in safety that come from acoustical research and technology. Often the need for improvement of these safety devices has led to significant fundamental research. Thus, the researches of John Tyndall, Joseph Henry, and Louis V. King on the propagation of sound in the atmosphere stemmed from the need for better and safer signaling between ships in fog.

I have no novel scientific discovery to report to this Society today. Furthermore, I shall make no attempt to review or praise what this Society has done in behalf of human comfort and safety, for which neglect I might be rightly blamed on this felicitous occasion of the Society's Twentieth Anniversary. I shall attempt, rather, to make a plea, supported by relevant data and by references to actual accomplishments in certain European countries, for quiet surroundings where people live, work, or seek refuge from the din of *homo mechanicus*.

I shall begin by referring to a chart of sound levels of common noises, including many that trespass beyond all reasonable bounds of comfort or even safety. (See Fig. 1.) Some of these data are from published reports and charts; others are selected from an extensive series of measurements I made with the new pocket-size Scott meter which I carried with me on a recent tour of Europe and a motor trip from New York to Savannah to Los Angeles.

Most of the sound levels in Fig. 1 are "spot" or "short-time average" readings, and no attempts were made to determine "long-time averages" or standard deviations. The standard deviations are given for a few types of location, namely for residences, offices, stores, and factories; these were obtained by Seacord of Bell Telephone Laboratories, and are based on several thousand spot readings.[1]

The level of 94 db given for the Paris Metro (subway) is the average value in eight different first class cars traveling at normal operating speeds. The actual readings in the different cars ranged from 86 to 100 db.

The level of 102 db for the Lexington Avenue bus—the greatest traffic noise encountered during a recent reconnaissance of New York—was taken on the sidewalk at an estimated distance of 12 feet from the bus, which was accelerating toward its

[1] D. F. Seacord, J. Acous. Soc. Am. 12, 183–187 (1940).

maximum speed. During comparative lulls in traffic at this site, the level dropped to 80 db.

The level of 65 db in the 4th floor hotel room in Dallas, Texas, was measured during the passing of street cars, which was occurring about 10 percent of the time from 6:00 a.m. until midnight. The window was open about 12 inches during these measurements. Levels in the room ranged from 65 to 68 db for different street cars, dropped to 50 db during traffic lulls, but mounted to 72 db for a passing truck and to 78 db for a passing airplane.

Although the average sound level of the noise in the ninth floor guest room in one of New York's most exclusive hotels was only 54 db (with window open 2 inches), the level exceeded 65 db 49 times during the hour from midnight to 1:00 a.m.; furthermore, and contrary to popular opinion and the assurances of the room clerk at this hotel, there is nearly as much traffic noise in the fourteenth floor rooms as in the ninth or even fourth—the average sound level on the fourth floor was only 3 db greater than that on the ninth floor, and there was no observable difference between the average levels on the ninth and fourteenth floors.

The valve-type toilet, which roared to 88 db at each flushing, was in a modern hotel in the South, recommended by a well known travel authority. Furthermore, the "sound-insulation" of the walls of the bathroom was so poor that the sound level in an adjacent guest room was 64 db.

The 118 db level in the electrical substation, which had been augmented to this high level by conversion from 50- to 60-cycle operation, was so great that it was regarded as a health hazard. The men who worked in the station complained of temporary deafness, tinnitus, dizziness, and other vestibular symptoms. Some of the men maintained that they could not hear ordinary conversation for several hours after a day's exposure to the noise. I experienced a temporary tinnitus in my left ear after a two-hour exposure, and was aware of a loss of hearing for two or three hours after leaving the noise.

It is interesting to compare these existing typical sound levels of Fig. 1 with a table of Acceptable Sound Levels for different types of rooms which Dr. Cyril Harris and I are proposing in a book now in press, *Acoustical Design in Architecture* (see Table II). These proposed levels are based on objective as well as subjective findings in rooms that are free from complaints or even acclaimed as highly satisfactory. The levels are somewhat lower than the comparable mean values reported by Seacord,[1] and are much lower than most of the selected ones given in Fig. 1.

Suppose it is required to provide residential rooms having an ambient noise level of not more than 40 db at a site such as 44th and Lexington Avenue, New York, where the noise level from the busses may reach levels of 102 db at a distance of 12 feet. Suppose, further, that the minimum distance from the busses to the rooms in question is 48 feet, at which distance the noise level has been reduced 12 db, that is, to 90 db, which is assumed to be the maximum level of noise incident upon the residential rooms. Then the reduction in sound level that must be provided by the combined effects of the insulation of the walls and the absorption of the interior of each room is 90–40, or 50 db. This would require costly construction, with few or no windows, and almost certainly no open windows. Practically, it would be much more feasible to establish traffic regulations that would reduce existing traffic noise by about 10 db. A moderate amount of acoustical designing and gadgetry, including some suitable sound filters, would suffice to reduce this traffic noise at least 10 db. Incidentally, the sound level of automobile horns could then be reduced 10 db, at least for city driving, without loss of signal to noise ratio, which would contribute greatly to the abatement of one of our most annoying noise nuisances. With such a reduction in the ambient traffic noise, the problem of constructing rooms so that the noise levels in these rooms would not exceed the acceptable values listed in Table II would be greatly simplified and could be solved practically at reasonable cost.

The levels proposed in Table II are realized, or even bettered, in most buildings recently completed in several European states. In most of these countries, traffic noise, even in the largest cities, is of the order of 10 db less than it is in our large cities. But even so, building layouts and acoustical designing in these foreign countries are establishing high standards of noise control and sound insulation. Sweden has been notably progressive in these matters, especially in the construction of schools, hospitals and apartment houses. Slums and substandard housing do not exist in Sweden's urban communities—they have been replaced by modern, attractive, sound-proofed buildings.

Figure 2 shows the layout of rooms in an apartment house at Finnboda, Nacka, Sweden, which incorporates many commendable features of acoustical design. Note the separation and insulation between the bedroom of one apartment and the living room of the adjacent one; the separation between the two bedrooms and between the two living rooms in adjoining apartments; the heavy party walls (solid brick), especially between adjacent bathrooms, which provide a sound-insulation of at least 50 db; the use of the hall as a "sound lock;" and many other apparent features of sound planning for the control of noise. The entrance doors are of solid panel construction and they fit tightly in their frames so that threshold cracks are

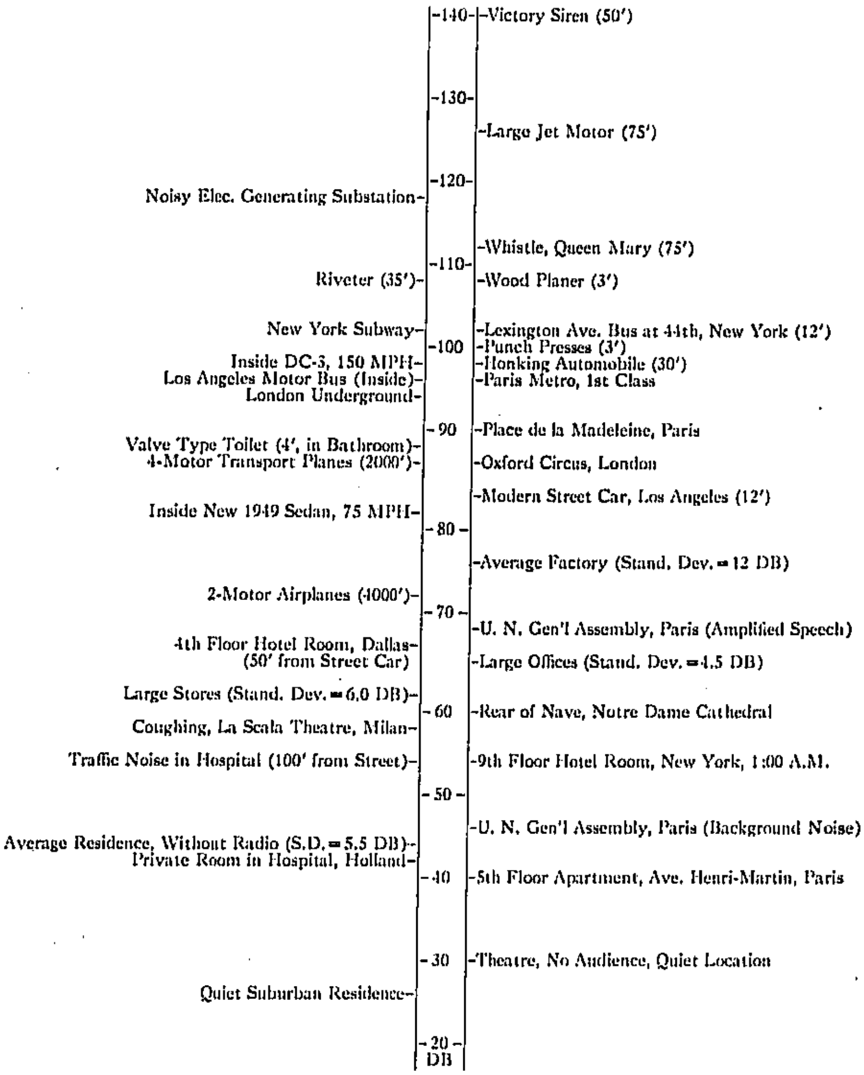


Fig. 1. Chart of sound levels of common noises.

eliminated. All floors above the ground level are of "floating" construction, on concrete slabs, so that impact sounds as well as air-borne sounds are thoroughly insulated. I recently visited several apartments of this general design in Sweden, and was most favorably impressed with the splendid results architects and builders have obtained in providing quiet homes, even in the low cost projects. The Swedes, Danes, Norwegians, Dutch, and British are creating examples of good sound-insulation in their apartment houses that we can well emulate.

The effective control of noise in the buildings recently constructed in these countries is no accident. It is deliberately planned; indeed it must be in order to meet the high standards of their building codes. Thus, the Swedish Code specifies that "buildings containing dwelling and working rooms shall be constructed according to directions given by the Building Authority for providing adequate sound insulation." The directions stipulate the upper limits of sound levels that will be tolerated in different types of buildings, and describe ways and means for attaining the required standards. A first edition of the Swedish Building Code appeared in 1946, and the second edition will appear about June, 1949, according to Dr. Ove Brand, who is one of the acoustical engineers engaged in the preparation of the new code, and to whom I am indebted for Fig. (2) and related material concerning sound-insulation in the buildings of Sweden. Table I gives the minimum sound-insulation, in db, that must be provided between rooms in hospitals, schools, dwellings, etc.

The new Swedish code will further specify that the sound level in certain rooms resulting from the transmission of sound into such rooms from rooms in adjacent apartments or buildings shall not exceed the following values: (a) in very noisy districts (to be designated by the Authority), 35 db in hospitals, 40 db in residential and school buildings, and 45 db in office and business buildings; (b) in very quiet districts, 25 db in hospitals, 30 db in homes and school buildings, and 35 db in office and business buildings. These proposed levels are somewhat lower than those Dr. Harris and I are proposing (see Table II), especially for hospitals, but also for other buildings in quiet districts. The Swedish ratings, the code states, are for continuous noises and not for peak sounds of short duration, such as result from the banging of doors, signals, etc.

Methods for measuring sound levels and the insulation of air-borne sounds are fairly satisfactory, although much remains to be done in perfecting present day sound level meters; methods for measuring impact sounds are less satisfactory, but techniques for providing adequate insulation of such impacts are well known and practiced in most European countries. Acoustical engineers in Great Britain and the Scandinavian countries are cooperating in devising suitable standards and techniques of measurement for the control of impact sounds. A progress report on this enterprise was reported to this Society a year ago by Dr. Jordan of Copenhagen.[2]

I have referred to the accomplishments of our associates across the Atlantic because I believe it will help us to go and do likewise. The problems of noise abatement and sound insulation in buildings have not received the attention they deserve in this country. The work of the Noise Abatement Commission of New York City, with the publication in 1930 of its book *City Noise*, was a good start. The National Noise Abatement Commission also deserves favorable mention for its management of yearly campaigns among our major cities. The efforts of these organizations have been beneficial, but much remains to be done.

It may not be the role of this Society to sponsor regulatory legislation for the control of city noise and of sound insulation in buildings. The provision of appropriate regulations is complicated by the existence of some 2000 building codes in the United States, mostly city or county codes. Local action probably is the required approach. Many of us can help our own communities undertake the necessary action. And we should not, as individuals or as a Society, shirk our responsibility in contributing the technical information on which proper regulatory codes should be based.

As one example of the type of technical information we should exploit, I refer to the findings of Dr. Steinberg, published in *City Noise*.[3] Steinberg found that at a distance of 23 feet the levels of 33 different automobile horns, which had been submitted to the New York Noise Abatement Commission by the manufacturers of the horns, varied from 72 to 102 db. Further tests of these horns, subjective as well as objective, indicated that, "In order to override the maximum street noise found in New York, the levels of the sounds emitted by automobile horns should be of the order of 88 to 93 db as measured with the noise meter for a reference distance of 23 feet between horn and microphone." Fourteen of the 33 horns tested exceeded this range of levels, and only five fell below it. No doubt Steinberg's results influenced favorably the subsequent design of horns and their use by the various manufacturers of automobiles, and we know some automobile companies continued acoustical investigations of the problem. But many of us also know from recent sample measurements we have made of the sound levels of automobile

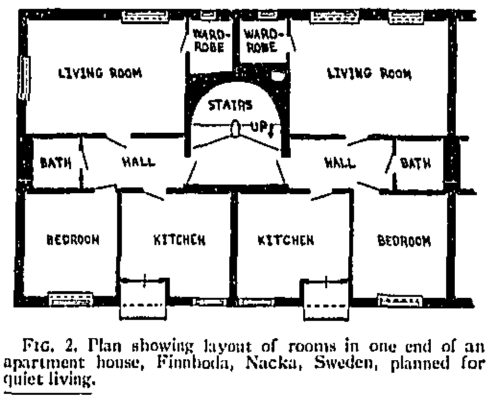


FIG. 2. Plan showing layout of rooms in one end of an apartment house, Finnboda, Nacka, Sweden, planned for quiet living.

[2] Vilhelm L. Jordan, J. Acous. Soc. Am. 20, 595 (1948) (abstract only).

[3] J. C. Steinberg, *City Noise*, pp. 161–187 (1930).

TABLE I.

Type of Building	Insulation for Air-Borne Sound	Insulation for Impact Sounds Floor-Ceiling Construction of: Concrete	Wood
Hospitals	48 db	60 db	48 db
Dwellings	48 db	55 db	46 db
Schools	44 db	50 db	44 db
Office Buildings	40 db	50 db	42 db

TABLE II.* Acceptable average noise levels.

Radio, recording, and television studies	25 to 30 db
Music rooms	30 to 35
Legitimate theatres	30 to 35
Hospitals	35 to 40
Apartments, hotels, homes	35 to 45
Motion picture theatres, auditoriums	35 to 40
Churches	35 to 40
Classrooms, lecture rooms	35 to 40
Conference rooms, small offices	35 to 45
Court rooms	40 to 45
Private offices	40 to 45
Libraries	40 to 45
Large public offices, banks, stores, etc.	45 to 55
Restaurants	50 to 55
Factories	45 to 80

* The levels given in this table are "weighted," i.e., they are the levels measured with a standard sound-level meter incorporating a 40 db frequency-weighting network.

horns that today many exceed the level suggested by Dr. Steinberg's study. It would be helpful to repeat the investigation made 20 years ago, and extend it to trucks, motor coaches, street cars, airplanes, and other offending sources of noise. It is encouraging to note that the Armour Research Foundation, cooperating with the Greater Chicago Noise Reduction Council, is making a survey of city noise in Chicago. With the instrumentation and measuring techniques now available, such surveys and investigations can be helpful in at least three ways: (1) they provide technical data which, when properly compiled, can be used for the reduction of noise among the worst offenders; (2) they arouse a too-indifferent public to the need for codes to control noise; and (3) they furnish indispensable data for the setting up of restrictive codes which will be in the best public interest.

Even with the data now available it is possible to draft codes for a reasonable reduction of traffic noise; for the insulation of air-borne and solid-borne noises in apartment houses, hotels, and hospitals; for the protection of the public against unnecessary and disturbing noise from such places as recreation centers, sub-stations, airports, factories, and proving grounds; and for the protection of workers exposed to noises that are hazards to health.

Several cities in the United States have codes for the regulation of noise, but they fail to specify quantitatively the noise levels that constitute violations. For example, the ordinance for Beverly Hills, California, declares that it is a nuisance and "unlawful for anyone to make, or cause, or permit to be made, any unusual, loud, penetrating, boisterous, or unnecessary noise or disturbance or commotion." And in similar non-quantitative language, the ordinance proceeds to prohibit a large variety of domestic, business, and industrial sounds if they can be "heard distinctly" on property other than that from which they emanate. The Los Angeles code requires that steam shovels, engines, and other mechanical apparatus used for excavating, breaking of pavement, demolishing of buildings, be equipped with mufflers of approved design. Sound-proofing is required for certain places of amusement; vehicles used for vending, soliciting, or advertising can make use of horns, bells, and musical instruments for such purposes only at certain hours during the day, and provided the sounds from such equipment are of such volume and character as do not "harass or annoy persons of reasonable sensibilities." The New York League for Less Noise, in a neat pamphlet bearing the title "Less Noise—More Safety—More Comfort," describes fifteen traffic, radio, and other noises that are against the law in the City of New York. The first of these is, "To sound any horn or signal device on any automobile, motorcycle, bus, street car or other vehicle while stationary except as a danger signal when an approaching vehicle is apparently out of control, or, if in motion, only as a danger signal after or as brakes are being applied." And, after describing the other fourteen offenses in similar legalistic but non-quantitative language, the pamphlet concludes by opining that "999 times in a thousand the sound of an automobile horn means 'Get out of my way—I'm coming.'"

In 1941, a Noise Abatement Commission in Los Angeles, of which the speaker was Chairman, attempted to prepare some quantitative regulations; it drafted for the Board of Building and Safety Commissioners a proposal for a building ordinance that would require dance halls, skating rinks, bowling alleys, night clubs, and other in-door places of recreation or amusement to be enclosed by structures so designed that the over-all noise reduction between the interior of the place of amusement and outside would be not less than 35 db. The proposal further stipulated that the sound level resulting from the noise issuing from such a building, measured at the boundaries of the lot on which the building is located, should not exceed the following values: (a) when surrounded only by business property, 75 db from 7:00 a.m. to 11:00 p.m., and 65 db from 11:00 p.m. to 7:00 a.m.; and (b) when any portion of the lot boundary is adjacent to a residential zone, 70 db from 7:00 a.m. to 11:00 p.m., and 60 db from 11:00 p.m. to 7:00 a.m. Similar proposals were to have been drafted for the regulation of other noises. The war intervened, the Noise Abatement Commission was disbanded, and has not yet been reactivated.

Efforts to control noise in the interest of human comfort have a long history, and, like recent attempts, the yield per unit of effort has been small but significant. Horace (Epist. ii. 2) inveighed against the noises that harassed the man of letters in the Eternal City:

"Festinat calidus mulis gerulisque redemptor;
Torquet nunc lapidem, nunc ingens machina tignum,
Tristia robustus luctantur funera plaustris;
Hac rabiosa fugit canis, hac lutulenta ruit sus:
I nunc, et versus tecum meditare canoros."[1]

English law (Act of 1864) allows a house-holder to send away street musicians, and to this day they are required to keep moving, albeit the motion often is at a snail's pace. James Sully, writing on Civilization and Noise in the Fortnightly Review (1878), discusses this and other legal aspects of noise control. Thus, he assures us that it is possible to restrain noise as a nuisance, and cites the "celebrated case of Soltan *vs.* DeHeld, in which plaintiff obtained an injunction to restrain the ringing of bells at unseasonable hours in a chapel near his dwelling." But in another case, a ruling of Lord Selborne says: "A nuisance by noise, supposing malice to be out of the question, is emphatically a question of degree. If my neighbor builds a house against a party wall next to my own, and I hear through the wall more than is agreeable to me of the sounds from his nursery or his music room, it does not follow (even if I am nervously sensitive or in infirm health) that I can bring an action or obtain an injunction." Sully concludes his paper with a discouraging footnote in which he protests against a "diabolical hooter" at a factory in the university town of Oxford which "shrieks its long, piercing wail every morning at 5:30, and again at 6:00."

Recent attempts in our own country at the control of noise by legal action and injunction have been, so far as I know, gloomily futile. Much of this futility, I believe, is attributable to the lack of proper regulatory codes, based upon sensible but quantitative requirements. Much of the data for formulating such requirements has been accumulated by the members of the Acoustical Society during the past twenty years. The Society can perform a much needed public service by encouraging its members to obtain the additional needed data and to help their communities in formulating sensible standards for the control of noise. Thus can acoustics make a further contribution to comfort and safety.

[1] The hot tempered contractor is hurrying about with his carriers and mules;
A mighty machine turns here a stone, lifts there a wooden beam,
Mournful funerals contend with heavy wagons [to see which makes more noise];
A mad bitch flees over that way, a filthy sow wallows around here.
But now, along with you; I am resolved to meditate on my songs.

Leo L. Beranek

Acoustics in Comfort and Safety

LEO L. BERANEK
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FOR the purposes of our twentieth anniversary meeting the activities of the Acoustical Society have been divided into four branches. The branch we are concerned with is Acoustics in Comfort and Safety. One of the topics under this heading is the quieting of dwellings. In this case our heading is particularly appropriate. Anyone who has lived in an apartment house will testify that from the sounds of neighbors' squabbles coming through the walls one can only conclude that there is no comfort in his own apartment and no safety in his neighbors'.

Our daily comfort is disturbed in many ways—by airplanes flying overhead, by streetcars, by auto horns, and by the roar of trucks on the highway. As for safety, we must include the contributions of sound to medicine, to the charting of the ocean floor and to the detection of enemy submarines.

In treating these various topics, one could describe new discoveries, or point the direction in which new development programs should head, or try to arouse interest in better engineering and in legislation for acoustic comfort. These aspects are all vital parts of our national activities in acoustics. Accordingly, let us treat as much of each as time will permit.

Acoustic comfort in buildings where people must live in close proximity to each other is one of our greatest national needs. Unfortunately, acoustic comfort is costly, and the achievement of it would deprive many people of some conveniences such as a television set, a new car as often as every four years, and so forth. Strangely enough, the biggest advances toward better quieting have been made in England, Holland and Sweden; countries where the income per capita is less than that in the United States. The explanation for this lies in the fact that the people of these nations prefer to spend more on comfort. They spend an estimated 5 percent more of their income on housing than we do. Also, in England and Holland, large areas of housing were destroyed by war, so that new construction is being financed, at least in part, by government funds.

To insure the best in housing, the English government has appointed a group, called the "Burt Committee," to approve new building designs and to encourage novel and promising building construction.[1] As part of this program, the Ministry of Works has expanded the activities of a World War I agency, the Building Research Station, which is located in a suburb of London. This station does research in an integrated manner on these essentials of building: structure, thermal insulation, acoustics and lighting.

Considering the acoustics aspect alone, we find that the Building Research Station has completed an extensive survey of noise conditions in London apartment and row houses. Stated very simply, they have found that in the type of construction now used in the United States (wood stud partitions, wood flooring laid on timber joists), two out of three families complain about noise from the neighbors. The noise reduction through partitions between apartments for this type of construction is about 35 decibels. No special means is provided for reducing the transmission of impact sounds from upstairs to the apartments below. The number of complaints decreases to one for each three families if the noise reduction is increased by eight to ten decibels and if the flooring overhead is floated resiliently. For fewer than one in four complaints, 55 decibels of noise reduction between apartments must be achieved and a floating floor plus a one-inch layer of sand poured on the laths beneath must be provided.

A survey of this type has not been made in the United States. However, the findings of the Building

[1] W. W. Allen, "Science in the construction of houses," undated paper presented before Architects Association in England.

Research Station sound reasonable. Some members of our laboratory at M.I.T. recently had an opportunity to investigate a new housing project in New England. A series of row houses has been constructed by an insurance company interested in long term rentals. The planning was careful—definitely not the result of the efforts of a get-rich-quick speculator. Nevertheless, the complaints from tenants about noise have been many and vociferous.

Measurement of the noise reduction between rooms of adjacent row houses showed 29 db between bathrooms, 37 db between bedrooms and 37 db between living rooms. The noise transmitted between bathrooms was completely intolerable. Both conversation and flushing sounds could be overheard. The principal leakage of sound was through the medicine cabinets which were back to back with no plaster layer between. Between bedrooms the complaint was that the sounds of closing closet doors was audible, and that speech was intelligible if people talked in a slightly raised voice. Between living rooms, the principal complaint was that radios were bothersome.

A quick survey in an apartment house in Cambridge, Massachusetts, confirmed these data. It is quite apparent that the 55 decibels between apartments recommended by the Building Research Station in England is needed. The difficulty of achieving this magnitude of noise reduction may be appreciated if we note that a six-inch cinder block wall plastered on both sides gives an attenuation of about 45 decibels. Structures providing for this amount of attenuation were described in a paper that was presented at these meetings last fall.[2] At least a cavity wall made of two heavy elements is required. Research on efficient structures of lighter weight and cost is clearly indicated.

Another aspect of the work of the Building Research Station is the construction and test of full-scale trial housing units. One way in which this is accomplished is to incorporate new ideas into government-sponsored housing developments. As stated above, government also encourages the construction of novel designs by promotors. The plans for new designs are turned over to the Building Research Station for study before the construction is approved. Suggestions are made to the promotor by the scientists. If all reports are favorable, a license to build is granted to the promotor. After the building is finished the Building Research Station performs physical measurements and from these measurements evaluates the physical suitability of the structure.

In public buildings the same types of problems arise. From the standpoint of acoustics, an important avenue of improvement is through the architect. Every architectural school in our country should require its graduates to take a course in building acoustics, prepared especially for them. This course should at least teach the student that good acoustics are not achieved by putting acoustic plaster on all surfaces of a room. The architect should appreciate that noisy functions should be separated from quiet ones and that this separation must be made on the drawing board at the outset of the planning. Conference rooms and auditoriums should be designed initially to produce good acoustics. This involves consideration of several major factors. The room should be *shaped* to guide the sound waves to all parts of the room uniformly, to eliminate echos and to prevent flutter echo. *After* the basic shape has evolved, absorbing materials should be introduced to control first the reverberation time as a function of frequency, and second, the fluctuations of the decay curve. The achievement of these needs is a complex matter. The successful design of a large auditorium is a job for a specialist, and the architect must become accustomed to employing his services just as he does those of heating, lighting and ventilating engineers.

If accompanied by a survey of the residents in these areas, it would also give us information on which conditions are tolerable and which are intolerable. These data should be assembled in such a way that they can be studied by other municipalities. They may then serve as a basis for a national movement to make city dwelling more pleasant.

Another source of interference with our comfort is airplane noise. Airplane noise casts a blight over the community adjoining an airport. So serious has this blight been that small airfields are located in outlying areas. This remote location results in potential fliers losing interest in private ownership of aircraft, because of the great distances involved in getting to and from the airfield. Studies carried out before and during the war reveal that, to a first approximation, the noise produced by a propeller varies as the sum of 20 $\log_{10}$ of the ratio of the horsepower plus 2.7 times each 100-ft./sec. increase in propeller tip speed. These relationships indicate that if the number of blades is doubled so that the power per blade is halved and if the propeller tip speed is reduced by about 150-ft./sec., the noise levels will drop by about 10 decibels. With this goal in mind, the National Advisory Committee of Aeronautics has contracted for the development of experimental planes with an increased number of blades, a lower propeller rotational speed, and improved engine exhaust muffling. The results of the development have been strikingly demonstrated throughout the country. Noise level reductions of 10 decibels or more are obtained with little or no loss of performance. In a recent demonstration in Cambridge, Massachusetts, one of these

[2] L. L. Beranek, J. Acous. Soc. Am. 21, 264–269 (1949).

quieted airplanes could not be heard above residential street noise, until the airplane was within a few hundred feet of the listener.

One of the most serious blank spots in our knowledge of acoustics is our inability to measure noises objectively in such a way as to yield readings that correlate well with our subjective reactions. For example, it would be desirable to have a meter that reads loudness, loudness level, speech interference level and annoyance of the noise, regardless of the nature of the noise. Results have been reported at these meetings which indicate that a single meter might be designed which would measure the first three quantities.

The noises that are produced outside our homes are the hardest to control. The increase in traffic in every large city has resulted in a din that borders on the intolerable. This statement is particularly true in cities where surface transportation and elevated trains are still in wide use. New York City has gradually changed from noisy to relatively quiet transportation. Their solution has been to replace elevated trains with subways and streetcars with buses or trolley buses. However, the buses in use today are still unnecessarily noisy. Levels of as high as 100 decibels are measured on the sidewalk near a bus as it accelerates. One large manufacturer of buses informs me that with optional equipment, they can now reduce these levels by 10 decibels for a cost equal to less than one percent of the cost of the bus. In Washington, D. C., the introduction of new style trolley cars has reduced the noise of surface transportation substantially. The public has properly responded to this change.

At the present time, a noise survey of Chicago is under way. A paper on this subject is being presented at these meetings. This study should reveal the principal sources and magnitude of noise in industrial and residential areas.

In essence, the measurement of loudness is accomplished by passing the outputs of each filter in a group of contiguous filter bands through a non-linear circuit of a special type and then summing the outputs of these filters. The non-linear circuit chosen for each frequency band should develop an output current proportional to the loudness in tones of the noise in that band. There now exists evidence that the same set of equal-loudness contours may be used for any pure tone or for a band of noise that is not too great in width.

The speech interference level may be obtained in a similar manner. Here the outputs of a set of contiguous filter bands with selected cut-off frequencies in the speech frequency range, are passed through a set of logarithmic amplifiers. The outputs of these amplifiers are combined linearly to produce a quantity that is proportional to the speech interference level of the noise. Unfortunately, there is no indication of how to approach the problem of objective measurement of annoyance. This remains a fertile, though complex, topic for further investigation.

As a final topic, let us look at a few developments related to acoustics in medicine. Sound has long played an important part in the diagnosis of illness. The physician listens to the heart and lungs with a stethoscope. He thumps the chest to learn if one lung has a different resonant frequency from the other. Obviously, a shift upward in resonant frequency would accompany a filling of that lung. A large part of his diagnosis of the illness of each patient is based on sound. Other than improvements in the stethoscope, the acoustical scientist has offered little to the general practitioner. I feel that this is a field in which we should attempt to apply our knowledge and skill.

The surgeon has fared slightly better in this regard. A little later this morning, we shall hear of an acoustic aid to the detection of gallstones. Extensive researches in one of our naval hospitals have led to a sonar-type device which aids in the detection of kidney stones, gallstones, and foreign objects in the body. One of the more thrilling applications of sound in medicine has been described in a recent paper in the German literature. Here, a pair of brothers, one a doctor and the other a physicist, joined together to produce a device that aids in the detection of brain tumors. This device consists of an ultrasonic source that transmits a pencil beam of ultrasound through the head. On the opposite side of the head, a microphone receives the transmitted energy. The output of the microphone is amplified and is used to modulate a source of light. This source of light radiates on a photographic paper. By moving the transducers backward and forward in a scanning motion, at the side of the head, and simultaneously, moving the light source backward and forward above the photographic paper, a photographic record of the attenuation of sound by the head is obtained which is similar to the presentation on a television screen. From the picture so obtained, distortions of the ventricles in the brain, produced by malignant growths, may be observed. This experiment, though crude at this time, points the way to much wider uses of sound for diagnostic purposes. We plan to initiate a program along these lines at M.I.T. this summer.

Ultrasound has been used with some success for producing a warming of tissues beneath the surface of the skin. Also, experiments on inhibiting the growth of tissue by ultrasound conditions are reported in several places in the literature.

In conclusion, I feel that acoustics will continue to play a great part in increasing comfort and safety. It is up to us, as scientists in this field, to pursue our endeavors with even greater vision and persistence.

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 21, NUMBER 4 JULY, 1949

Ralph Bown

Acoustics in Communication

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WHAT have acoustics and electrical communications done for each other up to now and what do telephone research engineers see them doing for each other in the present and in the future? These are the questions I have asked myself and shall now try briefly to answer.

There is no question but that great works have been accomplished during the past generation. One major step was the establishment of this society. There came out of these past activities the measurement technique and the acoustical development necessary to see the telephone through many thrilling years of growth, and to see broadcasting from a toy to a great network that blankets the country.

Acoustics contributed heavily to these achievements and might be said to have done its bit.

But we in the communications industry see no evidence that acoustics is running out of problems. The indications are that there will be many new and exciting problems, the character of which will be determined by new trends in communication, new developments in the areas of physiological and psychological research, and new techniques for analysis.

A new trend in communication that is already beginning to establish itself in broadcasting, is television.

Before the telephone came, human beings habitually used the integrated senses of sight and hearing as their primary means of communicating with each other. The joint use of ears and eyes in the passage of information was instinctive and—until the invention of the telephone—was almost universal. To see how innately our habit pattern combines aural and visual effects one has merely to attempt speaking emphatically while maintaining unchanged one's facial expression and bodily posture.

Dr. Bell's telephone instrument, in extending the distance of instantaneous sensory perception, divorced the two senses of sight and hearing by granting the extension to only one of them. The impact on acoustics was profound. It brought into the study of acoustics an entirely new set of factors and interests. It concentrated attention on the characteristics and capabilities of the ear alone, unaided by the eye, and resulted in an intensive study of these characteristics as matters of great economic importance in the engineering of electrical transmission systems of audible frequencies. Over the past forty or fifty years acoustics and telephony have had such a close relationship that in many divisions of these subjects they were a single study.

This is perhaps best illustrated by the fact that one of the largest and most fruitful research programs in the acoustics field was carried on by a group of scientists associated with the telephone industry. This work, which was mostly under the supervision of Harvey Fletcher, was concerned for one thing with the ways in which understandability of speech is impaired by modification of its frequency content and by changing its energy level both absolutely and in relation to accompanying sounds of an interfering character. In the course of these studies there has come about a great body of factual knowledge together with a formulation of results known as Quality Theory which makes them more available for engineering uses.

A truly scientific attack on the acoustics of speech and hearing required the establishment of methods of evaluation or measurement and the existence of standards of reference. In this, telephony has been of great service to acoustics because most of the instruments used have turned out to be, in essence, merely highly perfected telephone instruments, using the word instruments, in this case not in the usual narrow telephone sense of transmitter and receiver but in the broader sense of instrumentalities to include, also, amplifiers, filters, level indicators, and the like. In this work acoustics and telephony jointly set the stage for the next great advance in sound technology which was radio broadcasting.

The coming of broadcasting added another factor. Telephony thereby became distant, one-way, mass communication as distinct from interchange of words between single individuals. This was a medium suited to the uses of drama and music as well as speech, and the demand for artistic fidelity of transmission became dominant.

The precision instruments which had been developed for laboratory measurements became the operating equipment of the studio and the control room. Acousticians and electrical researchers went to work on the embryonic loudspeaker. The development of a body of art and science for dealing with the problems of the capture, transportation, and reproduction of musical sounds which occurred in the fifteen years just preceding the war is too recent and too well known to require detailed citation. The rejuvenation of the phonograph and the emergence of successful sound motion pictures stemmed also from this scientific development of high fidelity electroacoustic devices, which got its start in the telephone laboratory with important contributions by Edward Wente.

The recent war brought new acoustics problems and fostered the establishment of new or enlarged laboratory groups for the study of these problems. The subject expanded both in scope and in magnitude.

And now we find ourselves today standing again at a time when the relation between acoustics and electrical transmission is undergoing or is about to undergo another material change. The telephone spanned distances, but without vision. Now vision is literally coming back into the picture. The emergence of television into commercial importance has—at least for broadcasting—restored the sense of sight and reunited the ear and the eye of the distant observer. Now the visual medium must be studied and catered to with the meticulous precision which the aural medium has already enjoyed.

This is not to say that acoustics is to be displaced by optics, or that it is of any less importance to telephony than heretofore, or that the program of research in acoustics is of any less vital interest. But it is undergoing reorientation and for reasons which basically are associated with television or at least with the same forces that have been effective in bringing about television. Spurred on by the philosophical needs of such recent transmission developments as pulse modulation, time division multiplex, and television, the basic theory of intelligence transmission, having its beginnings in the philosophy that Ralph Hartley initiated about twenty years ago, is in process of extension, revision, and perfection. This is evidenced in the recent work of Claude Shannon, Norbert Wiener, and others.

Several pertinent items may be mentioned. Hartley's work indicated that a relation exists between the band width for a message channel and its capacity to convey intelligence. The wide swing frequency modulation experiments of Edwin Armstrong directed engineering attention to the interchangeability between band width and power as alternative means of dominating the deleterious effects of interfering noise. More recently there has appeared pulse code modulation, commonly known as PCM, by means of which any form of communication can be reduced essentially to the dimensions of a simple telegraph signal of dots, dashes, and spaces. A related concept is the idea of redundancy of information transmitted which is being recognized as having powerful possibilities—for example, why devote the same amount of band width, time, and power to transmitting successively the identical signals representing a blank wall as are devoted to transmitting the changing visual detail of the like area of a human face or other object to which the blank area is adjacent?

Is it not natural to query whether there exists an analogous factor of redundancy in speech or even in music? And if so how can it be practically isolated and what uses made of it? Information theory analyzes and illuminates the interrelations between these various notions and facts, and in doing so suggests new approaches to the study of speech and hearing. The resemblance, for example, between pulse modulation and the process of nerve transmission by the movement of electrical pulses is so striking as inevitably to lead to renewed and reoriented attempts to understand the fundamental nature of perception.

New developments of interest to acoustics are going on in the areas of physiological and psychological research. Despite the vast amount of measurement that, during past years, has been done on hearing, our understanding of the hearing mechanism is still filled with speculations. But a great deal of brilliant work in many laboratories is bringing us closer to a solution of some of the mysteries surrounding this subject. These include work on the ear, on signals over the nerve transmission system, on the patterns of stimulation produced in the brain, and recent advances in brain wave research.

New techniques for analysis are yielding new information concerning the structure of speech and other sounds. You will see results of some of the things which have been done with sound analysis when you visit our laboratories tomorrow at Murray Hill, New Jersey. The flexibility of modern magnetic tape recording makes it of great value as an experimental tool. Ralph Potter's methods of presenting speech sounds in a multiplicity of dimensions as, for instance, by the sound spectrograph, are even more potent. These techniques, when combined with the philosophical transmission ideas just mentioned, give bases for new experimental approaches to identifying the location of significant elements of speech. Present trends are toward the treatment of speech and other sounds as patterns in both frequency and time and these patterns are being scrutinized in terms of the same basic theory of intelligence transmission that is being applied to broad band multiplex communication problems. In effect, we have new measurement and computational tools that will permit microscopic examination of the structure of speech and other sounds.

The electromagnetic wave guide of George Southworth led us into microwave techniques which later drew upon geometrical optics for aid in solving problems of directive radio transmission. The electromagnetic wave-lengths involved are of the same order as acoustic wave-lengths and already there is the beginning of evidence that acoustics can make use of this electrical communications technology.

An so once more, as in an earlier generation, advances over the broad front of electrical communications are bound to have a profound impact upon acoustics. From the communications standpoint, I am inclined to predict that those who will carry on research and development in the field of acoustics for the generation to come will find it quite as exciting as it has been to those who have brought this science to its present status.

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Charles Kittel

The High Frequency Region of the Acoustic Spectrum in Relation to Thermal Conductivity at Low Temperatures

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THIS talk is concerned with our knowledge of the propagation of sound waves at microwave frequencies. There is at present no direct experimental information in this field. The highest frequency to which a crystal has been excited by electronic means[1] is in the neighborhood of 1000 Mc/sec. or 10^9 cps. The theoretical upper limit to the vibrational spectrum of a solid occurs when neighboring atoms in the lattice are vibrating 180° out of phase with each other—this corresponds to an acoustic frequency of $\sim 5\times10^5/5\times10^{-8}=10^{13}$ cps. This mode of motion has been observed as an infra-red absorption line. Information regarding the intermediate region between 10^9 and 10^{13} cps, which occurs in thermal vibrations in solids, may be inferred from x-ray and optical scattering, infrared (Reststrahlen) absorption, and thermal conductivity at low temperatures.[2]

Electronic power sources are available up to around 50,000 Mc/sec., so that one may say that ultrasonic work is lagging behind available power supplies by a factor of fifty in frequency. What is the reason for this lag? Let us consider the means of conversion of electrical energy to acoustical energy. In the high frequency region only quartz crystal transducers have been used. There is every reason to believe that the piezoelectric properties of quartz hold up to the infra-red; one evidence is the fact that the dielectric constant of quartz is the same in the microwave region as at lower frequencies.

What then is the problem? The problem is the mechanical tolerance on the thickness of the quartz crystal. It is pertinent to consider the situation at 3000 Mc/sec. The wave-length of sound in quartz at this frequency is about 2×10^{-4} cm, or 20,000A, which is just 3 or 4 times the wave-length of visible light. It is clear that *optical tolerances* will be required.

A quartz crystal whose fundamental frequency is 3000 Mc/sec. will be only one micron thick. This is much too thin to prepare and to handle, so that it is necessary to use a thicker crystal driven in a high harmonic, for example, a 30-Mc/sec. crystal (about 0.01-cm thick) driven near its hundredth harmonic. In principle nothing is lost at the same electric-field intensity by using a high harmonic of a thick crystal, but in practice there may be interference between adjacent overtones if the crystal is not of uniform thickness to a high degree of accuracy.

[1] Ringo, Fitzgerald, and Hurdle, Phys. Rev. 72, 87 (1947); R. A. Rapuano, M.I.T. Research Laboratory of Electronics Progress Report for January 15, 1948, p. 38.
[2] C. Kittel, Phys. Rev. 75, 972 (1949).

In receiving sound waves the crystal will only respond effectively to a beam arriving within the "main lobe" of the crystal. If the beam arrives outside of the main lobe, different portions of the crystal face will be out of phase and the electrical pulse will be greatly weakened. For a crystal 1 cm in diameter the angular positions of the first minimum in the diffraction pattern at 3000 Mc/sec. occurs at about 0.01 degrees from the normal. This means that the alignment of the reflector relative to the transducer is very critical.

Let us now consider indirect means of obtaining information about the behavior of sound waves at microwave frequencies. At 1°K heat conduction in a non-metallic solid occurs principally through sound waves (phonons) of frequencies in the micro-

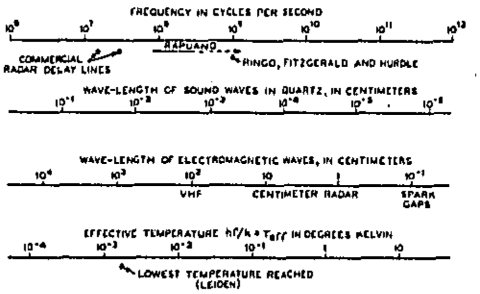

FIG. 1. Chart showing orders of magnitudes of quantities of interest in the hypersonic region.

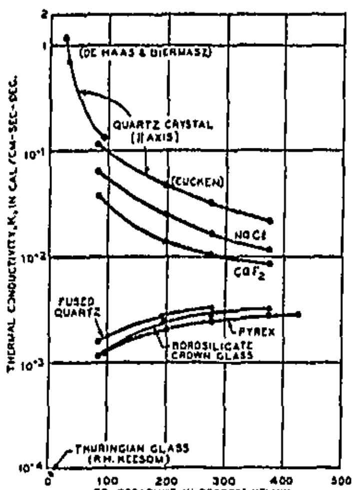

FIG. 2. Comparison of thermal conductivity of glasses and crystalline substances.

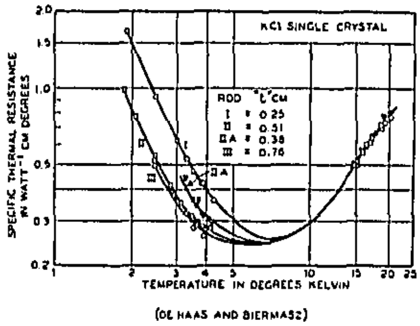

FIG. 3. Thermal resistivity of single crystal of potassium chloride as measured by Biermasz and de Haas. Below 5°K the resistivity is a function of the crystal thickness "t".

wave range. The order of magnitude relationships are illustrated by Fig. 1.

By analogy with the corresponding expression in the kinetic theory of gases the thermal conductivity K of a solid is written $K = \frac{1}{3}cv\Lambda$, where c is the heat capacity per unit volume, v is average velocity of sound, and Λ is the mean free path of the sound waves which participate in the conductivity. The behavior of the conductivity in crystals and glasses is illustrated by Fig. 2.

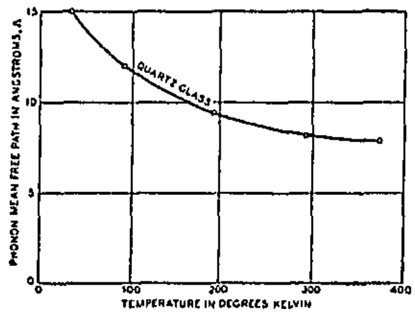

FIG. 4. Phonon mean free path Λ as a function of absolute temperature, for quartz glass.

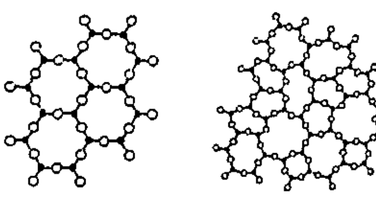

FIG. 5. Schematic two-dimensional figures, after Zachariasen, illustrating the difference between: (a) the regularly repeating structure of a crystal; and (b) the random network of a glass.

The thermal resistivity of several single crystals has been found by de Haas and Biermasz[3] to pass through a minimum in the liquid helium range—the position of the minimum depending on the diameter of the test specimen (Fig. 3). Casimir[4] pointed out that the maximum occurs when the phonon mean free path becomes of the same order as the specimen diameter. This result means that sound waves of the given frequency range may propagate for at least 10^4 wave-lengths, so that the absorption per wave-length is very low.

The thermal conductivity of glasses decreases with decreasing temperature, while the conductivity of crystalline substances increases with decreasing temperature. The behavior of glasses is interpreted in terms of an approximately constant free path for the lattice phonons, so that the conductivity decreases roughly with the specific heat. The value of the phonon mean free path at room temperature (Fig. 4) is of the order of magnitude of the scale of the disorder in the structure of glasses (Fig. 5) as determined from x-ray evidence—that is, of the order of 7A. Here we are concerned with frequencies of the order of 10^{13} cps. This process is analogous to the scattering of ultrasonic waves in polycrystalline materials.[5] At low temperatures the mean free path increases, as here the wave-length becomes larger than the scale of the disorder.

We therefore see that the behavior of sound waves of microwave frequency as deduced from thermal evidence is consistent with the behavior at lower frequencies where direct ultrasonic measurements have been made.

[3] W. J. de Haas and T. Biermasz, Physica **2**, 673 (1935); **4**, 752 (1937); **5**, 47, 320, 619 (1938).

[4] H. B. G. Casimir, Physica **5**, 495 (1938).

[5] W. P. Mason and H. J. McSkimin, J. Acous. Soc. Amer. **19**, 464 (1947).

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The Contributions of Acoustics to the Arts

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IF one looks up the word "art" in the dictionary, he will find mention of various activities, including such pursuits as etching, painting, sculpture, bookbinding, weaving, and needlework. It is difficult to see how acoustics can contribute to such arts except for providing suitably quiet workrooms for the artists, and suitably reverberant galleries for the public display of their works.

We agree, of course, that when we speak of the contributions of acoustics to the arts we mean the sounding arts; i.e., public speaking, the drama, the sound theater, music, and of course, as a very important by-product, the art of architecture. Speech or music in any enclosed space is subject to the effects of reflections, boundary shapes, and materials, as has been under investigation since the days of Wallace Clement Sabine. The science of architectural acoustics has aided in improving the characteristics of auditoriums, large and small, and in tailoring them for the specific demands to be made by such purposes as lecturing, play production, broadcasting, individual and class music instruction, liturgical worship, opera, sound picture recording and reproduction, chamber music, and orchestral concerts. Existing auditoriums are being corrected for acoustical faults through the aid of high speed level recorders which make possible a rapid survey of the behavior of sound throughout the frequency range. Much has been learned about insulating such rooms from outdoor noise or from transmitted machinery noise. All of this has been reflected in the art of architecture itself, adding new possibilities for functional beauty never realized in past centuries.

It is completely to be expected, however, that the greatest contributions of acoustics to the arts lie in the specific art of music, since acoustics has to do with sound, and music is the art most dependent on sound. All the other sounding arts carry a certain amount of meaning in the word-content. This is also true of sung music. Absolute music, however, divorced even from programmatic implications, is wholly dependent on sound.

In addition to the contributions from the field of architectural acoustics already mentioned, music has profited in other ways. Foremost, perhaps, are the broad general research and measurement programs of the past twenty and more years, which have so greatly increased our knowledge about music from its origins in primitive scales and in the vibrations of vocal cords, strings, reeds, membranes, and air-columns to its reception as sensations in the brain. Naturally, these programs have been made possible only as suitable apparatus became available. The history of the contributions of acoustics to music is thus largely the history of the development and constant improvement of the microphone, the vacuum tube amplifier, the loudspeaker, and their various combinations and permutations in reproducing and measuring equipment of all types. These useful devices have made possible detailed analyses of the vibration characteristics of all musical instruments, and of the orchestra as a whole, while the frequency and intensity spectra of the most important instruments, and especially of the human voice, have been exhaustively studied.

Not only has the production of sound been studied, but also its reception, and we note great developments in the fields of physiological and psycho-acoustics. We have learned much about the physical operation of the ear mechanism, and about the psychological aspects of hearing which lie beyond the basilar membrane. Music directors have become decibel-conscious, and perhaps realize more clearly why a doubling of the number of performers does not mean a doubling of the loudness of the chorus or orchestra. Some directors realize the implications of masking, and wish that composers did also.

The education of us musicians is a slow process, however. In 1937, a writer in the Journal tentatively prophesied a quantitative scale of loudness for musicians, in 5-db steps, with a sound level meter at the conductor's stand. I don't believe this has occurred as yet!

One hesitates to mention names of those who have contributed to this large amount of research for fear of omitting those who should not be omitted. However, in the educational field, the pioneer work of Dayton Miller at Case School, of Seashore and his followers at the University of Iowa, of Ortmann and his associates at the Peabody Conservatory of Music, and of Saunders at Harvard, and Stanley in New York, should be mentioned. Also, in the industrial field, the equally pioneer work of Fletcher and his associates at the Bell Laboratories, and the technical advances of RCA-Victor, Columbia, Conn, and the various makers of electronic equipment, should be recorded.

An outgrowth of this research has been the advance in all forms of sound recording and reproducing, which in addition to giving us sound pictures of good frequency and intensity range, has so greatly stimulated music and its appreciation in the home and school. Whether it is now to be wire, or tape, or disks, and at what speed of revolution, is for the thirtieth anniversary meeting to tell.

An interesting field of development, given great impetus some years ago by the Bell Laboratories, is that of the stereophonic transmission, recording, and reproduction of sound, in which the spatial aspect is preserved through the use of two or more independent channels. This improves greatly the realism of any reproduced sound that depends to any extent on a perception of left-right differences, and for this reason should be of great value, at least in the transmission of plays and operas.

One might think that acoustic research and improved technical methods would have given music greatly improved pianos, violins, and other instruments, but this has not occurred. The improvements in the traditional instruments have been mostly in minor details, and in time-saving methods of manufacture which in some cases have actually lowered the quality. True, we have new instruments in the electronic field. One would like to say they have ushered in the dawn of a new day. In fact, some do say so. Their various faults, however, give rise to the reactionary view that perhaps the cheapest and most efficient way to imitate a reed tone with transients fore and aft is to use a reed. A reed, after all, is a fairly inexpensive gadget, and will actually sound more like a reed than a synthetic and imperfect imitation of one by means of vacuum tubes and loudspeakers. One is also driven to the view that there is no cheap and simple way to imitate an ensemble of musical sources short of providing at least two and preferably more loudspeakers, with each one fed by slightly different signal material. The peculiar satisfaction produced by any ensemble, such as a large pipe organ, chorus, or orchestra, is largely a matter of the spreadness of the sources in space, and of the "fringes" and richness produced by many sources not precisely in tune and not precisely simultaneous in onset and release. These new instruments have, of course, given us many new and heretofore never-heard timbres. New aesthetic experiences are in store for us as they develop, and in time they may improve sufficiently to influence the trend of musical composition, or even to create a new literature conceived in their own particular idiom.

Since acousticians are to some extent mathematicians, a certain amount of research is always being carried on toward the theoretical development of new scales, and of instruments to produce their tones. The long struggle for the alleged perfection of the just scale goes on today as it did twenty, fifty, a hundred years ago, even though musicians continue to demonstrate that they do not often use just intervals when they are able to do so. The fascination that these just ratios hold for academic acousticians could be lessened if they were to realize the horizontal, motional, melodic significance of tones, usually a far more important matter artistically than their roughness or lack of it in combination with other tones. A more fruitful field, perhaps, would be the development of a scale based on the functional characteristics of the ear, as proposed by Knudsen. Such a scale would have smaller-sized steps in the upper part of the musical range where the ear is more sensitive to change. Twelve steps per octave would be enough in the lower range, more divisions in the middle and upper.

Perhaps the most important, and in the long run most significant, contributions of acoustics to music and the other arts are in less tangible matters. Thus, for example, one who has watched the growth of the Society during the twenty years of its existence notes an increasing recognition of the importance of the aesthetic factors, those hazy borderline phenomena which the pure scientist would like to ignore and often does. One of the early ones was the vibrato. Others were found in the transients of tone onset and decay, and the pitch and intensity contour of the bridge between connected tones in many orchestral instruments and in the voice. I have mentioned the important effect for musical "blend" of multiple sources slightly out of tune either by virtue of mistuned constant pitches or of tones modulated by vibrati, preferably at differing rates. Such aesthetic factors become of great importance in the field of ecclesiastical architecture. The textbooks give us "optimum reverberation time" values for various types of auditoriums, and for churches of non-liturgical and of liturgical character, the curve for the latter being higher because of the lessened importance of the sermon articulation to the whole. However, more is involved here than the mere ability to attain a certain syllable articulation or to hear the music with best appreciation of its contrapuntal or harmonic structure. A long, slowly-dying reverberation of music or spoken liturgy, particularly in the higher reaches of a large church, is an effective aid to a spirit of meditation and worship. This effect on people can even be noticed in certain highly reverberant structures of non-religious character. In the case of churches and cathedrals, the persistence, and the very indefiniteness of localization except in the upward direction, may cause such reverberant sound to become a symbol for the omnipresence of the Holy Spirit, subconsciously or even consciously experienced. If this be true, and there is evidence that it is, a long reverberation becomes a desirable thing, perhaps even more desirable than the complete understanding of the spoken word. We have difficulty in giving most church musicians enough reverberation to satisfy them. This has come to light again in measurements made recently on the Riverside Church, where according to published optimum curves the church is, if anything, too reverberant even when filled, at least at low frequencies, although church musicians generally join in condemnation of its acoustics and of what they term the "remoteness" of the large choir which does not sound out as it should. The usual attempted remedy in such cases is to cut down the reverberation of the lows and step up the highs. One even wonders if this is the best solution, however, since the use of the words "cathedral roll" by musicians in a laudatory sense implies the long continuation of low frequencies.

As one reviews the course of development of the Society, he notes a greater understanding and cooperation, or at least attempts in that direction, between acousticians and musicians. In 1937 Knudsen wrote, "Throughout the centuries, until recently, music and acoustics have been closely allied. To be a musician, it was necessary to know thoroughly the science of sound, and the acoustician pursued his theories and experiments almost wholly for the benefit of music. Today, musicians as a group know far too little about acoustics, and acousticians know less about music." We would like to think that situation has been improved. The attempt to arrive at a clarification of definitions satisfactory to all is a hopeful sign, as is also the setting up of liaison committees, joint symposia, concerts at acoustical meetings, acousticians speaking to music societies, and the subtle interpenetration of each other's camps from an increasing use of each other's terminology. Occasional papers in the Journal even touch lightly the fields of the psychology and the pedagogy of music. Music journals have references and whole papers on acoustic matters. Conservatories of music are starting to teach acoustics and conduct acoustic laboratories. The Juilliard's work branches out in this field, and helps to balance the discontinuance of the acoustics work at the Peabody Conservatory some years ago.

At anniversaries one is tempted to look into the crystal ball, and by extrapolation of present tendencies attempt to predict the future. So frequently one can be mistaken, or overly-optimistic, as can perhaps be seen by reference to the programs at the tenth anniversary meeting. It is certain, however, that we will see further developments in architectural acoustics, particularly in the development of materials; and in general acoustic research, particularly in the field of physiological and psychoacoustics. In the field of musical instruments I personally am able to see little in the crystal ball except a possible wedding between the pipe organ and some electronically produced stops. In the recording field all I can see are wheels revolving at 33⅓, 45, and 78, with an ominous cloud of magnetic tape approaching. Aided perhaps by wishful thinking, I see increasing cooperation and understanding between musicians and acousticians. I see an extension of the frequency and intensity ranges used in music, and an increasing amount of electrical creation and manipulation of sound for special effects, particularly in theatrical presentations. The work of Burris-Meyer points the way here. Selective amplification, or timbre modification by means of filters, of certain instruments or of sections of an orchestra is a possibility. Sepulchral reverberation chamber effects, and the modulation of one timbre by another, as when a locomotive whistle is made to speak words, give still more possibilities for dramatic, if not for musical enhancement. We soon come to very real boundaries, however, in the physiological limits of the ear, which are not likely to be increased even in the next twenty thousand years. How much more would just one additional octave or ten more db add! But our musicians would at once use it as they did with the Bell Laboratories amplified orchestra system, and pine for more.

Optimists sometimes make statements like this: "There can be little doubt that the music of the future will be revolutionized as the result of modern developments in acoustics." Perhaps this is true, but the crystal ball seems not so sure. Do scientific advances, improved instruments, and scientifically designed scales produce a new and superior musical art? Or does art invent its own media, instruments and scales, and go its own merry and uninhibited way while the acousticians try to catch up with it, explain it, and improve it?

Beats and Nodal Meridians of a Loaded Bell

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A small load on a bell usually changes the rapidity of beats and shifts the positions of nodal meridians. A study of the first three partials of one bell leads to the following conclusions. If the antinodal meridian nearest the position at which the load is to be applied is associated with the lower [higher] pitched of two beating components, the addition of the load increases [decreases] the rapidity of the beats, and also shifts the nearest antinodal meridian of the lower component toward the position of the load. Small increases in the load increase these effects.

INTRODUCTION

THE nodal lines of a bell consist of meridians, which run up and down the bell at different azimuths, and circles, which lie at different levels. If a bell were perfectly symmetrical, the positions of the nodal meridians would depend on the position at which the bell was excited. If the bell is not perfectly symmetrical, the positions of the nodal meridians are determined by the distribution of matter in the bell. When there are slight deviations from symmetry, each of the various natural pitches given by the bell is likely to be affected with beats. The beats arise from two components which have the same number of nodal lines and have these lines distributed alike, except that the meridians for one component lie halfway between those for the other. Thus, for the "hum note," which is the lowest note given by a bell, each component has nodal meridians 90° apart, and the nodal meridians of one component coincide with antinodal meridians of the other.

Each of the various natural pitches of a bell is therefore likely to be in reality a doublet, the motion consisting of two normal modes of vibration, each of which has its own effective inertia and effective stiffness, but which have nearly the same frequency. Mounting a small load on a bell is likely to change the effective inertia for one component more than it does for the other, and thus to change the rapidity of the beats. It also changes somewhat the distribution of material, and so is likely to produce a shift in the positions of the nodal meridians.

This paper reports a study of the changes in the rapidity of beats and in the positions of nodal meridians under the action of a series of small loads mounted successively at a given position on the bell.

METHOD OF STUDY

This investigation is restricted to the first three partial tones of a bell which has a diameter of about 54 cm at the mouth, and a weight of about 80 kg. On this bell the first three partials are clear, are easy to examine, and have frequencies of about 400, 670, and 810 ~/sec. Any chosen one of the partials is readily brought out alone by pressing against the bell, through a piece of cloth, the stem of a vibrating tuning fork of suitable frequency, and immediately removing the fork. The bell then sings out that particular partial tone, and continues to sing for some ten to forty seconds before the sound dies out. When the bell is not loaded, and is thus excited, each of the three partials is affected with beats—which come respectively at rates of about 0.49, 0.74, and 0.56 per second. There are no beats when the point at which the fork is applied lies on a nodal meridian for either component, and this fact provides the means employed in this study for finding the locations of nodal meridians. The circumference of the mouth of the bell was marked off in centimeters, and it was often possible to determine a nodal position to a fraction of a centimeter.

In order to load the bell, sixteen equally spaced holes about 4 mm in diameter were drilled around the soundbow—the thickened part on which a clapper strikes. These holes extended to a depth of a few millimeters and were tapped to receive screws. The principal part of each load was a steel cylinder about 25 mm in diameter, drilled with a longitudinal hole through which the screw slipped easily. The results of loading were erratic until a short brass ring of about the same diameter as the cylinder was inserted between the cylinder and the bell. This ring had three short legs that rested against the bell, and so provided a firm contact at the periphery of the cylinder. Eleven loads were used, running up to a maximum of nearly 140 grams, which is less than 0.2 percent of the mass of the bell.

RESULTS

Nodal Meridians

The addition of a load shifts the pattern of nodal and antinodal meridians around the bell, and shifts it in such a direction as to bring an antinodal meridian nearer to the position of the load. With an increasing load, the antinodal meridian comes nearer and nearer to the position of the load, as is shown by the curves in Fig. 1. In this figure a curve is given for each of the three partials, and each curve shows the positions found for a nodal meridian of the higher pitched component, and therefore, for an antinodal meridian of the lower component. The vertical line at about 43 cm shows the position at which the loads were applied when the observations for the first partial were taken. The vertical line at about 32 cm shows the position of the loads when observations for both the second and third partials were taken. The vertical arrows that point to the axis of distance show the positions at which nodal meridians for the lower pitched component were found when the bell was not loaded. The shifts in the positions of nodal meridians for the lower component would be given by curves parallel to those shown, but starting upward from the positions of the arrows.

When the position at which loads are added is not too far from an antinodal meridian for the lower component of the unloaded bell, the curve becomes steeper and steeper as it rises. When the loads are far enough from such an antinodal meridian, the lower part of the curve shows a knee like those in the curves of Fig. 1. Where there is such a knee, the part of the curve that is most nearly horizontal usually occurs when the load is about halfway between nodal and antinodal meridians.[1] This result may be expected because a load applied close to an antinodal meridian would certainly not produce much shift; and if the load is applied at a nodal meridian, there would be nothing to determine the direction in which a shift would start, so that a load in the neighborhood of a nodal meridian may perhaps be expected to give rise to only a small shift.

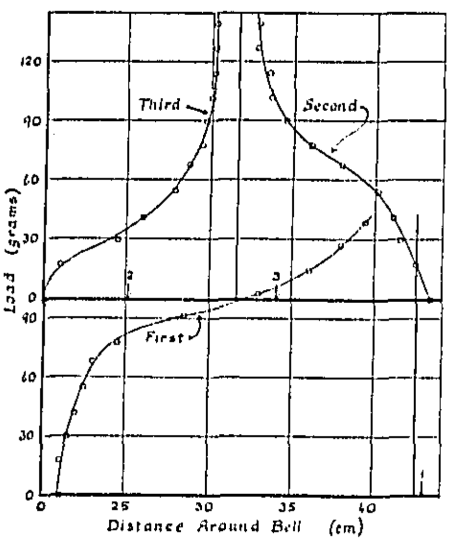

Fig. 1. Positions of antinodal meridians for the lower pitched component of the first three partial tones of a loaded bell. The curves show how these meridians shift around the bell under the action of an increasing load applied at one point on the bell.

The height of the knee is usually greater for the first partial than for the second, and greater for the second than for the third. This difference may be associated with the different masses of the vibrating segments. Each component of the first partial has four nodal meridians and no nodal circle, and consequently has four vibrating segments. For the second partial, each component has four nodal meridians and also a nodal circle, and therefore has eight vibrating segments. For the third partial, there are six nodal meridians and one nodal circle, and therefore twelve vibrating segments. The vibrating segments for the first partial are larger than those for the second, and those for the second might at first thought seem to be larger than those for the third. But the nodal circle for the second partial is lower than that for the third, so that there is not much difference in the sizes of the lowest segments of the second and third partials. However, the thickening of the bell in the soundbow provides more mass in the lower segments of the second partial than in those of the third. It follows not only that the mass of one vibrating segment at the bottom of the bell is greater for the first partial than for the second, but also that it is greater for the second than for the third. For the first and third partials, the loads at which the curves are most nearly horizontal in Fig. 1 are about average values. For the second partial, the knee is usually lower than in Fig. 1, but on the average I find the knee of the second partial something like half again as high as that for the third. These results seem to fit the idea that the knee occurs for a greater load when the mass of a vibrating segment at the bottom of the bell is greater.

Beats

The curves in Fig. 2 show the rapidity of beats as a function of load. All three curves are for loads applied at one position on the bell. The curves for the first and third partials are nearly straight, but that for the second is decidedly curved, with a minimum at a load in the neighborhood of 40 grams.

For the first and third partials, the load was rather close to a position at which there was an antinodal meridian for the lower component when

[1] In Fig. 1 the curve for the second partial is exceptional in this respect.

the bell was not loaded. The load therefore produced very little shift in the pattern of nodal lines. Since the load was near a nodal meridian of the higher pitched component, the frequency of that component was not much affected by the load. The principal change brought about by the load was an increase in the effective inertia of the lower component, with a consequent decrease in the frequency of that component. The beats therefore became more rapid, and for the small changes in load that were involved, the relation between load and rapidity of beats was nearly linear.

For the second partial, the position at which the load was applied was not far from a nodal meridian of the lower component on the unloaded bell. The effect of a small enough load was therefore to lower the pitch of the higher component without greatly affecting the lower. This decreased the rapidity of the beats. But the load also shifted the meridians, and when the load was in the neighborhood of 40 grams, it was about halfway between a nodal and an antinodal meridian for each component. Under these circumstances a small increase in the load lowered both components to about the same extent, and so had little effect on the rapidity of the beats. As the load increased further, the effect approached that described in the preceding paragraph, so that the curve rose and tended to straighten out.

CONCLUSION

A study of the shifts of nodal patterns and the rapidity of beating, when increasing loads were applied at each of the sixteen holes around the bell,

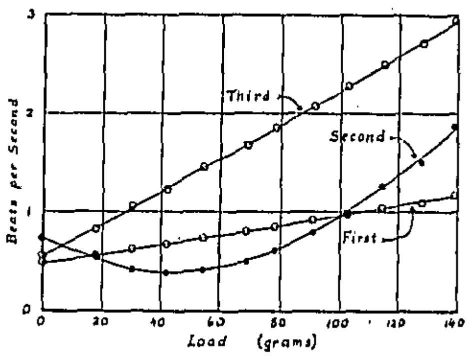

Fig. 2. Rapidity of beats in the first three partial tones of a bell. The curves show how the rapidity of the beats changes under the action of an increasing load applied at one point on the bell.

showed that the above statements seemed to fit the facts in general. A small load applied to a bell rotates the pattern of nodal and antinodal meridians in such a sense as to bring closer to the load an antinodal meridian of the lower pitched component. The addition of the load increases the rapidity of beats when the load is closer to an antinodal than to a nodal meridian of the lower of the beating components, and in the opposite case it decreases the rapidity of the beats.

The bell used in this study was kindly lent by the Meneely Bell Company, and I wish to express my appreciation of their courtesy, and my hearty thanks.

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A Proposed Loading of Piano Strings for Improved Tone

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An ideally stiff string has overtones ν_n which are sharper than multiples of the fundamental, the inharmonicity being proportional to (n^2-1). This well known theoretical result has been verified by Schuck and Young [J. Acous. Soc. Am. 15, 1, (1943)] for typical strings. It is proposed to improve the tone of a piano string by attaching a small mass, thus lowering the frequency of each normal mode except those for which the mass is at a node. It turns out that for an ideally stiff string, approximate correction of a large number of overtones can be obtained with a single mass suitably located. In the limit of a large mass near the end of the string, the correction is exact for all overtones. A mass of the order of 0.1 g placed a few cm from the end of a typical string adjusts the first eight overtones to within a few hundredths of a semitone, a negligible inharmonicity. Improved tone is expected since the subjective fundamentals derived from difference tones between adjacent partials will show greatly reduced dispersion. The effect of the loading upon tuning would reduced the observed stretching of the octaves to a negligible amount. Deviations from ideal stiffness and the effect of adding two masses are also considered.

INTRODUCTION

THE modal frequencies of a perfectly flexible string are integral multiples of the fundamental frequency, but each partial, including the fundamental, is raised in frequency if stiffness is not negligible. Seebeck showed that

$$\nu_n = n\nu_0(1+\alpha+\alpha^2+\tfrac{1}{8}\pi^2\alpha^2 n^2+\cdots), \qquad (1)$$

where ν_n is the frequency of the nth partial, $\nu_0 = \frac{1}{2}(T/ML)^{\frac{1}{2}}$ is the fundamental frequency calculated from the total mass M, length L, and tension T of the string, and α is a constant depending upon the string.[1-3] For a circular string of radius a and specific gravity ρ and Young's Modulus Q, $\alpha = (a/2\nu_0 L^2)(Q/\rho)^{\frac{1}{2}}$, which is a small quantity for the strings of a piano. For convenience, let $\beta = \frac{1}{2}\pi^2\alpha^2(1+\alpha+\alpha^2)^{-1}$ and $\nu_0' = \nu_0(1+\alpha+\alpha^2)$; then

$$\nu_n = n\nu_0'(1+\beta n^2). \qquad (2)$$

We will call a string for which stiffness is the only perturbation an "ideally stiff" string. The higher terms in n^4, etc., which were omitted from Eq. (1) are negligibly small for actual piano strings.

Experimental investigations of the modal frequencies of strings have been made by several authors,[4-7] all of whom found the partials to be definitely inharmonic. The most comprehensive work has been that of Schuck and Young, who showed that for many strings a remarkably accurate square-law inharmonicity does indeed exist; β was found to be about 0.000139 for a typical F_3 string (the F below middle C). The very lowest two octaves showed a hybrid behavior. Schuck and Young found that for any given piano, the inharmonicity was lowest for strings in the low middle register. In comparing pianos they found the inharmonicity to be less for pianos with longer strings, as is to be expected from the formula for α. They showed quantitatively how the measured inharmonicities cause the "stretching of the octaves" which is commonly found in the tuning of pianos. They attributed the mellow tone of longer strings to the fact that the smaller inharmonicities of such strings cause less dispersion among the frequencies of the subjective fundamentals derived as difference tones between adjacent partials.

It is apparent that it would be desirable to reduce the inharmonicities of the partials of the vibrating piano string. It is well known that a small mass attached to the string will lower each modal frequency except those for which the mass is at a node; in this paper the effects of various loadings upon the inharmonicities are considered. It turns out that approximate correction of many partials can be obtained by a single small mass suitably located.

EFFECT OF VARIOUS LOADINGS

Case 1. Continuous loading

We shall apply the usual first-order perturbation theory[8] and assume throughout that the mass perturbation and the stiffness perturbation are independent. If the linear density is $\epsilon_0[1+b(x)]$, where $b(x)$ is small, then

$$\nu_n = n\nu_0'\left[1+\beta n^2-(1/L)\int_0^L b(x)\sin^2(\pi nx/L)dx\right]. \qquad (3)$$

[1] A. Seebeck, Abh. d. Math. Phys. Cl. d. K. Sächs. Gesellschaft d. Wiss. Leipzig, 1852.
[2] Lord Rayleigh, *Theory of Sound* (Macmillan, New York, 1894), second edition, Vol. I, p. 301.
[3] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., 1948), second edition, p. 170.
[4] R. S. Shankland and J. W. Coltman, J. Acous. Soc. Am. 10, 161–166 (1939).
[5] A. W. Nolle and C. P. Boner, J. Acous. Soc. Am. 13, 145–148 (1941).
[6] O. H. Schuck and R. W. Young, J. Acous. Soc. Am. 15, 1–11 (1943).
[7] R. Jouty and Y. Rocard, Rev. Sci. Paris 84, 283–285 (1946).

[8] Reference 2, p. 118; reference 3, p. 124.

The location of each node is in general shifted by the mass perturbation, but this fact does not affect the derivation of Eq. (3). As was shown by Rayleigh,[9] a sinusoidal density perturbation will affect only one of the partials; in fact, if $b(x)$ is expanded into a Fourier cosine series, the successive coefficients in the expansion determine the correction to the frequencies of the successive partials. In practice, a perturbation such as

$$b(x) = -4\beta\sum_n n^2 e^{-kn}\cos(2\pi n x/L) \qquad (4)$$

could be used to correct the first ten or twenty partials, the factor e^{-kn} being adjusted to damp out the higher terms of the series for $b(x)$. This method of compensation, although theoretically attractive because it could be applied to a non-ideal string, would be prohibitively complicated to carry out experimentally.

Case 2. Single mass, two partials harmonized

Passing now to the case of discrete masses m_i placed at distances x_i from one end of the string, Eq. (3) becomes

$$\nu_n = n\nu_0'(1+\beta n^2 - \beta\sum_i \mu_i \sin^2 n\theta_i), \qquad (5)$$

where we define $\mu_i = m_i/M\beta$ and $\theta_i = \pi x_i/L$. Thus, for a single mass,

$$\nu_1 = \nu_0'(1+\beta-\beta\mu\sin^2\theta), \qquad (6)$$

and

$$\nu_n = n\nu_0'(1+\beta n^2-\beta\mu\sin^2 n\theta). \qquad (7)$$

The inharmonicity D_n of the nth partial is defined as $D_n = 1200\log_2(\nu_n/n\nu_1)$, whence

$$D_n = 1731(\nu_n - n\nu_1)/n\nu_1, \qquad (8)$$

if ν_n is almost equal to $n\nu_1$. The unit for D_n is the logarithmic cent, or 0.01 of a semitone (about a 0.06 percent change in frequency). Inharmonicities of a few cents are detectable; for example, the tempered fifth is about 2 cents flatter than a true fifth, and this inharmonicity is perceived by piano tuners. Since β and $\mu\beta$ are small quantities for actual strings, Eqs. (6) and (7) then yield

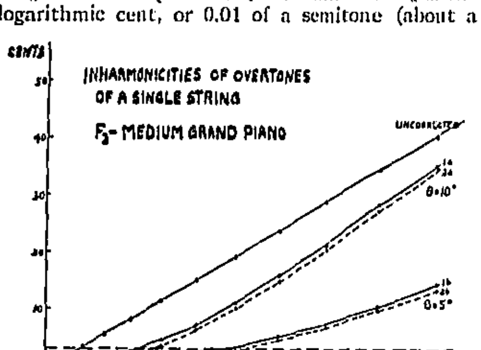

FIG. 1. Effect of various loadings upon the overtones of an ideally stiff F_3 string. Curves 1a and 1b are single-mass corrections; curves 2a and 2b are two-mass corrections.

[9] Reference 2, p. 215.

TABLE I. Correction of overtones of an F_3 string for which $\beta = 0.000139$, for various loadings. The table shows the residual inharmonicity D_n in logarithmic cents for each partial whose nominal frequency is $n\nu_1$. The corrections 1a, 1b, 2a, 2b are described in the text.

		Single mass		Two masses	
n	uncorr.	corr. 1a	corr. 1b	corr. 2a	corr. 2b
1	0	0	0	0	0
2	0.7	0	0	0	0
3	1.9	0.1	0.0	0	0
4	3.6	0.5	0.1	0	0
5	5.8	1.1	0.3	0.7	0.2
6	8.4	2.5	0.7	1.8	0.4
7	11.5	4.5	1.3	3.5	1.0
8	15.1	7.3	2.2	6.5	1.8
9	19.2	11.1	3.5	10.2	3.0
10	23.8	16.0	5.3	15.1	4.8
11	28.8	21.7	7.7	20.9	7.0
12	34.3	28.4	10.5	27.7	10.0
13	40.3	35.6	14.4	35.4	13.7

$$D_n = 1731\beta[(n^2-1)-\mu(\sin^2 n\theta - \sin^2\theta)]. \qquad (9)$$

Setting $D_n = 0$, we arrive at the condition

$$n^2 - 1 = \mu\sin(n-1)\theta\sin(n+1)\theta. \qquad (10)$$

Since there are two adjustable constants, μ and θ, two equations of the type (10) may be set up, and two upper partials may be harmonized (i.e., adjusted for $D_n = 0$). For example, let us adjust ν_2 and ν_3. Equation (10) yields

$$3 = \mu\sin\theta\sin 3\theta, \qquad (11a)$$

and

$$8 = \mu\sin 2\theta\sin 4\theta. \qquad (11b)$$

These equations may be solved by algebraic means, giving $\mu = -54/5$ and $\theta = \sin^{-1}(5/6)^{\frac{1}{2}} = 65.9°$. This single-mass loading suffers from two defects. First, μ is negative, and controlled removal of mass would offer technical difficulties and might weaken the string. Second, while it is true that it makes $\nu_2 = 2\nu_1$ and $\nu_3 = 3\nu_1$, calculation shows that the remaining partials are affected irregularly. Some are lowered, some are raised, but all are about as inharmonic as before. This is a consequence of the fact that the negative loading at $\theta = 65.9°$ is far from the end of the string, $\theta = 90°$ corresponding to the center of the string.

For a low tone of a piano, the fundamental is often weak or missing. A tuner then adjust octaves by relying upon beats between ν_2 and ν_4. For such a string it would be advantageous to harmonize ν_1, ν_2, and ν_4. Unfortunately, the appropriate equations analogous to Eqs. (11) have no real solution for θ.

TABLE II. Inharmonicities of subjective fundamentals derived from neighboring partials, calculated for the same string as Table I.

		Single mass		Two masses	
n	uncorr.	corr. 1a	corr. 1b	corr. 2a	corr. 2b
1	1.4	0	0	0	0
2	4.3	0.3	0.0	0	0
3	8.7	1.6	0.4	0	0
4	14.4	3.8	1.1	3.6	0.9
5	21.6	9.4	2.5	7.2	1.8
6	30.2	16.0	4.9	13.7	4.1
7	40.3	27.4	8.6	27.5	7.9
8	51.8	41.8	13.2	39.4	12.6
9	64.7	59.0	21.5	59.8	20.5
10	79.2	79.4	32	78.7	30
11	95.0	114	42	102	42
12	112.2	123	61	126	58

Case 3. Single mass, one partial harmonized

By giving up the requirement of Case 2 that $\nu_3=3\nu_1$ and retaining only $\nu_2=2\nu_1$, we may arbitrarily fix either μ or θ so that $D_2=0$. It follows from Eq. (11a) that the smallest positive mass which will do the job is $\mu=16/3$ placed at $\theta=\sin^{-1}(3/8)^{\frac{1}{2}}=37.8°$. However, by placing a larger mass closer to the end of the string, the fluctuations in inharmonicities can be largely removed since the mass will be near a node only for relatively high partials. All of the lower partials will be improved, in a somewhat regular fashion, and the 2nd partial will be exact. Typical results are given in Table I and plotted in Fig. 1. Two single-mass corrections are calculated for the medium grand piano F_2 string of Schuck and Young. This is an ideally stiff string, with $K(=1731\beta)$ having a value of 0.24 as determined from the slope of the D_n *vs.* n^2 curve in Fig. 8 of reference 6. The residual inharmonicities remaining after correction are computed from Eq. (9), using the experimental value of K, and values of μ and θ consistent with Eq. (11a). If we arbitrarily assume that an inharmonicity of ± 3 cents is tolerable, Table I shows that only the first 3 partials of the uncorrected string are within the 3 cent limit. It is seen that correction 1a ($\mu=35.55$ at $\theta=10°$) renders the first 6 partials tolerable, while correction 1b ($\mu=132.99$ at $\theta=5°$) similarly adjusts the first 8 partials. In the limit, a large mass placed very near the end of the string gives exact correction for all partials (until the approximation of Eq. (1) breaks down). This is apparent from the fact that for small θ, Eq. (10) reduces to $\mu\theta^2=1$, and this condition does not involve the mode number n.

Case 4. Two masses, 3 partials harmonized

By loading the string at two points with masses μ_1 and μ_2 placed at θ_1 and θ_2, there are four adjustable constants, and hence in theory, at least, we can adjust four of the upper partials. To adjust ν_2, ν_3, ν_4, and ν_5, we must find a real solution for the equations

$$3=\mu_1 \sin\theta_1 \sin 3\theta_1+\mu_2 \sin\theta_2 \sin 3\theta_2, \quad (12a)$$

$$8=\mu_1 \sin 2\theta_1 \sin 4\theta_1+\mu_2 \sin 2\theta_2 \sin 4\theta_2, \quad (12b)$$

$$15=\mu_1 \sin 3\theta_1 \sin 5\theta_1+\mu_2 \sin 3\theta_2 \sin 5\theta_2, \quad (12c)$$

and

$$24=\mu_1 \sin 4\theta_1 \sin 6\theta_1+\mu_2 \sin 4\theta_2 \sin 6\theta_2. \quad (12d)$$

For constructional reasons, we will limit discussion to positive loadings. A careful study of Eqs. (12) shows that there is no real solution for θ_1 and θ_2 with positive masses, nor is there a real solution if Eq. (12d) is replaced by the corresponding equation for adjustment of ν_6, ν_7, or ν_8. Therefore we shall consider only the first three of Eqs. (12), treating θ_2 as an adjustable parameter. It turns out that θ_1 is real only for $0\leqslant\theta_2\leqslant 52.2°$ and $69.2°\leqslant\theta_2\leqslant 90°$ and for corresponding intervals in the other half of the string. For both μ_1 and μ_2 to be positive and finite, θ_1 is further restricted, so that one mass must be placed in the region $0<\theta<36°$ and the other mass in the region $69.2<\theta<72°$. The inharmonicities for two choices of θ_2 are listed in Table I, and plotted in Fig. 1. For each of these loadings $D_2=D_3=D_4=0$. Curve 2a is for $\mu_1=0.857$ at $\theta_1=69.43°$ and $\mu_2=38.93$ at $\theta_2=10°$. This may be compared with the single-mass correction 1a. Likewise, placing a larger mass at 5°, we have curve 2b for which $\mu_1=0.203$ at 69.33°, $\mu_2=136.92$ at 5°.

In general, each two-mass correction involves a mass near the end of the string of the same order as that for the corresponding single-mass correction, and, in addition, a much smaller mass near the 69° position. Because of its position, the small mass introduces some irregularities in the inharmonicities, but these are small. The two-mass correction does not materially improve the 5th and higher partials,

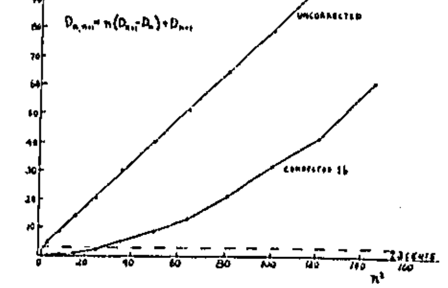

FIG. 2. Effect of a single-mass loading upon the derived fundamentals of the string of Fig. 1.

although it does of course render the first 4 partials exactly harmonic. It is doubtful whether a two-mass correction would justify the technical complexities introduced.

THE DERIVED FUNDAMENTALS

As pointed out by Schuck and Young, the derived fundamentals may play an important role in the mellowness of the tone quality of a string. It would be desirable to reduce the spread among the frequencies of the subjective difference tones arising from adjacent partials. Let $D_{n,n+1}$ be the inharmonicity of $(\nu_{n+1}-\nu_n)$ with respect to ν_1. Since, from Eq. (8), $\nu_n=n\nu_1(1+D_n/1731)$, we may compute the actual difference frequency $(\nu_{n+1}-\nu_n)$ and thence find $D_{n,n+1}$. The result is

$$D_{n,n+1}=n(D_{n+1}-D_n)+D_{n+1}. \quad (13)$$

For an ideally stiff string for which $D_n=K(n^2-1)$, we obtain

$$D_{n,n+1}=3Kn(n+1). \quad (14)$$

For any chosen value of n, the inharmonicity of the derived fundamental is proportional to the factor K for that string. Therefore, as proposed by Schuck and Young, the slope K of the D_n *vs.* n^2 plot should be intimately related to the "mellowness" of the string. Incidentally, Eq. (14) shows that for an ideally stiff string $D_{n,n+1}$ is almost a straight line when plotted against n^2.

The inharmonicities of the derived fundamentals have been calculated for the uncorrected F_3 string of reference 6, and are surprisingly large, being of the order of 3 times the inharmonicities of the partials. Table II shows that none of the derived fundamentals is exactly harmonic for the uncorrected string, and only the first one is within 3 cents of the fundamental. The single-mass correction 1*b*, which is plotted in Fig. 2, exactly adjusts the first derived fundamental, and reduces the first 5 inharmonicities to less than 3 cents. The improvement

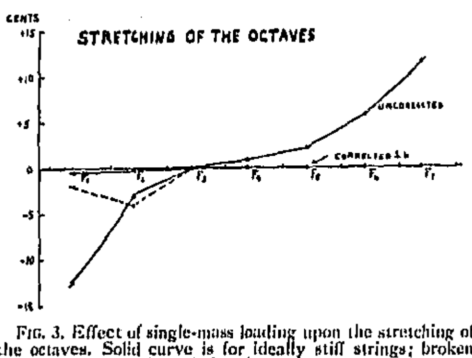

FIG. 3. Effect of single-mass loading upon the stretching of the octaves. Solid curve is for ideally stiff strings; broken curve is for actual F_1 and F_3 strings.

TABLE III. Calculated tuning for a medium grand piano, following the usual tuning procedure. All strings assumed ideally stiff with various constants K as listed. Inharmonicities of the fundamentals are given, expressed in logarithmic cents relative to F_3.

Note	Frequency c.p.s.	K	uncorr.	Single mass corr. 1*a*	Single mass corr. 1*b*
F_1	43.7	0.69	−12.5	−1.8	−0.3
F_2	87.3	0.24	− 2.9	−0.5	−0.1
F_3	175	0.24	0	0	0
F_4	349	0.48	+ 0.7	0	0
F_5	698	1.25	+ 2.2	0	0
F_6	1396	1.9	+ 5.9	0	0
F_7	2792		+11.6	0	0

is greatest for the fundamentals derived from the lower partials, and it is just these partials which, because of their intensity, would produce the strongest subjective tones.

The two-mass correction adjusts the first 3 derived fundamentals exactly, but Table II shows that the over-all result would not be noticeably better than the simpler single-mass correction.

THE EFFECT UPON TUNING

Let us assume that a tuner starts with F_3 and adjusts F_4, F_5, F_6, and F_7 by beating fundamentals against 2nd partials. Let us also assume that the tuning of F_2 and F_1 proceeds by the beating of 2nd partials against 4th partials. As shown by Schuck and Young, the inharmonicities of the first few partials causes stretching of the octaves which is in quantitative agreement with the observations of Railsback[10] upon many pianos if the above tuning scheme is followed. The sharpness of the upper octaves presents a very real difficulty to the tuner, and is a source of distress to the sensitive performer or listener. Table III and Fig. 3 show the calculated effect of the single-mass correction 1*b* upon the tuning of the medium grand piano of reference 6, assuming all strings to be ideally stiff. The stretching of the upper octaves is eliminated identically, and that of the lower octaves reduced to a negligible amount. It is obvious that the two-mass correction would eliminate stretching of all octaves, but again the single-mass correction seems entirely adequate.

STRINGS THAT ARE NOT IDEALLY STIFF

The foregoing analysis applies only to strings that are ideally stiff, or, in general, to strings for which $D_n=K(n^2-1)$ for whatever reason. However, a string that is not ideally stiff can usually be improved by a suitable single-mass correction. Consider the F_1 string (low F) of the medium grand piano of reference 6. The D_n *vs.* n^2 curve consists of two linear segments of slope $K_1=0.69$ and $K_2=0.32$. A single-mass correction at $\theta=5°$ has

[10] O. L. Railsback, J. Acous. Soc. Am. 9, 274 (1938); 10, 86 (1938).

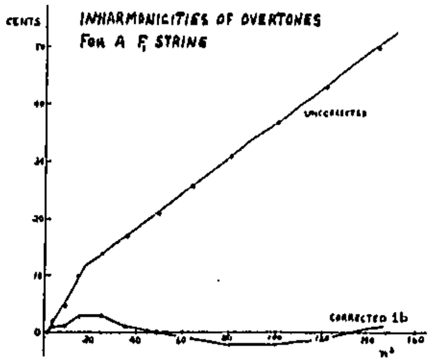

FIG. 4. Effect of a compromise single-mass loading upon the overtones of an actual F_1 string.

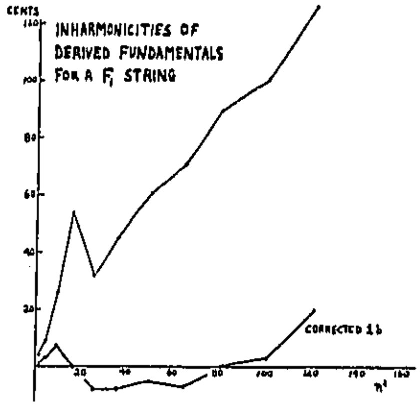

FIG. 5. Effect of a compromise single-mass loading upon the derived fundamentals of an actual F_1 string.

been computed, using a β based upon a compromise $K=0.50$; the results of applying this correction to the observed (non-ideal) inharmonicities are shown in Figs. 4 and 5. It is seen that the inharmonicities both of the partials and the derived fundamentals have been considerably improved, at least up to $n=12$.

A tuning curve has been calculated based upon actual F_1 and F_2 strings and is shown as the broken curve in Fig. 3. For the F_2 string a compromise $K=0.40$ was used. While not as good a correction as if the strings had been ideally stiff, nevertheless an improvement is noted. It would, of course, be possible to adjust the tuning curve at the expense of the derived fundamentals. It is probable that the improvement of tone is more desirable for the lower piano strings than is elimination of octave stretching, although this is a matter of conjecture. For the middle low strings and above, this dilemma need not be faced, since experiment has shown such strings to be ideally stiff.

CONCLUSION

The calculations described in this paper indicate several ways in which a single-mass loading might be expected to improve the tuning of a piano and the tone of the individual strings. The practical details of the realization of such a loading remain to be worked out. A small, controlled loading might be applied by electrolysis, but it would probably be desirable to use a movable mass to allow for adjustment as the string stretches in the initial tuning. The problem of firm attachment of the mass to the string would have to be solved. In production, a relatively few stock masses could be used, the position θ for each string being adjusted according to Eq. (11a). It might be possible to design an inharmonicity meter for routine factory adjustment as the piano is strung. As a concrete example, the correction 1b for a string of length 120 cm and mass 7.2 g would amount to only 0.133 g placed 3.33 cm from one end of the string. The load could be subdivided between the equivalent points near each end of the string. Because of its great density, gold would be a most suitable material for the added mass.

The final criterion of the desirability of loadings such as described in this paper would, of course, lie in listening tests with actual pianos.

Generalized Solutions of Webster's Horn Theory*

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Webster's equation for the approximate formulation of the propagation of sound waves in horns is solved using two methods of approach. The first method considers a transmission line with variable parameters as the electrical analogue of the horn. This approach is specially useful in yielding generalized solutions for horns of finite length. The second method, based on an investigation of the singularities of Webster's differential equation, leads to the discovery of a great number of new families of horns.

1. INTRODUCTION

A RIGOROUS solution for the propagation of sound waves down horns of arbitrary shapes is still outside the realm of the existing methods of mathematical physics. To reduce the complexity of the mathematics, a number of assumptions are introduced. Some of these, however, are in some instances in contradiction to physical concepts. The resulting "simplified" solutions[1] which have been developed are thus the mathematical description of an idealized case. All these solutions formulate the propagation as a one dimensional problem and in lieu of the wave equation the Webster equation, incorporating the data of the problem, is found.

The previous assumptions are plausible when the horn does not flare too quickly and when the curvature of the wave front is small. The range of validity of the solutions is thus restricted to low frequencies. Consideration of curved wave fronts, although more in accordance with physical reality, leads to considerable complication in the mathematics. The additional labor required to solve the curved wave front problem is not justified, since the Webster equation is already an approximate formulation to the phenomenon of propagation. Consequently, the wave front is assumed to be plane.

The preceding discussion would lead one to believe that Webster's equation is a crude approximation valid only in a very restricted number of cases, but otherwise giving results of doubtful value. To a certain extent this is true. The justification of its use is that, for want of a better solution and from a practical point of view, the plane wave theory yields a design basis of comparison—at low frequencies—for horns of different shapes. The high frequency transmission characteristics are not so important, since the behavior of all horns at high frequencies is very nearly the same.

* This research has been aided by funds made available under a contract with the ONR.

[1] A. G. Webster, Proc. Nat. Acad. Sci. 5, 275 (1919). C. R. Hanna and J. Slepian, AIEE 43, 393 (1924). G. W. Stewart and R. B. Lindsay, *Acoustics* (D. Van Nostrand Company, Inc., New York, 1930), p. 332.

2. WEBSTER'S EQUATION, ITS SOLUTION

A search in the literature[2] reveals the fact that the horn contours which have been studied are very few in number. This is due to the difficulty of solving Webster's equation exactly when the horn contours are of arbitrary shape. Salmon[3] has made use of numerical methods of integration when the solution in closed form is unfamiliar. But his method is not suitable for solving horns of finite length. The purpose of the present paper is to obviate these difficulties and to develop generalized methods of solution. Two lines of approach have been followed to achieve this aim. The first method, based on the analogy between electrical and acoustical systems, is very effective in dealing with horns of finite length. Its importance as a generalized method of study, however, is secondary. The second approach has a wider scope of generality and has the great merit of discovering new families of horns.

3. THE ELECTRICAL ANALOGUE

The idealized horn described by Webster's approximations has for analogue an electric transmission line with variable parameters. The validity of the preceding statement can be argued on physical reasoning. The transmission line is visualized as the limiting case of infinitesimal lumped inductances and capacitances connected in a recurrent pattern. Similarly in the horn, by virtue of the plane wave theory assumptions, each infinitesimal slice of air is allowed to vibrate only in a direction parallel to the axis of the horn. Both infinitesimal systems having one degree of freedom and both behaving alike in a linear manner, the analogy is established.

The original problem of the solution of the characteristics of a horn by Webster's approximate solution is reduced to a discussion of the properties of special classes of electric transmission lines.

(i) The Infinite Line

Let the line be first thought as built from a large number of quadripoles in cascade numbered

[2] (a) S. Ballantine, J. Franklin Inst. 203, 85 (1927). (b) V. Salmon, J. Acoust. Soc. Am. 17, 212 (1946).
[3] See reference 2(b), p. 199.

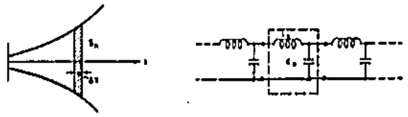

FIG. 1. Horn and its electrical analogue.

1, 2, ⋯, n, ⋯. For any one quadripole there exists a set of relations between the input and output voltages and currents. These relations are of the form:

$$V_{n-1}=a_{11,n}V_n+a_{12,n}I_n$$
$$I_{n-1}=a_{21,n}V_n+a_{22,n}I_n \qquad (3.1)$$

or in the notation of matrix algebra:

$$\begin{pmatrix} V_{n-1} \\ I_{n-1} \end{pmatrix}=(a_{ij,n})\begin{pmatrix} V_n \\ I_n \end{pmatrix}. \qquad (3.2)$$

Since the lines considered are to be physically realizable and are to be constructed from linear passive elements, each quadripole n can be represented by a T network having for branch impedances $(Z_n^{11}-Z_n^{12})$, Z_n^{12}, $(Z_n^{22}-Z_n^{12})$. These impedances are related to the elements of the characteristic matrix $(a_{ij,n})$ in the following manner:[4]

$$\begin{aligned} a_{11,n}&=Z_n^{11}/Z_n^{12},\\ a_{12,n}&=[(Z_n^{12}+Z_n^{22})/Z_n^{12}]-Z_n^{12},\\ a_{21,n}&=1/Z_n^{12},\\ a_{22,n}&=Z_n^{22}/Z_n^{12}. \end{aligned} \qquad (3.3)$$

The branch impedances depending on the geometry of the horn (the terms horn and line are freely interchanged in this discussion), the coefficients $a_{ij,n}$ are uniquely defined for a specific horn shape. Since any one quadripole n represents the behavior of an infinitesimal section of horn, comparison of the two analogous systems of Fig. 1 leads to:

$$L_n=\frac{\rho\Delta x}{S_n}; \quad C_n=\frac{S_n\Delta x}{\rho c^2}, \qquad (3.4)$$

S_n being the cross-sectional area of the "element" n. Δx is the width of the infinitesimal section of the horn, ρ is the density of the air, and c is the velocity of propagation of sound in free space. By means of relations (3.3) the coefficients $a_{ij,n}$ are identified as:

$$\begin{aligned} a_{11,n}&=1-\omega^2L_nC_n=1-(\omega\Delta x)^2/c^2\\ a_{12,n}&=j\omega L_n=j\omega\rho\Delta x/S_n\\ a_{21,n}&=j\omega C_n=j\omega S_n\Delta x/\rho c^2\\ a_{22,n}&=1. \end{aligned} \qquad (3.5)$$

A functional relation for the input impedance at the throat of the horn can now be determined from the fundamental pair of relations (3.1). Thus, dividing the first by the second of these relations, it is found that,

$$Z_{n-1}=\frac{a_{11,n}Z_n+a_{12,n}}{a_{21,n}Z_n+a_{22,n}} \qquad (3.6)$$

where $V_n/I_n=Z_n$ and $V_{n-1}/I_{n-1}=Z_{n-1}$. As Δx is made to approach zero, all terms in (3.6) become functions of the continuous independent variable x. The above equation then reduces to:

$$Z-\frac{\partial Z}{\partial x}=\frac{a_{11}(x)Z+a_{12}(x)}{a_{21}(x)Z+a_{22}(x)}. \qquad (3.7)$$

Substituting the values of the coefficients $a_{ij}(x)$ from (3.5) and neglecting infinitesimals of higher order, the previous relation may be put in the form:

$$\frac{\partial Z}{\partial x}-\frac{j\omega}{\rho c^2}SZ^2+\frac{j\omega\rho}{S}=0. \qquad (3.8)$$

The latter expression is a generalized Ricatti equation. This relation must be equivalent to Webster's equation. The equivalence is necessary to confirm the preceding physical argument establishing the analogy between the line and the horn. By means of the two successive transformations $Z=1/Y$ and $Y=-(S\phi'/j\omega\rho\phi)$ (Y defines the input admittance) it is readily found that Eq. (3.8) reduces first to:

$$\frac{\partial Y}{\partial x}-\frac{j\omega\rho}{S}Y^2+\frac{j\omega S}{\rho c^2}=0 \qquad (3.9)$$

then to the Webster equation:

$$\frac{d^2\phi}{dx^2}+\left(\frac{S'}{S}\right)\frac{d\phi}{dx}+\left(\frac{\omega}{c}\right)^2\phi=0. \qquad (3.10)$$

The dependent variable ϕ is identified with the velocity potential.

The Ricatti-impedance equation is not easier to solve than Webster's original equation. As a matter of fact, the discussion of the former equation is usually performed on its transform, the second order linear differential equation.[5] One of the aims of this study being the investigation of possible methods of solution, a brief discussion of the impedance equation will be made.

The use of the admittance Eq. (3.9) is sometimes more convenient. Writing $y=Y\rho c$, this same equation becomes:

$$S\frac{dy}{dx}-\frac{j\omega}{c}y^2+\frac{j\omega}{c}S^2=0. \qquad (3.9')$$

[4] E. Guillemin, *Communication Networks* (John Wiley and Sons, Inc., New York, 1935), Vol. II, p. 140ff.

[5] E. L. Ince, *Ordinary differential equations* (Dover Publications, New York, 1944), p. 295.

Solving the previous expression (3.9′) as a quadratic equation in S, it is found:

$$S=y\left[-\frac{jc}{2\omega}\frac{\partial y}{\partial x}\cdot\frac{1}{y}\pm\left\langle 1-\left(\frac{c}{2\omega}\frac{1}{y}\frac{\partial y}{\partial x}\right)^2\right\rangle^{\frac{1}{2}}\right]. \quad (3.11)$$

The expression S is reduced to a more condensed form when $-(jc/2\omega)(1/y)(\partial y/\partial x)$ is substituted by $\sinh\theta$; Eq. (3.11) is then

$$S(x)=ye^{j\theta}. \quad (3.12)$$

Since $S(x)$ is not a function of the frequency $\omega/2\pi$, then:

$$\frac{\partial S}{\partial\omega}=0=\frac{\partial y}{\partial\omega}+y\frac{\partial\theta}{\partial\omega}. \quad (3.13)$$

The relation (3.13) can be considered as the generalized equation for the propagation of sound waves in horns. This relation has for intermediate integral the Ricatti-impedance (or admittance) equation. A direct use of Eq. (3.13) is to determine all families of horns having admittances of the form:

$$y(x,\omega)=\Theta(x)\cdot\Omega(\omega). \quad (3.14)$$

It is expected that Eq. (3.13) should become tractable for the above case. When (3.14) is substituted in (3.13), then using the method of separation of variables, the equations for Θ and Ω are:

$$\frac{\partial\Theta}{\partial x}-\Theta k_0=0 \quad (3.15)$$

and

$$\left(\frac{\partial\Omega}{\partial\omega}\right)^2\left(1-\frac{c^2}{4\omega^2}k_0^2\right)=-k_0^2\frac{c^2\Omega^2}{4\omega^4} \quad (3.16)$$

where k_0 is an arbitrary constant. The second of these relations integrates to:

$$\log\Omega\cdot A=j\cos^{-1}\left(\frac{ck_0}{2\omega}\right), \quad (3.17)$$

A being a new arbitrary constant. A more convenient way of expressing (3.17) is to write:

$$\begin{aligned}\Omega A&=e^{j\cos^{-1}\left(\frac{ck_0}{2\omega}\right)}\\&=\cos\left(\cos^{-1}\left(\frac{ck_0}{2\omega}\right)\right)+j\sin\left(\cos^{-1}\left(\frac{ck_0}{2\omega}\right)\right)\\&=\frac{ck_0}{2\omega}+j\left(1-\left(\frac{ck_0}{2\omega}\right)^2\right)^{\frac{1}{2}}\\&=\frac{\omega_0}{\omega}+j\left(1-\left(\frac{\omega_0}{\omega}\right)^2\right)^{\frac{1}{2}}\end{aligned} \quad (3.18)$$

$ck_0/2$ having been substituted by ω_0. The arbitrary constant A can be evaluated from the condition imposed on horns to be purely resistive at higher frequencies, i.e., for ω tending to infinity Ω approaches unity. Whence $A=j$, and

$$\Omega=\frac{\omega_0}{j\omega}+\left(1-\left(\frac{\omega_0}{\omega}\right)^2\right)^{\frac{1}{2}}. \quad (3.18')$$

The former Eq. (3.15) integrates to:

$$\Theta=\Theta_0e^{k_0x}, \quad (3.19)$$

and the admittance of horns satisfying 3.14 is:

$$y=\Theta_0e^{k_0x}\left(\frac{\omega_0}{j\omega}+\left(1-\left(\frac{\omega_0}{\omega}\right)^2\right)^{\frac{1}{2}}\right). \quad (3.20)$$

The previous relation (3.19) shows that the exponential horn is the only horn whose frequency characteristics remain unchanged along its length.

Another useful application of the admittance equation is to consider the cases for which the equation is integrable in finite terms. When $S=S(x)$ is of the form x^r, Eq. (3.9′) is:

$$x^r\frac{dy}{dx}-by^2=-bx^{2r};\quad b=\frac{j\omega}{c}. \quad (3.21)$$

The same expression can be rewritten in the form:

$$x^{(1-a)}\frac{dy}{dx}-by^2=-bx^{(2-2a)} \quad (3.22)$$

where $r=1-a$.

It is shown in treatises on differential equations[6] that for the particular case of (3.22), the equation is integrable in finite terms whenever $(1\pm a)/2$ is a positive integer. This sets for the exponent $r=1-a$ the values 2, 4, 6, $\cdots$. The solutions integrable in finite terms are of practical importance since they will give an idea of the rate of variation of the impedance with the flare of the horn without performing elaborate computations.

The solution for a few values of r have been computed and are given below:

$$r=2,\quad S=x^2,\quad y=\left(\frac{1}{bx}+1\right)x^2,$$

$$r=4,\quad S=x^4,\quad y=\frac{x^4\left[\frac{3}{bx}\left(1+\frac{1}{bx}\right)+1\right]}{\left(1+\frac{1}{bx}\right)}. \quad (3.23)$$

[6] A. R. Forsyth, *A treatise on differential equations* (MacMillan Company, Ltd., London, 1914), p. 190.

It is easy to check that the first example corresponding to the conical horn yields for the specific impedance at the throat:

$$Z_{sp\infty}=\frac{(\rho c)}{y}S=\rho c\left(\frac{j\omega x}{c+j\omega x}\right)$$

which is the known answer.

The horn contours having for solutions (3.23) have been specially chosen to show the superiority of the Ricatti equation over Webster's equation in determining the impedance whenever the equation is easily solvable.

The function S occurring in the impedance equation is a continuous function of the independent variable x. Furthermore, for flaring horns S grows monotonically with x. It is easily deduced from Eq. (3.8) that as $S(x)$ grows indefinitely with x, Z tends to zero. As a result, Poincaré's asymptotic series[7] are useful in solving the equation. The procedure, however, is not always easy because of the non-linear character of (3.8). As an illustration of the method, the case of $S=x^2$ is again considered. The impedance equation is rewritten for convenience as:

$$\frac{\partial z}{\partial x}=b\left(Sz^2-\frac{1}{S}\right) \tag{3.8'}$$

with

$$z=\frac{Z}{\rho c};\quad b=\frac{j\omega}{c}$$

z is now substituted by an asymptotic series expansion:

$$z=\frac{a_1}{x}+\frac{a_2}{x^2}+\frac{a_3}{x^3}+\cdots.$$

Comparison of the coefficients of equal powers of x yields for the a_n:

$$a_1=0,\quad a_2=1,\quad a_3=-\frac{1}{b},\quad a_4=\frac{1}{b^2},\quad \cdots,$$

$$a_n=\frac{(-1)^n}{b^{n-2}},\quad \cdots.$$

Hence:

$$z=\frac{1}{x^2}\left(1-\frac{1}{bx}+\frac{1}{(bx)^2}\cdots\right)$$

$$=\frac{1}{x^2}\left[\frac{1}{1+\dfrac{1}{bx}}\right]=\frac{b}{x(1+bx)}$$

yielding the known result.

[7] T. J. I'A. Bromwich, Introduction to the theory of infinite series (MacMillan Company, Ltd., London, 1908), p. 344.

The use of asymptotic series is also effective for $S=x^n$, n any integer. But it becomes very laborious when S is a polynomial expression.

It is seen from the previous discussion that the electrical analogy is not of much use in solving the infinite horn. This has already been pointed out in Section 2. The method, however, is very powerful for the horn of finite length. This will now be considered.

(ii) The Horn of Finite Length

The line is again visualized as a number of quadripoles (a_1), (a_2), $\cdots$, (a_n) in cascade. The characteristic matrix for the finite length of line l is then readily given, as is known in the theory of matrix algebra, by the expression:

$$(a)_l=(a_1)(a_2)\cdots(a_n). \tag{3.24}$$

Neglecting all infinitesimals of higher order, the matrix of an individual quadripole r is:

$$(a_r)=\begin{bmatrix}1 & \dfrac{j\omega\rho\Delta x}{S_r}\\ \dfrac{j\omega S_r\Delta x}{\rho c^2} & 1\end{bmatrix}$$

$$=(1)+\begin{pmatrix}0 & j\omega\rho/S_r\\ j\omega S_r/\rho c^2 & 0\end{pmatrix}\Delta x$$

$$=(1)+(\epsilon_{ij,r})\Delta x, \tag{3.25}$$

where

$$\epsilon_{11,r}=0=\epsilon_{22,r}\quad\text{and}\quad \epsilon_{12,r}=j\omega\rho/S_r,\quad \epsilon_{21,r}=j\omega S_r/\rho c^2.$$

On substituting the value of (a_r) in (3.24), a new expression for $(a)_l$ is found. This is:

$$(a)_l=((1)+(\epsilon_{ij,1})\Delta x)((1)+(\epsilon_{ij,2})\Delta x)\cdots((1)+(\epsilon_{ij,n})\Delta x)$$

$$=(1)+\frac{1}{1}\sum_i(\epsilon_i)\Delta x+\frac{1}{2!}{\sum_{i,j}}'(\epsilon_i)(\epsilon_j)\Delta x^2+\frac{1}{3!}{\sum_{i,j,k}}'(\epsilon_i)(\epsilon_j)(\epsilon_k)\Delta x^3+\cdots. \tag{3.26}$$

In the above expression (ϵ_r) has been written for $(\epsilon_{kl,r})$ and the sign ′ indicates that the $i=j$ term (or $i=j=k=\cdots=n$) has been omitted.

In the limiting case when Δx tends to zero, n grows indefinitely, indicating a transition from a discontinuous to a smooth line. The matrix $(a)_l$

ultimately becomes:

$$(a)_l=(1)+\int_0^l(\epsilon(x))dx+\frac{1}{2!}\times\int_0^l\int_0^x(\epsilon(x))(\epsilon(x'))dx\cdot dx'+\cdots. \quad (3.27)$$

It will be now proved that the infinite series of matrices leads to a converging process.

Let E_{ij} be an upper bound for the modulus of a typical element ϵ_{ij}, then $|\epsilon_{ij}|\leqslant E_{ij}$. Let E be a positive number such that $E_{ij}\leqslant E$ for all elements. Then,

$$\int_0^l(\epsilon(x))dx\leqslant E\cdot l(1).$$

Similarly

$$\int_0^l\int_0^x(\epsilon(x))(\epsilon(x'))dx\cdot dx'\leqslant\int_0^l\epsilon(x)dx\int_0^x\epsilon(x')dx'$$
$$\leqslant\int_0^l E(1)\cdot E\cdot l(1)dl$$
$$\leqslant\frac{E^2l^2}{2!}(1).$$

Hence

$$\Sigma=(1)+\int_0^l(\epsilon(x))dx+\int_0^l\int_0^x(\epsilon(x))(\epsilon(x'))dxdx'+\int\int_0^l\int\cdots\leqslant(1)+\frac{El}{1!}(1)+\frac{E^2l^2(1)}{2!}+\frac{E^3l^3(1)}{3!}+\cdots.$$

Each of the series defining the elements of Σ is less than $S=e^{El}$ which is bounded. The series Σ is then an absolutely converging series of matrices. Since $(a)_l$ is smaller than Σ, then the series defining the individual $a_{ij}(l)$ are also converging series.

The individual elements a_{ij} are thus given by:

$$a_{11}(l)=1+\frac{1}{2!}\int_0^l\int\frac{j\omega\rho}{S(x)}\frac{j\omega S(x')}{\rho c^2}dxdx'+\frac{1}{4!}\int\int_0^l\int\int\cdots,$$

$$a_{12}(l)=\int_0^l\frac{j\omega\rho}{S(x)}dx+\frac{1}{3!}\int\int_0^l\int\frac{j\omega\rho}{S(x)}\times\frac{j\omega S(x')}{\rho c^2}\frac{j\omega\rho}{S(x'')}dxdx'dx''+\cdots,$$

$$a_{21}(l)=\int_0^l\frac{j\omega S(x)}{\rho c^2}dx+\frac{1}{3!}\int\int_0^l\int\frac{j\omega\rho}{S(x)}\times\frac{j\omega S(x')}{\rho c^2}\frac{j\omega S(x'')}{\rho c^2}dxdx'dx''+\cdots,$$

$$a_{22}(l)=1+\frac{1}{2!}\int_0^l\int\frac{j\omega\rho}{S(x')}\frac{j\omega S(x)}{\rho c^2}dxdx'+\frac{1}{4!}\int\int_0^l\int\int\cdots. \quad (3.28)$$

The previous series are very rapidly converging and are consequently useful for numerical computations. There is no restriction on the horn contours which can be investigated as long as they satisfy the original requirements of Webster's approximations.

To determine the input impedance at the throat of the horn, use is made of the pair of relations (3.1) which are rewritten in the more convenient notation:

$$V_0=a_{11}(l)V_l+a_{12}(l)I_l,$$
$$I_0=a_{21}(l)V_l+a_{22}(l)I_l. \quad (3.29)$$

Dividing these two equations and substituting for $V_0/I_0=Z_0$, the input impedance at the throat, and for $V_l/I_l=Z_l$, the load impedance, it is then found,

$$Z_0=\frac{a_{11}(l)Z_l+a_{12}(l)}{a_{21}(l)Z_l+a_{22}(l)}. \quad (3.30)$$

The load impedance Z_l is taken as a Rayleigh piston of area equal to that of the mouth of the horn, or for a slightly better approximation, as a spherical cap having for base the mouth of the horn. Expression (3.30) yields the required result. The formulation of the elements of $(a)_l$ in closed form is sometimes possible to be found without much labor; this has been done in a companion paper.

4. THE SECOND APPROACH: THE SINGULARITIES OF WEBSTER'S DIFFERENTIAL EQUATION

The relative easiness with which a differential equation can be solved depends a great deal on the number and kind of its singularities. The present state of the theory allows the possibility of successfully coping with any second order linear differential equation having three or less singular points. The presence of irregular singularities increases the complexity of the solution to such an extent that very few equations with more than two irregular singularities have been investigated. Very little is known about equations having four or more singularities.

The proposed scheme of study in this section is to

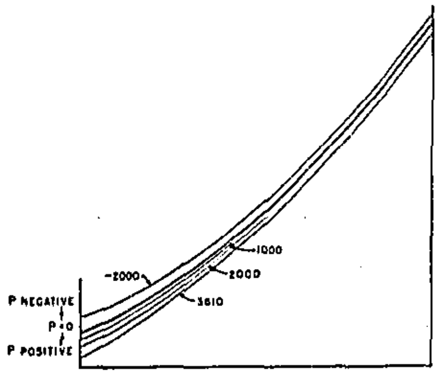

FIG. 2. Horn contours for $m=2$ and $N_0=7.4\times10^{-2}$.

cast the Webster equation in a suitable form reducible to canonical types representative of equations with pre-assigned numbers of singular points. Only standard types yielding known solutions have been considered. The conditions required to reduce the Webster equation to any of the canonical equations will define families of horn contours. It is readily seen that the study is very general, since it deals with families of horns instead of an individual horn contour such as is the case in Webster's equation. The new relations defining the horns are usually simpler to solve than the original Webster equation.

A transform of the Webster equation

$$\Phi''+(S'/S)\Phi'+k^2\Phi=0$$

lending itself to better manipulations is determined by substituting the dependent variable Φ by the product $\psi\cdot\phi$. The above then reduces to

$$\phi''+\phi'\left(\frac{S'}{S}+2\frac{\psi'}{\psi}\right)+\phi\left(\frac{S'}{S}\frac{\psi'}{\psi}+\frac{\psi''}{\psi}+k^2\right)=0. \quad (4.1)$$

The primes refer to differentiation with respect to the complex quantity z, which can take any value on the whole complex plane.

The original Webster equation having an irregular singular point at infinity, the cases considered must also have infinity as an irregular singular point. Any attempt at removing this singular point will lead to the physically meaningless result of the horn contour S depending on the frequency parameter k. The truth of this statement will become evident later in the discussion.

The second method of approach is now illustrated on the standard type of equations with one regular and one irregular singular point. The most general form of this class is:

$$\frac{d^2\phi}{dz^2}+\left(A_0+\frac{A_1}{z}\right)\frac{d\phi}{dz}+\left(B_0+\frac{B_1}{z}+\frac{B_2}{z^2}\right)\phi=0, \quad (4.2)$$

the A's and B's being constants.

An investigation of all possible combinations which make the previous Eq. (4.1) equivalent to (4.2) will lead to the horn contours allowing Webster's equation to reduce to the form (4.2).

Comparison of the coefficients of the derivatives of equal order of (4.1) and (4.2) gives:

$$\frac{S'}{S}+2\frac{\psi'}{\psi}=A_0+\frac{A_1}{z} \quad (4.3)$$

and

$$\frac{S'}{S}\frac{\psi'}{\psi}+\frac{\psi''}{\psi}+k^2=B_0+\frac{B_1}{z}+\frac{B_2}{z^2}. \quad (4.4)$$

The former Eq. (4.3) has for solution

$$\psi^2=\frac{z^{A_1}e^{A_0z}}{S}. \quad (4.5)$$

Substituting the above value of ψ in the left-hand side of Eq. (4.4), then

$$\frac{S'}{S}\frac{\psi'}{\psi}+\frac{\psi''}{\psi}+k^2=-\frac{1}{2}\frac{S''}{S}+\frac{1}{4}\left(\frac{S'}{S}\right)^2+k^2$$
$$+\frac{1}{4}\left(A_0+\frac{A_1}{z}\right)^2-\frac{1}{2}\frac{A_1}{z^2}. \quad (4.6)$$

If this study is restricted to horns made from surfaces of revolution, the S can be replaced by $S=\pi\xi^2$. The function $\xi=\xi(x)$ is then the horn contour and the rotation of $\xi(x)$ about the x axis generates the surface S. Equation (4.4) ultimately becomes:

$$-\frac{\xi''}{\xi}+k^2+\frac{1}{4}A_0^2+\frac{A_1}{2z^2}\left(\frac{A_1}{2}-1\right)$$
$$+\frac{1}{2}\left(\frac{A_0A_1}{z}\right)=B_0+\frac{B_1}{z}+\frac{B_2}{z^2}. \quad (4.7)$$

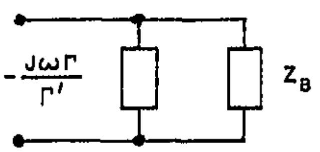

FIG. 3. Equivalent circuit for input impedance of new horns. Z_B is the impedance of the Bessel horn and $(-j\omega\Gamma/\Gamma')$ is the incremental impedance due to the change of flare at the throat.

Consideration of (4.7) shows that the equation in ξ has been reduced to its normal form, i.e., the most general the differential equation for ξ can have. For this reason Eq. (4.7) will be referred to as the generating equation.

Except for B_0, the constants appearing in the generating equation:

$$G(\xi'', \xi, z, A_0, A_1, B_0, B_1, B_2)=0 \qquad (4.8)$$

can be arbitrarily chosen. To every selection there corresponds one *family* of horns. The number of families which can thus be generated is of the order of ∞^4. It is also apparent that B_0 cannot be assigned any value at will since ξ must be independent of k to be physically realizable. Thus there are only a finite number of selections of B_0 which will make $\partial G/\partial k \equiv 0$.

The preceding discussion will be illustrated on a number of examples.

(1) Let the following choice of arbitrary constants be made:

$$B_1=\frac{A_0A_1}{2},$$

$$B_2=\frac{A_1}{2}\left(\frac{A_1}{2}-1\right),$$

$$B_0=k_0^2. \qquad (4.9)$$

The generating equation becomes

$$\xi''-\tfrac{1}{4}A_0^2\xi=0 \qquad (4.10)$$

whose general solution is:

$$\xi=\frac{1}{(\pi)^{\frac{1}{2}}}\left(\cosh\frac{A_0}{2}z+T\sinh\frac{A_0}{2}z\right)$$

when $S(0)=1$. This family of horns has already been discussed by Salmon[8] and will not be considered here. T is the characteristic parameter of the family.

(2) As a second example, let the following choice be made:

$$B_0=k^2,$$

$$B_1=\frac{A_0A_1}{2},$$

$$A_1=2,$$

$$A_0=0. \qquad (4.11)$$

The above will lead to the generating equation:

$$\xi''+\frac{\xi}{z^2}B_2=0. \qquad (4.12)$$

[8] See reference 2(b), p. 199.

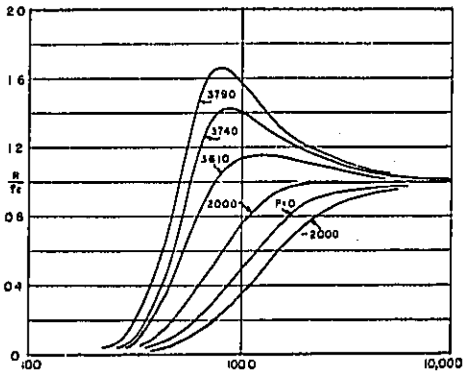

Fig. 4. Specific input impedance for new family with $m=2$ and $N_0=7.4\times10^{-2}$.

It is noticed that $\xi_1=z^m$ is a solution. The exponent m must satisfy the condition:

$$m(m-1)=-B_2,$$

whence

$$m=\tfrac{1}{2}\pm(\tfrac{1}{4}-B_2)^{\frac{1}{2}}. \qquad (4.13)$$

For m to be a real quantity (yielding physically realizable horns) $-B_2+\frac{1}{4}\geqslant 0$. The second solution of (4.12) can now be found from

$$\xi_2=\xi_1\int\frac{dz}{\xi_1^2}$$

$$=z^m\int\frac{dz}{z^{2m}}. \qquad (4.14)$$

The general solution of the generating equation is:

$$\xi=N_0z^{\frac{1}{2}}(1+P\log z) \quad \text{for } m=\tfrac{1}{2}$$

or

$$\xi=N_0\left(z^m+\frac{P}{(1-2m)z^{m-1}}\right) \quad \text{for } m\neq\tfrac{1}{2}. \qquad (4.15)$$

P, one of the two arbitrary constants of (4.15) is the characteristic parameter of the family of horns represented by m; N_0 is the other constant. Horn contours corresponding to different values of P for $m=2$ have been drawn in Fig. 2. P can be positive or negative. The family of contours thus vary about a mean corresponding to the contour with $P=0$.

When the values of the constants defined by (4.11) are substituted in (4.2) and (4.5), these equations become:

$$\left.\begin{aligned}&\frac{d^2\phi}{dz^2}+\frac{2}{z}\frac{d\phi}{dz}+\left(k^2+\frac{B_2}{z^2}\right)\phi=0,\\ \text{and}\quad&\\ &\psi=\frac{z}{(S)^{\frac{1}{2}}}.\end{aligned}\right\}\qquad(4.16)$$

The first of the above expressions is identified as a Bessel equation. Using the conventional procedure of discarding solutions of (4.16) representing converging cylindrical waves,[9] it is found:

$$\phi=\frac{A'}{(z)^{\frac{1}{2}}}(J_p(kz)-jY_p(kz))\qquad(4.17)$$

where A' is an arbitrary constant and $p^2=(-B_2+\frac{1}{4})$.

The general solution of Webster's equation for these horns is:

$$\Phi=\psi\cdot\phi=A'\left(\frac{z}{S}\right)^{\frac{1}{2}}(J_p(kz)-jY_p(kz)).\qquad(4.18)$$

Substituting the value of S as determined from (4.15), the former Eq. (4.18) reduces to:

$$\begin{aligned}\Phi&=\frac{A'}{(\pi)^{\frac{1}{2}}}\frac{z^{-(m-\frac{1}{2})}}{\Gamma}(J_p(kz)-jY_p(kz))\\ &=\frac{A}{\Gamma}\cdot z^{-p}(J_p(kz)-jY_p(kz))\\ &=A(\Gamma^{-1}\cdot\Delta)\end{aligned}\qquad(4.19)$$

where $p=(m-\frac{1}{2})$ by virtue of 4.13, $A=A'/(\pi)^{\frac{1}{2}}$,

$$\begin{aligned}\Gamma&=N_0\left(1+\frac{P}{(1-2m)z^{2m-1}}\right),&&\text{for } m\neq\tfrac{1}{2}\\ &=N_0\left(1+\frac{P}{\log z}\right),&&\text{for } m=\tfrac{1}{2}\end{aligned}$$

and

$$\Delta=z^{-p}(J_p(kz)-jY_p(kz)).$$

The input impedance of the horn is deduced from

$$Z=\frac{j\omega\Phi}{\Phi'}=\frac{1}{\dfrac{\Delta'}{j\omega\Delta}-\dfrac{\Gamma'}{j\omega\Gamma}}.\qquad(4.20)$$

The advantage of rewriting (4.18) in the form (4.19) is now apparent. The above Eq. (4.20) shows that the impedance of the new "m" horns can be considered as the parallel combination of $j\omega\Delta/\Delta'$ and $-j\omega\Gamma/\Gamma'$ (Fig. 3). The former impedance can be identified with the solution for the conventional Bessel horns $S=N_0\xi^m$.

The resistive part of the impedance Z has been numerically evaluated for the horn contours of Fig. 2. The results of the computations are drawn in Fig. 4. It is noticed that horns with positive values of the parameter P show an improved response at the low frequencies.

The preceding detailed discussion has fully illustrated the use of the second method of approach and further examples have been deemed unnecessary. The formal solution of differential equations reducible to the form of (4.2) is known.[10] Both of Eq. (4.2) and the generating equation yield solutions expressible in terms of confluent hypergeometric functions. Special choices of the arbitrary constants, however, can reduce the solutions to simpler functions. The generating equations resulting from differential equations having two essential singularities are harder to solve and will usually involve Mathieu functions.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the constant encouragement and helpful criticism of Professor F. V. Hunt in connection with this investigation.

[9] See reference 2(a).

[10] P. Morse and H. Feshbach, *Methods of theoretical physics* (Technology Press, Cambridge, 1946), p. 82.

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Diffraction of Sound by a Circular Disk*

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The near and distant diffraction fields of a circular disk of zero thickness are plotted according to an exact theory. The results are compared with the Kirchhoff approximation and recent experimental data.

1. INTRODUCTION

BOUWKAMP,[1] Spence,[2] and Storruste and Wergeland[3] have independently announced an exact theory of diffraction of sound by circular disks and apertures, based on the wave functions of the oblate spheroid. Subsequently, Spence[4] published graphs of the near and distant diffraction field for the case of the aperture, and discussed the merits of the classical Kirchhoff solution in the light of the exact values. Thus it was shown that this approximation held good even at wave-lengths greater than the radius a of the aperture, although usually expected to hold only for very short wave-lengths. The present note contains analogous graphs for the exact values of the field diffracted by the *disk* and reaches equivalent conclusions regarding the Kirchhoff theory. Certain simple observations presented here generalize these conclusions to all *plane scatterers.*

Wiener[5] recently performed measurements on the surface of a thin circular metal disk scattering sound waves, in order to check an approximate theory for the field on the surface of diffracting obstacles.[6] Now that an exact solution is finally available, it is easy to check the experimental results against it.

2. PLANE SCATTERERS. THE KIRCHHOFF ASSUMPTIONS

Consider a rigid scatterer lying entirely in a plane, say $z=0$. The total velocity potential may be split into incident and scattered parts

$$\psi=\psi^{(inc)}+\psi^{(s)}. \qquad (1)$$

Now the value of $\psi^{(s)}$ on either side of the plane is determined by

$$\psi^{(s)}=-(1/2\pi)\iint_{z=0}\psi^{(s)}(\partial/\partial n)(e^{ikR}/R)dxdy, \qquad (2)$$

or

$$\psi^{(s)}=(1/2\pi)\iint_{z=0}(\partial\psi^{(s)}/\partial n)(e^{ikR}/R)dxdy, \qquad (3)$$

that is, in terms of its values, or those of its outward normal derivative, on the plane.

The familiar Kirchhoff assumptions here assert that $\psi=\psi^{(inc)}$ in the parts of the plane not occupied by the scatterer, right up to the edge of the scatterer, i.e., $\psi^{(s)}=0$ there; that $\psi^{(s)}=\psi^{(inc)}$ on the illuminated side of the scatterer (perfect reflection) and $\psi^{(s)}=-\psi^{(inc)}$ on the shadow side (total shadow).

For the case of normal incidence let us take

$$\psi^{(inc)}=e^{-ikz},^{**} \qquad (4)$$

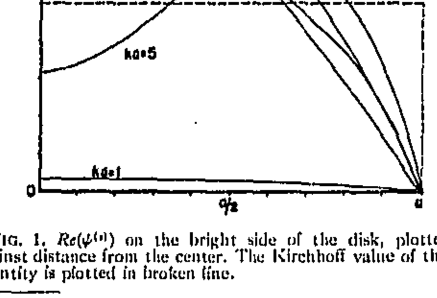

FIG. 1. $Re(\psi^{(s)})$ on the bright side of the disk, plotted against distance from the center. The Kirchhoff value of this quantity is plotted in broken line.

** Time dependence $e^{-i\omega t}$ removed.

* This work is based on a report prepared at New York University under the sponsorship of the Geophysical Research Directorate of the Cambridge Field Station, AMC, U. S. Air Force, under Contract No. AF-19(122)42.

1 C. J. Bouwkamp, Dissertation, Groningen (1941).

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3 Storruste and Wergeland, Phys. Rev. 73, 1937 (1948).

4 R. D. Spence, J. Acous. Soc. Am. 21, 98–100 (1949).

5 F. M. Wiener, J. Acous. Soc. Am. 21, 39 (1949). Additional material in process of publication. Data reproduced by kind permission of author and Bell Telephone Laboratories, Murray Hill, New Jersey.

6 L. J. Sivian and H. T. O'Neil, J. Acous. Soc. Am. 3, 483 (1932); Muller, Black, and Davis, J. Acous. Soc. Am. 10, 6 (1938).

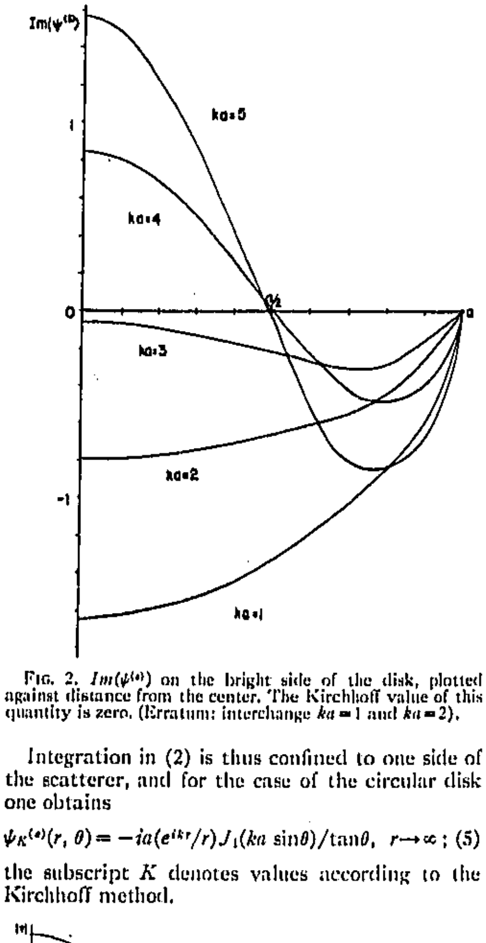

FIG. 2. $Im(\psi^{(s)})$ on the bright side of the disk, plotted against distance from the center. The Kirchhoff value of this quantity is zero. (Erratum: interchange $ka=1$ and $ka=2$).

Integration in (2) is thus confined to one side of the scatterer, and for the case of the circular disk one obtains

$$\psi_K^{(s)}(r,\theta) = -ia(e^{ikr}/r)J_1(ka\sin\theta)/\tan\theta,\quad r\to\infty; \quad (5)$$

the subscript K denotes values according to the Kirchhoff method.

The *exact* boundary condition, however, is

$$(\partial\psi/\partial n)=0 \text{ on the surface of the scatterer.} \quad (6)$$

In general $\psi^{(\text{inc})}$ and its first derivatives are finite and continuous except at the source, which we will assume not to lie on the scatterer, and $\psi^{(s)}$ and its first derivatives are finite and continuous except, perhaps, on the scatterer. As a consequence

$$[\partial\psi^{(s)}/\partial n]=0 \text{ across the plane of the scatterer,} \quad (7)$$

$[\,.\,]$ denoting a discontinuity. Here $\partial/\partial n = \pm\partial/\partial z$ depending on which side of the plane is under consideration. From (3) follows the general result, true also for an infinite screen containing an aperture of any shape:

$$\psi^{(s)} \text{ is odd across the plane of the scatterer.} \quad (8)$$

Thus $\psi^{(s)}=0$ in the plane of, but off the scatterer, exactly as assumed in the Kirchhoff solution.

The error in the approximation therefore lies only in the assumptions on the surface of plane scatterers. In our case the assumed values are $\psi_K^{(s)}=\pm 1$ on the bright and shadow side, respectively. Figures 1 and 2, in which we plot the real and imaginary values of the exact $\psi^{(s)}$ on the circular disk, show that the Kirchhoff assumptions are approached as averages as the ratio of diameter to wave-length increases ($ka=2\pi a/\lambda$); the average of $Re(\psi^{(s)})$ oscillates about unity and that of $Im(\psi^{(s)})$ about zero with decreasing amplitude as a function of the parameter ka.

In Figs. 3 and 4 are plotted the exact and Kirchhoff values of the angular part of $|\psi^{(s)}|$. At $ka>3.81$, zeros occur in $|\psi_K^{(s)}|$ at $\theta<\pi/2$, and near these zeros the curves for $|\psi^{(s)}|$ are noticeably flattened out. When $ka=4$ ($\lambda=1.57a$) and $ka=5$ ($\lambda=1.25a$) the Kirchhoff theory agrees very well with the exact theory, at values of θ below those for which $\psi_K^{(s)}=0$, the region into which most of the scattered energy is radiated.

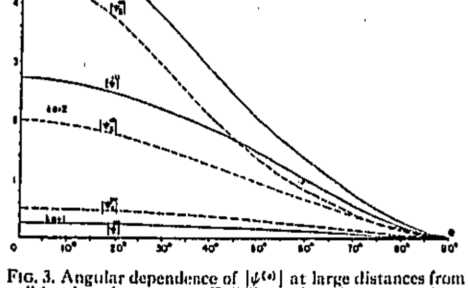

FIG. 3. Angular dependence of $|\psi^{(s)}|$ at large distances from the disk when $ka=1, 2, 3$. Full lines give the exact values, broken lines the values according to the Kirchhoff solution.

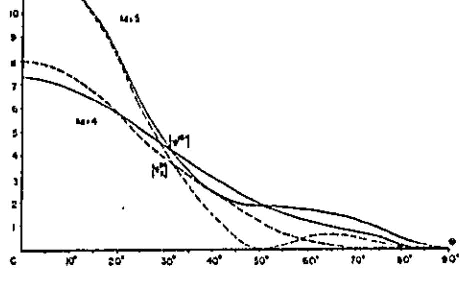

FIG. 4. Angular dependence of $|\psi^{(s)}|$ at large distances from the disk when $ka=4, 5$.

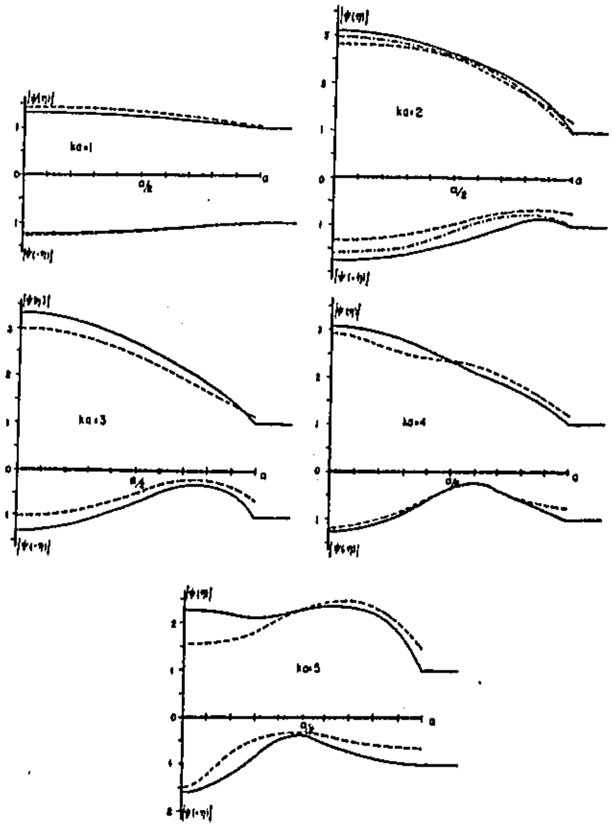

FIG. 5. Values of $|\psi|$ on the surface of the disk, plotted against distance from the center. Values on bright side ($\eta>0$) plotted upward, those on shadow side ($-\eta>0$) plotted downward; η is the angular spheroidal coordinate. Full lines represent the exact theoretical values, broken lines the experimental data; the dot-dash line is the result of a correction discussed in section 3.

3. COMPARISON WITH EXPERIMENT

Wiener determined the ratio of excess pressure on the surface to excess pressure at the same points in the absence of a thin metal cylinder. This quantity is identical to our $|\psi|$. Its values on the bright side ($\eta>0$) and on the shadow side ($-\eta>0$) are plotted in Fig. 5.***

The Kirchhoff values for this quantity are $+2$ and 0, respectively, regardless of ka. A point of particular interest about the exact values is the increasingly sharp central bright spot on the shadow side, as ka increases.

There is very good agreement with experimental values, considering that the estimated experimental error is relatively large (±1 to 2 decibels). The discrepancy may be reduced by theoretical argument: the disk of the theoretical problem is of zero thickness and only modes odd in η (or z) are generated in the scattered field; the experimental disk, however, had the dimensions $a=7.5$ cm, thickness 0.25 in.

Were we to consider a non-zero spheroid, the value of ψ on its surface would be

$$\psi(\eta,\xi_0)=e^{-ika\eta\xi_0}+\sum_{l=0}^{\infty}[2(-1)^{l+1(1)}v_l'(\xi_0)^{(3)}v_l(\xi_0)/ q_lN_l^{(3)}v_l'(\xi_0)]u_l(\eta), \quad (9)$$

where we have used the notation of Spence.[8]† The summation now is over all positive integers l, and represents the scattered field $\psi^{(s)}$; ξ_0 is the coordinate identifying the diffracting oblate spheroid. The value of ξ_0 such that its average thickness corresponds to the proportions of the experimental disk is 0.054.

*** η is the angular coordinate of the oblate spheroidal system, $-1\leqslant\eta\leqslant1$.

† See this paper for details.

This is a small value so that the even modes (l even) make a small contribution and the odd modes are changed by little from their values for $\xi_0=0$. If, furthermore, ka is small, it is reasonable to suppose that among all even modes the $l=0$ mode predominates. This is the only mode which does not change sign over the entire spheroid since $u_0(\eta)$ is the only even angular function without zeros in the range of η. Consequently the values of $\psi^{(s)}(\eta,0)$ and of $\psi^{(s)}(\eta,\xi_0)$ will not intersect when plotted against η. Just such a situation prevails at $ka=2$, 3 in Fig. 5 when we compare the exact and experimental curves. As an example we have plotted the result of our theoretical correction for $ka=2$. It is seen that a substantial part of the discrepancy is removed by such a correction.

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The Diffraction of Sound by Rigid Disks and Rigid Square Plates*

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A rigid circular plate was exposed to an essentially plane progressive sound wave, and the sound pressure p at various points on the surface measured relative to the free-field pressure p_0 in the undisturbed incident wave by means of a small probe microphone. The diffraction effect $|p/p_0|$ was determined as a function of angle of incidence over a range of frequencies beginning with "long" wave-lengths and extending into the region where the radius a of the obstacle approximately equals the wave-length. Expressed in customary notation, $\frac{1}{2}<ka<8$, where k is the wave number of the incident wave. Data were obtained for angles of incidence $\theta=0$, 45, 135, and 180 degrees, where θ is measured with respect to the axis of the obstacle. Similar measurements for $\theta=0$ and 180° were made for a rigid square plate with side $2a$.

Approximate contour maps of the quantity $|p/p_0|$ in decibels have been prepared from the experimental data portraying the pressure distribution on the surface of the plates.

The experimental results are compared with computed values of $|p/p_0|$ obtained from an approximate theory in which an attempt is made to solve the problem in terms of a scattered potential calculated as if the face of the obstacle were surrounded by an infinite baffle. The agreement is quite good on the "illuminated" side of the plates, i.e., for $\theta=0$ and 45° and on the "shadow" side for $\theta=180°$. The agreement for 135 degree incidence is generally poor, although the computed values show the trends of the experimental data in many instances. At low frequencies the theory gives values which are somewhat too high on the illuminated side and too low on the shaded side.

The values of $|p/p_0|$ obtained from the exact expression of the diffraction of a plane wave by a disk of zero thickness and for perpendicular incidence are found to be in good agreement with experiment and the approximate theory on the illuminated side ($\theta=0$) and they agree reasonably well on the shaded side ($\theta=180°$) for $1\leq ka\leq 5$. The region near the edge shows discrepancies which are to be expected from the finite thickness of the circular plate (approx. $a/12$).

It is concluded that the approximate theory mentioned above is capable of predicting the diffraction effect $|p/p_0|$ on the illuminated side of the obstacles in the frequency range covered by this study for the angles of incidence investigated. On the shadow side the theory can be expected to yield usably approximate answers only for $\theta=180°$. There are reasonable grounds for the assumption that similar predictions can be made for points on or "near" the surface of "thin" plane obstacles of arbitrary shape and for other acute angles of incidence not too close to $\theta=90°$.

I. INTRODUCTION

In a recent note[1] in the Letter to the Editor column a brief report was made on a series of measurements designed to explore the sound pressure at various points on the surface of a rigid circular plate in an approximately plane wave as a function of frequency and angle of incidence. Similar measurements were performed on a rigid square plate but for the case of perpendicular incidence only. It is the purpose of this paper to present these results in detail and to compare them with theory.

The case of the disk of zero thickness can be solved exactly by means of spheroidal wave functions.[2] Although interest in the diffraction of acoustic and electromagnetic waves by disks and circular apertures has increased recently and a number of theoretical papers have been published on the subject,* only the field at large distances is generally considered. Furthermore, the tabulation of the spheroidal wave functions is as yet incomplete.[3] No exact solution is known at present for the case of the square.

Leitner[4] has obtained numerical results for the

* The experimental data in this study were obtained at the Psycho-Acoustic Laboratory, Harvard University, Cambridge, Massachusetts, under contract with the ONR.

[1] F. M. Wiener, J. Acous. Soc. Am. 21, 39 (1949).

[2] Stratton, Morse, Chu, and Hutner, *Elliptic Cylinder and Spheroidal Wave Functions* (John Wiley & Sons, Inc., New York, 1941).

* See Appendix A.

[3] G. Blanch, Math. Tables, Aids Comp. 3, 99 (1948).

[4] A. Leitner, Mathematics Research Group, New York University, New York, N. Y.

sound pressure on the surface of a rigid disk of zero thickness for perpendicular incidence for $ka=1$, 2, 3, 4, 5. His results are compared with the experimental data discussed in this study.

Sivian and O'Neil[5] and Muller *et al.*[6] have used an approximate method to predict the sound pressure at the center of the plane face of rigid obstacles of various shapes. The same method is used here in an attempt to predict the sound pressure over the *whole plane surface* of a rigid disk and a rigid square plate.**

This method consists essentially of the following: The total pressure at the surface of the obstacle in question is separated formally into an incident pressure and a scattered pressure. The following two suppositions then are made: (1) The scattered pressure is computed by assuming that the plane surface of the obstacle oscillates (fictitiously) in an infinite rigid baffle with a certain normal velocity distribution equal and opposite to the distribution of the particle velocity component of the incident wave normal to the surface. This makes the normal particle velocity of the total field on the surface equal to zero, in accordance with the required boundary condition. (2) The effect of all other surfaces of the obstacle is neglected. Note that the method imposes no restriction on the shape of the surface of the obstacle on which the pressure is to be computed except that the surface be plane.*** That the method is approximate may be seen from the fact that the calculated scattered pressure does not vanish, as it should, in the plane of the surface of the obstacle outside its boundary, except for grazing incidence.

Despite these seemingly rough approximations surprisingly good agreement is obtained on the illuminated side of the plates with the results of the measurements at hand and with the results of the exact theory as far as available. Agreement on the "shadow" side is generally poor except for $\theta=180°$. Since there is good reason to believe that this state of affairs should also hold for plane obstacles of "small" thickness† and arbitrary shape, a valuable tool is therefore at hand to handle, within limits, the wide variety of cases where no exact solutions are available or possible with known methods.

II. EXPERIMENTAL TECHNIQUE

The technique is essentially the same as used in the tests on spheres and cylinders described in an earlier paper.[7]

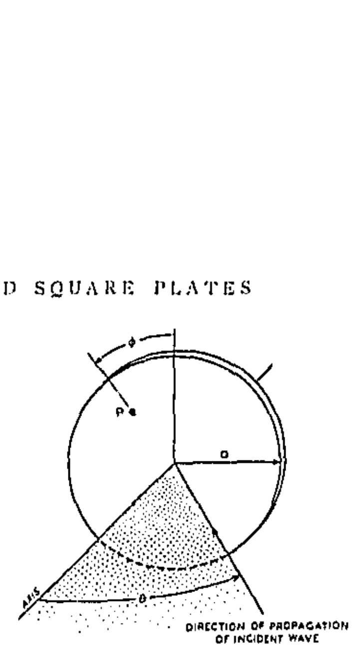

FIG. 1. Geometry.

The sound pressures p at the surfaces of the plates and in the free field were determined, in magnitude, by means of a small probe microphone whose effective area was that of a circle less than 0.1 cm in diameter. They are expressed throughout in terms of the free-field pressure p_0. The ratio $|p/p_0|$ was taken to be a measure of the diffraction effect. The incident sound field was produced by a sound source of conventional design in a sound chamber essentially free from acoustic wall reflections. The testing geometry satisfied the commonly cited criteria for an approximately plane wave field. In addition, measurements were made to determine the actual variations of the free-field pressure near the location of the plates. It was found that these variations measured inside a spherical region whose diameter was approximately $2a$ did not exceed ± 2 db for frequencies up to $ka=5$ and were less than ± 3 db for the remainder of the range. A limited number of measurements of $|p/p_0|$ in a larger chamber with a larger distance between sound source and obstacles showed only small differences attributable to the changed testing geometry. The data reported in this study can therefore be considered as having been obtained under plane-wave conditions to a reasonable degree of approximation.

The experimental errors in general are expected to increase with frequency, to increase at points near the edge due to the large pressure gradient existing there, to decrease with increasing absolute value of $|p/p_0|$. As a consequence, the frequency range at which valid measurements could be obtained on the shadow side was not quite as large as for measurements on the illuminated side. In addition, the number of positions used to explore the pressure on the shadow side become rapidly inadequate with increasing frequency due to the large fluctuations of the pressure with position.

The obstacles were made from carefully machined brass plates of $\frac{1}{4}$-inch thickness with a diameter or side of $2a=15$ cm. Radial lines from the center were

[5] L. J. Sivian and H. T. O'Neil, J. Acous. Soc. Am. 3, 483 (1932).
[6] Muller, Black, and Davis, J. Acous. Soc. Am. 10, 6 (1938).
** See Appendix B.
*** H. T. O'Neil has successfully performed similar calculations for the case of slightly curved surfaces.
† Right cylinders of arbitrary cross section may be included here for the case of perpendicular incidence.
[7] F. M. Wiener, J. Acous. Soc. Am. 19, 444 (1947).

ruled on their surfaces at intervals of 45 degrees. The distance from the center to the edge along these lines was divided into four equal parts marking the points at which the pressure measurements were made, i.e., 33 points for each angle of incidence. The pressures at acoustically symmetrical points were averaged. The free-field pressure was determined at the position of the center of the obstacle. No evidence of disturbing vibrations of the plates was observed.

A good measure of validity of the experimental procedure has already been established by comparisons with the theoretical results for the sphere in the earlier paper.[7] Further weight is added by the favorable results of the comparison of the present measurements with the exact theory as discussed below.

III. RESULTS

Disk

Figure 1 will serve to explain the geometry. The direction of propagation of the incident wave makes an angle θ with the axis of the disk of radius a. The wave front intersects the disk along a vertical diameter. A point P on the disk is fixed in angular position by the angle ϕ.

Figure 2 shows the pressure distribution for normal incidence ($\theta=0$) together with the values computed from the approximate theory.†† The agreement is seen to be remarkably good. At the

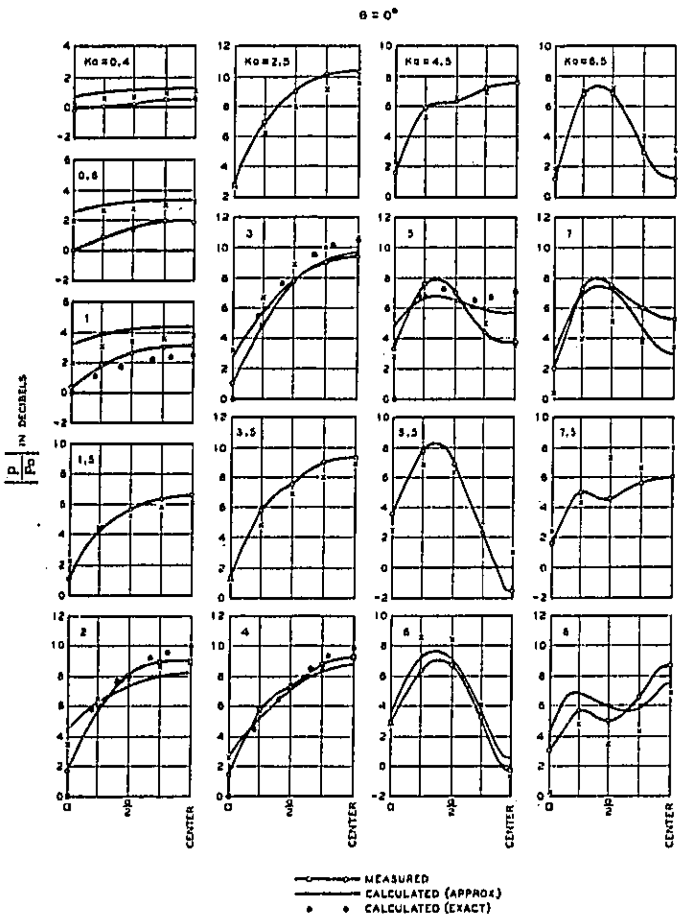

FIG. 2. Showing the ratio of the sound pressure p to the free-field pressure p_0 on the surface of a rigid disk in the field of a plane wave of wave number k. The magnitude of this ratio, in decibels, is plotted along a radius, as a function of ka for $\theta=0$. The calculated values from the approximate theory are indicated as well as those obtained from the exact one for a disk of zero thickness. In addition, the crosses (X) denote the experimental points obtained for a rigid circular cylinder of radius a and length $2a$.

†† In view of the considerable labor involved computations have not been made for all values of ka shown in this and the following graphs.

low frequencies, the theoretical values are consistently somewhat too high. This is also true for other angles of incidence $\theta<90$ degrees. Conversely, as can be seen from the graphs presented later in this paper, the theoretical values are too low for low frequencies and angles of incidence $\theta>90$ degrees. A similar situation exists at all frequencies at the edge. Such differences are not too surprising considering the assumption of an infinite baffle implicit in the theory.

The agreement of the experimental values with those computed from the exact theory is likewise good, including the low frequencies. At other frequencies, some of the largest deviations occur at the edge, as expected, since the theory applies to a disk of zero thickness. According to Fig. 2 and the data shown below in Fig. 4 for $\theta=180°$ the plate acquires "zero thickness" for all practical purposes for wave-lengths at least two orders of magnitude larger than its thickness.

The comparatively large disagreement for the center region at $ka=5$ can be most likely ascribed to the fact that the pressure there changes very rapidly with frequency. An error in the frequency setting of the oscillator of 1–2 percent may result in an error in $|p/p_0|$ of 1–2 db.

To obtain an estimate of the role which is played by the back surface of the disk, the measurements were repeated with a similar obstacle with the back surface "removed," namely a circular cylinder with the same diameter and length $2a$. While a good part of the differences between disk and cylinder must be due to experimental errors, it is clear that at frequencies up to about $ka=5$ the approximate theory predicts the results for the cylinder somewhat better than for the disk. This is not unexpected, especially at the low frequencies, in view of the assumptions underlying the approximate theory. To help visualize the pressure distribution for normal incidence, Fig. 2 of an earlier paper[8] may be consulted which applies here to a first approximation.

Figure 3 shows the diffraction effect plotted as a function of frequency for three points on the disk for $\theta=0$ together with the corresponding theoretical values. It was in the form of graphs such as this one that the experimental data were first plotted and the pressure distributions derived therefrom.

The pressure distribution for $\theta=180$ degrees is shown in Fig. 4. Note again, that the approximate theory agrees somewhat better with the data for the cylinder than with the results for the disk. Comparatively large pressure fluctuations as a function of position occur here, and for angles of incidence $\theta>90$ degrees in general, with pressure minima as low as 10 db or more below the free-field

[8] F. M. Wiener, J. Acous. Soc. Am. 20, 367 (1948).

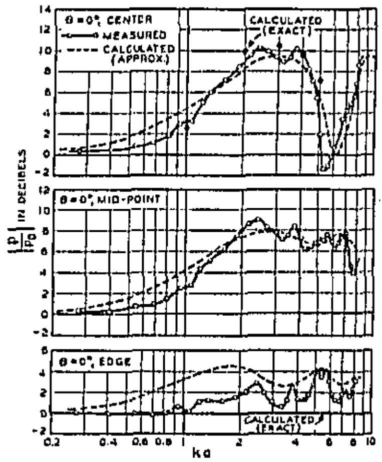

FIG. 3. Showing $|p/p_0|$ in decibels *vs.* frequency for three points on the disk and $\theta=0$, together with the theoretical values.

values. The agreement with the exact theory is not quite as good as before, especially at $ka=2$ and 3. The obvious explanation of an error in the measurement of p_0 is possible but not very probable in view of the presence of the independently measured cylinder data.

The results for $\theta=45$ degrees are depicted in Figs. 5–7. Agreement with theory is not as good as in the previous cases, especially at high frequencies and near the edge. It should be kept in mind that the results computed from theory for high frequencies are in themselves approximate as pointed out in Appendix B.

This theory is of not much help for $\theta=135$ degrees except in a general way, as shown, for example, in Fig. 8 for $\phi=0$. The data for $\phi=45$ to 270 degrees are shown in Figs. 9 and 10.

It is interesting to note the comparatively large difference in the pressures at a point on the edge of the illuminated face and the opposite point across the thickness of the plate lying in the acoustic shadow by examining a corresponding pair from the preceding set of figures obtained for supplementary angles of incidence. It will be seen that in most cases the pressure on the illuminated edge is above and the pressure on the edge in the shadow below the free-field value. It is reasonable to assume that, had the pressure been measured at points midway between the two points, it would have been found to be closer to the free-field value predicted by the theory for a disk of zero thickness.

The experimental data for oblique incidence as presented so far were used to derive an approximate picture of the pressure distribution on the surface

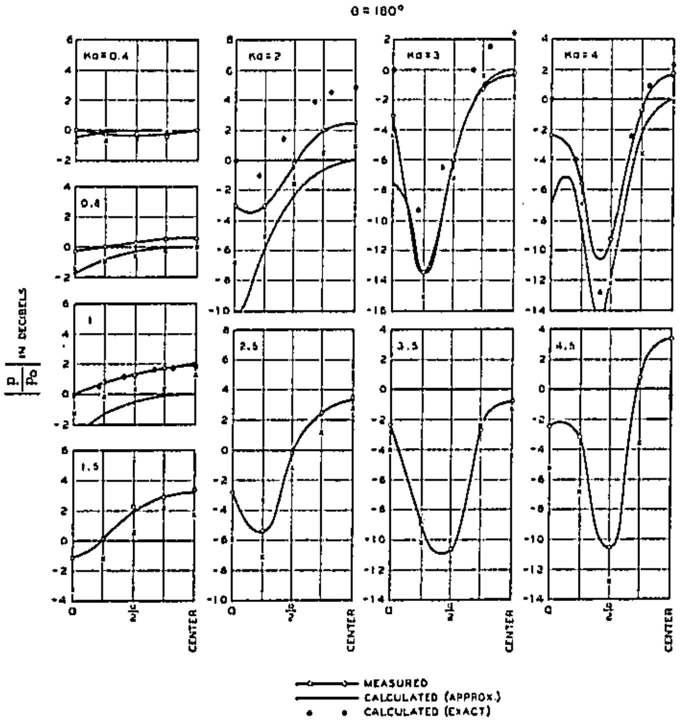

FIG. 4. Similar to Fig. 2 for $\theta = 180°$.

of the disk in the form of isobars $20 \log|p/p_0| = \text{const}$. These are presented in Fig. 11 for $\theta = 45$ and 135 degrees. In both cases, the sound wave approaches from the right,—down and into the plane of the paper for $\theta = 45$ degrees,—up and out of the plane of the paper for $\theta = 135$ degrees. The values of $|p/p_0|$ in decibels are plotted perpendicularly upward towards the observer in all cases.

It may be worth noting that, by reciprocity, the variation of the magnitude of the sound pressure measured at large distances in the θ-direction due to a point source describing an arbitrary path on the surface of the disk (or square) is given by the isobars crossed by that path.

Square

Data for a rigid square plate with sides of length $2a$ were obtained in a manner similar to the one described above for the disk. Measurements were made for perpendicular incidence only.

Figures 12 and 13 show the results of the measurements for $\theta = 0$ and $\theta = 180$ degrees, respectively. Comparison with the approximate theory shows reasonably good agreement. The theory yields again high values on the illuminated side ($\theta = 0$) and low values on the shadow side ($\theta = 180°$), at low frequencies. Again this is true for all frequencies at the edge. Essentially the same considerations as discussed in connection with the disk apply to the experimental values at the edge. At the corners the measured values are very nearly equal to the free-field pressure in all cases.

The corresponding approximate isobars are shown in Fig. 14.

ACKNOWLEDGMENT

The author wishes to express his thanks to Dr. A. Leitner of New York University for making available his computations of spheroidal wave functions prior to publication. Thanks are also due Dr. S. P. Morgan, Jr., for his assistance with the exact theory, and Miss C. L. Froelich and her staff for their painstaking numerical evaluation of the integrals of the approximate theory.

APPENDIX A

Exact Solution for a Rigid Disk of Zero Thickness in a Plane Wave Field

The diffraction of a plane wave of single-frequency sound by a disk of zero thickness can be solved exactly by the use of spheroidal wave functions. A number of theoretical papers and reports[9-13] have appeared recently dealing with the problem in this fashion, including the complementary one of diffraction by circular apertures; hence the discussion of the problem will be limited in accordance with the scope of this paper. The notation of Stratton, Morse, *et al.*[2] will be used throughout with only minor deviations.

The velocity potential Ψ describing the propagation of sound in an homogeneous, isotropic medium without friction satisfies the scalar wave equation $\nabla^2\Psi-(1/c^2)(\partial^2\Psi/\partial t^2)=0$, where c is the velocity of propagation of the sound waves of small amplitudes. Assuming a sinusoidal time dependence[14] $\Psi=\psi\exp(-i\omega t)$ where ω is the angular frequency and t the time, ψ is a solution of the equation $(\nabla^2+k^2)\psi=0$, where $k=\omega/c$. This equation is separable in the oblate spheroidal coordinate system ζ, η, φ. The surfaces $\zeta=$const. form a set of confocal oblate spheroids generated by rotation of confocal ellipses around their minor axes which are assumed to be in the z-direction. The focal circle of diameter $2a$ lies then in the x–y plane, and forms the boundary of the surface $\zeta=0$. The surfaces $|\eta|=$const. are confocal hyperboloids of one sheet whose axis is the z axis and which are rotationally symmetrical. In particular, the surface $\eta=0$ is the whole x–y plane outside the focal circle.

The surfaces $\varphi=$const. are half-planes containing the z axis and making the angle φ with the x axis. The transformation relating the Cartesian coordinates to the spheroidal ones is then given by

$$\left.\begin{array}{l} x=a(1+\zeta^2)^{\frac{1}{2}}(1-\eta^2)^{\frac{1}{2}}\cos\varphi \\ y=a(1+\zeta^2)^{\frac{1}{2}}(1-\eta^2)^{\frac{1}{2}}\sin\varphi \\ z=a\zeta\eta \end{array}\right\}. \quad (1)$$

The wave equation is separable in these coordinates and the solutions can be written as follows[2]:

$$\psi=S_{ml}(-ika,\eta)R_{ml}(-ika,i\zeta)\exp(\pm im\varphi) \quad (2)$$

where m is an integer indicating the order of the "angular" and "radial" spheroidal wave functions S_{ml} and R_{ml}. The integer l denotes the index of the characteristic values of the separation constant.

To solve the diffraction problem, assume an incident plane wave of velocity potential ψ_0 whose direction of propagation makes an angle θ with the

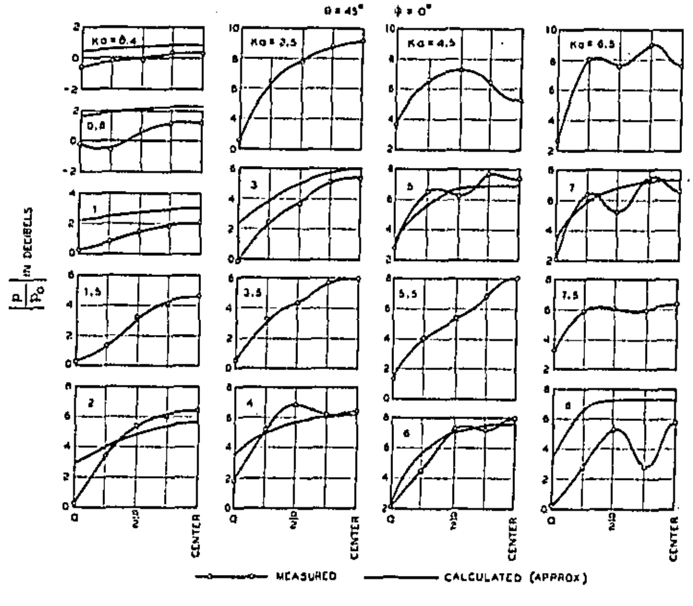

FIG. 5. Similar to Fig. 2 for $\theta=45°$ and $\phi=0$.

[9] C. J. Bouwkamp, "Theoretische en Numerieke Behandeling van de Buiging door een Ronde Opening," Thesis, University of Groningen, Holland (1941).
[10] R. D. Spence, J. Acous. Soc. Am. 20, 380 (1948).
[11] A. Storruste and H. Wergeland, Phys. Rev. 73, 1397 (1948).
[12] R. D. Spence and A. Leitner, Phys. Rev. 74, 349 (1948).
[13] R. D. Spence, J. Acous. Soc. Am. 21, 98 (1949).
[14] C. J. Bouwkamp, Philips Res. Reports 1, 251 (1946).

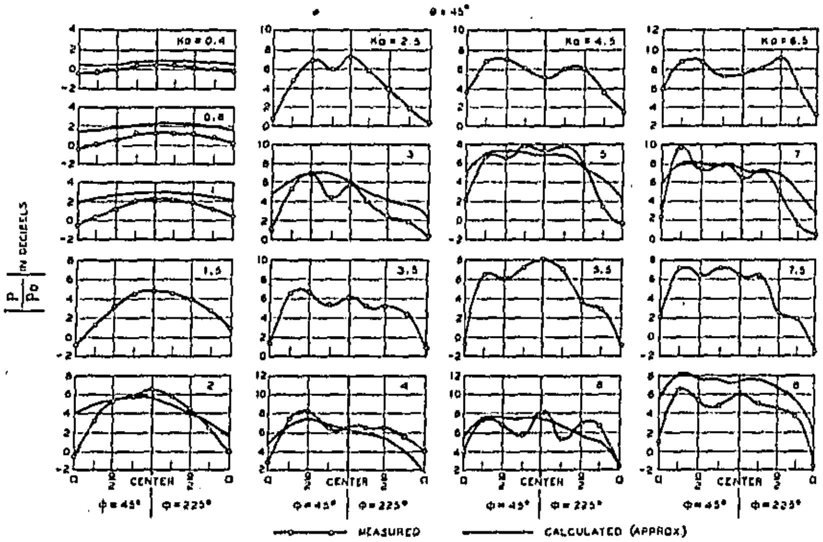

FIG. 6. Similar to Fig. 2 for $\theta=45°$ and $\phi=45°$, 225°.

negative z axis.

$$\psi_0=\exp[ik(x\sin\theta-z\cos\theta)]$$
$$=\exp\{ika[(1+\zeta^2)^{\frac{1}{2}}(1-\eta^2)^{\frac{1}{2}}\cos\varphi\sin\theta-\zeta\eta\cos\theta]\}. \quad (3)$$

This can be expanded in terms of spheroidal wave functions of the first kind as follows:

$$\psi_0=\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}A_{ml}S_{ml}^{(1)}(-ika,-\cos\theta)S_{ml}^{(1)}(-ika,\eta)\times R_{ml}^{(1)}(-ika,i\zeta)\cos m\varphi, \quad (4)$$

where the constant A_{ml} contains the normalization factor.

Similarly, the wave scattered by the disk can be expanded in terms of spheroidal wave functions of the first and third kinds as

$$\psi_1=\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}A_{ml}B_{ml}S_{ml}^{(1)}(-ika,-\cos\theta)\times S_{ml}^{(1)}(-ika,\eta)R_{ml}^{(3)}(-ika,i\zeta)\cos m\varphi, \quad (5)$$

where $R_{ml}^{(3)}$ behaves at large distances from the obstacles like an outgoing spherical wave, as i should. At the disk, assumed to be rigid, the norm particle velocity must vanish. Hence we hav $\partial/\partial\zeta(\psi_0+\psi_1)|_{\zeta=0}=0$, which leads to

$$B_{ml}=-\left[\frac{dR_{ml}^{(1)}(-ika,i\zeta)}{d\zeta}\right]_{\zeta=0}\times\left[\frac{dR_{ml}^{(3)}(-ika,i\zeta)}{d\zeta}\right]_{\zeta=0}^{-1}. \quad (6)$$

Since the sound pressure $p=-i\omega\rho\psi$, where ρ is the density of the medium, we have for the sound pressure on the surface of the disk, expressed in terms of the incident sound pressure $p_0=-i\omega\rho\psi_0$

$$p/p_0|_{\zeta=0}=\exp[-ika(1-\eta^2)^{\frac{1}{2}}\cos\varphi\sin\theta]\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}A_{ml}S_{ml}^{(1)}(-ika,-\cos\theta)S_{ml}^{(1)}(-ika,\eta)\cos m\varphi$$
$$\times\left\{R_{ml}^{(1)}(-ika,0)-\frac{\left.\dfrac{dR_{ml}^{(1)}(-ika,i\zeta)}{d\zeta}\right|_{\zeta=0}}{\left.\dfrac{dR_{ml}^{(3)}(-ika,i\zeta)}{d\zeta}\right|_{\zeta=0}}R_{ml}^{(3)}(-ika,0)\right\}. \quad (7)$$

For perpendicular incidence, $\theta=0$ or π, and the field becomes independent of φ. Hence $m=0$ and

$$p/p_0\Big|_{\substack{\zeta=0\\ m=0}}=\sum_{l=0}^{\infty}A_{0l}S_{0l}^{(1)}(-ika,\mp 1)S_{0l}^{(1)}(-ika,\eta)$$

$$\times\left[R_{0l}^{(1)}(-ika,0)-\frac{\left.\dfrac{dR_{0l}^{(1)}(-ika,i\zeta)}{d\zeta}\right|_{\zeta=0}}{\left.\dfrac{dR_{0l}^{(3)}(-ika,i\zeta)}{d\zeta}\right|_{\zeta=0}}R_{0l}^{(3)}(-ika,0)\right]. \quad (8)$$

This is essentially the expression which is given in Spence's[10] analysis and which was used by Leitner[4] to compute the values shown by the full circles in Figs. 2–4.

It is of interest to examine the scattered potential ψ_1 in more detail. Since B_{ml} vanishes for even values[2] of l the summation for the scattered potential is carried out over odd values of l only. A typical term in this summation is from Eq. (5), proportional to $S_{ml}^{(1)}(-ika,\ \eta)R_{ml}^{(3)}(-ika,\ i\zeta)\times\cos m\varphi$. In the plane of the disk outside its boundary, i.e. for $\eta=0$, the scattered wave vanishes, since $S_{ml}^{(1)}(-ika,0)=0$ for odd values[2] of l. On the disk ($\zeta=0$) the above term becomes $S_{ml}^{(1)}(-ika,\eta)\times R_{ml}^{(3)}(-ika,0)\cos m\varphi$ which goes continuously to zero as the edge is approached. Hence p/p_0 is continuous as one proceeds from the center of the disk towards the edge and becomes unity there and everywhere in the plane $\eta=0$.

The particle velocity has a singularity at the edge. It can be shown that it becomes infinite, as the edge is approached, as $\sigma^{-\frac{1}{2}}$, where σ is the distance between the field point and the edge. Singularities of this type are encountered also in electromagnetic diffraction theory and are discussed by Bouwkamp.[15]

It may be pertinent to remark that the exact solution for the sound field generated by a vibrating

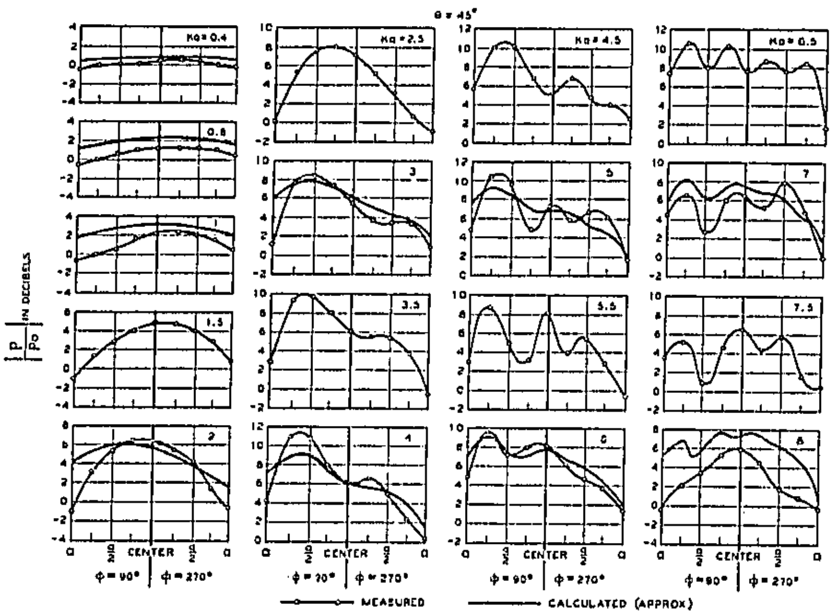

FIG. 7. Similar to Fig. 2 for $\theta=45°$ and $\phi=90°$, 270°.

[15] C. J. Bouwkamp, Physica 12, 467 (1946).

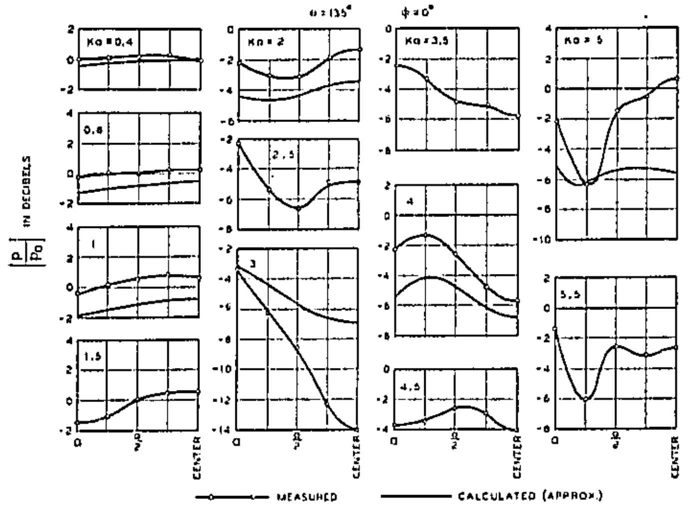

FIG. 8. Similar to Fig. 2 for $\theta = 135°$ and $\phi = 0$.

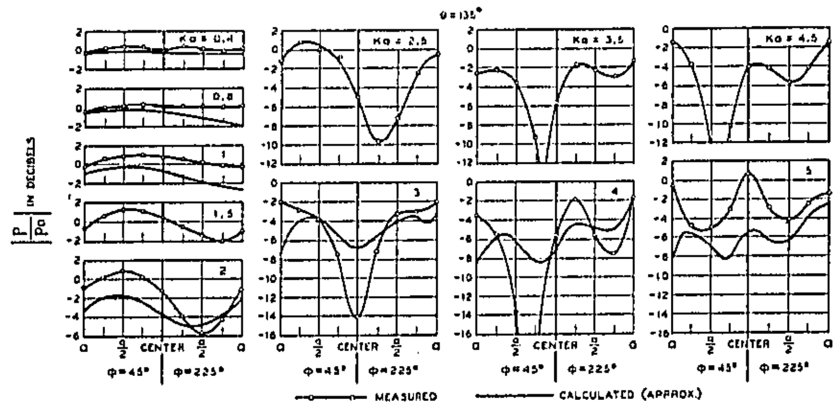

FIG. 9. Similar to Fig. 2 for $\theta = 135°$ and $\phi = 45°, 225°$.

disk of zero thickness is of the same form as the scattered potential of the present problem[9] for $m = 0$.

APPENDIX B

Approximate Solution for Rigid Plates of Arbitrary Shape in a Plane Wave Field

It is well known that the expression for the velocity potential due to a membrane set in a rigid wall of infinite extent can be computed exactly. If the membrane is oscillating sinusoidally with a prescribed normal velocity distribution v the velocity potential is given by the so-called Rayleigh formula[11]

$$\psi = (2\pi)^{-1} \int_F v\,[\exp(ikr)/r]\,dS. \qquad (9)$$

The usual assumptions, as listed at the beginning of Appendix A, are presumed to hold. The integration is to be carried out over the surface F of the membrane whose shape is arbitrary, and r is the distance between the surface element dS and the point at which the velocity potential ψ is to be determined.

If one wishes to determine the velocity potential on F, r lies in the plane of F and is conveniently measured from the origin where ψ is to be determined (see Fig. 15).

$$\psi(0)=(2\pi)^{-1}\int_0^{2\pi}\int_0^{r_c} v(r)\exp(ikr)drd\varphi, \quad (10)$$

where $v(r)$ is the normal velocity distribution on F and r_c is the value of r on the boundary C of F and dS is conveniently expressed in polar coordinates r, φ.

Sivian and O'Neil[5] and later Muller *et al.*[6] have suggested the following procedure to solve the diffraction of a sound wave by an obstacle whose plane face coincides with F, for points on F: The velocity distribution $v(r)$ on F is to be adjusted so as to be equal and opposite to the component of the particle velocity perpendicular to F of the incident wave. If we assume the incident plane wave to be

$$\psi_0=\exp[-ik(x\sin\theta+z\cos\theta)],$$

the particle velocity in the z direction is

$$-\frac{\partial\psi_0}{\partial z}=ik\cos\theta\exp[-ik(x\sin\theta+z\cos\theta)]. \quad (11)$$

Hence at the surface of the obstacle

$$v(r)=\frac{\partial\psi_0}{\partial z}\bigg|_{z=0}=-ik\cos\theta\exp(-ikr\sin\theta\cos\varphi) \quad (12)$$

where $v(r)$ is taken to be positive in the z direction.

Inserting Eq. (12) in Eq. (10) and identifying ψ with the scattered velocity potential $\psi_1(0)$ at the origin, we have

$$\psi_1(0)=-(2\pi)^{-1}ik\cos\theta\int_0^{2\pi}\int_0^{r_c} \times\exp[ikr(1-\sin\theta\cos\varphi)]drd\varphi. \quad (13)$$

The ratio of the total sound pressure p to the incident pressure p_0 at the origin becomes then

$$p/p_0=1-(2\pi)^{-1}ik\cos\theta\int_0^{2\pi}\int_0^{r_c} \times\exp[ikr(1-\sin\theta\cos\varphi)]drd\varphi. \quad (14)$$

This is identical, except for slight differences in notation, with Eq. (14) of reference 6.

Carrying out the integration over r one obtains

$$p/p_0=1+\frac{\cos\theta}{|\cos\theta|}-(2\pi)^{-1}\cos\theta \times\left.\int_0^{2\pi}\frac{\exp[ikr_c(1-\sin\theta\cos\varphi)]}{1-\sin\theta\cos\varphi}d\varphi\right\}_{\theta\neq\pi/2}. \quad (15)$$

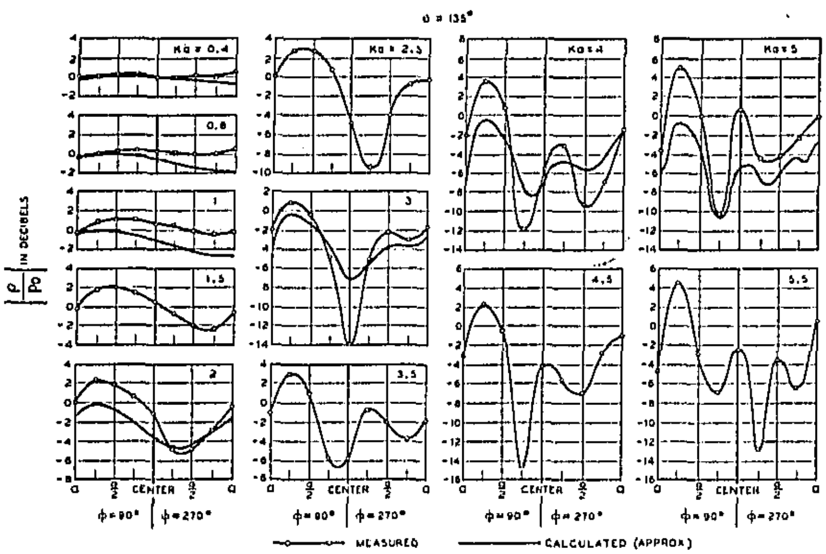

FIG. 10. Similar to Fig. 2 for $\theta=135°$ and $\phi=90°$, 270°.

This expression was used to calculate the approximate values of $|p/p_0|$ for the disk and the square as shown in Figs. 2–10 and 12, 13. Along the values of ϕ (see Fig. 1) marked on the graphs computations were made at 8 equal intervals from the center to the edge. The evaluation of the integral was carried out by numerical methods, for selected values of ka, using 20 equal steps in the interval of integration, after expressing r_e in terms of the variable of integration φ. In some cases, especially for high frequencies, the results may be in error as much as perhaps 2 db due to the rapid fluctuations of the integrand. In view of the considerable labor involved and the approximate nature of Eq. (15) to start with, it was not deemed advisable to repeat the computations with a finer subdivision, except for a few spot checks.

In conclusion, there is presented a list of the more explicit expressions into which Eq. (15) was transformed before numerical evaluation. The position of the origin with respect to the center of the square and the disk is determined by the parameters α and β where α is the x coordinate and β the y coordinate, both expressed relative to the radius a of the disk and half the length of the side of the square, respectively. For perpendicular incidence Eq. (15) can be reduced to simple expressions for the center ($\alpha=\beta=0$) and the edge ($\alpha^2+\beta^2=1$) of the disk.

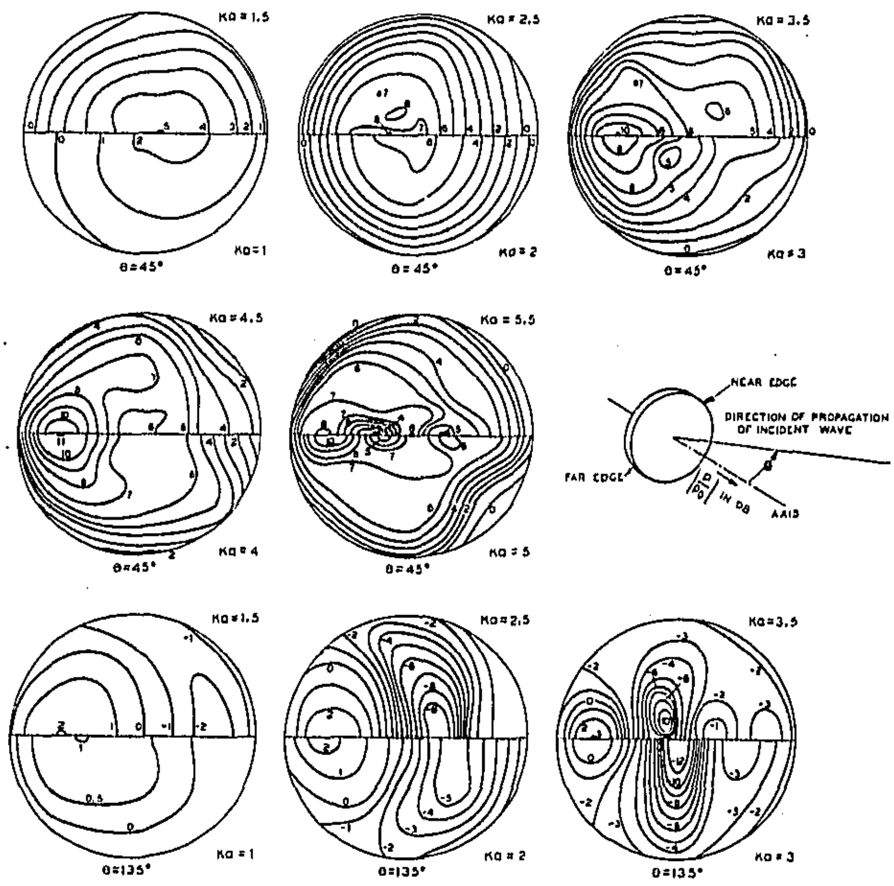

FIG. 11. Showing the approximate pressure distribution on the surface of the disk obtained from the experimental data for $\theta=45°$ and $135°$ in the form of isobars $20\log|p/p_0|=$const. The wave approaches from the right and the wave front intersects the plane of the disk along the vertical diameter. The pattern is symmetrical about the horizontal diameter.

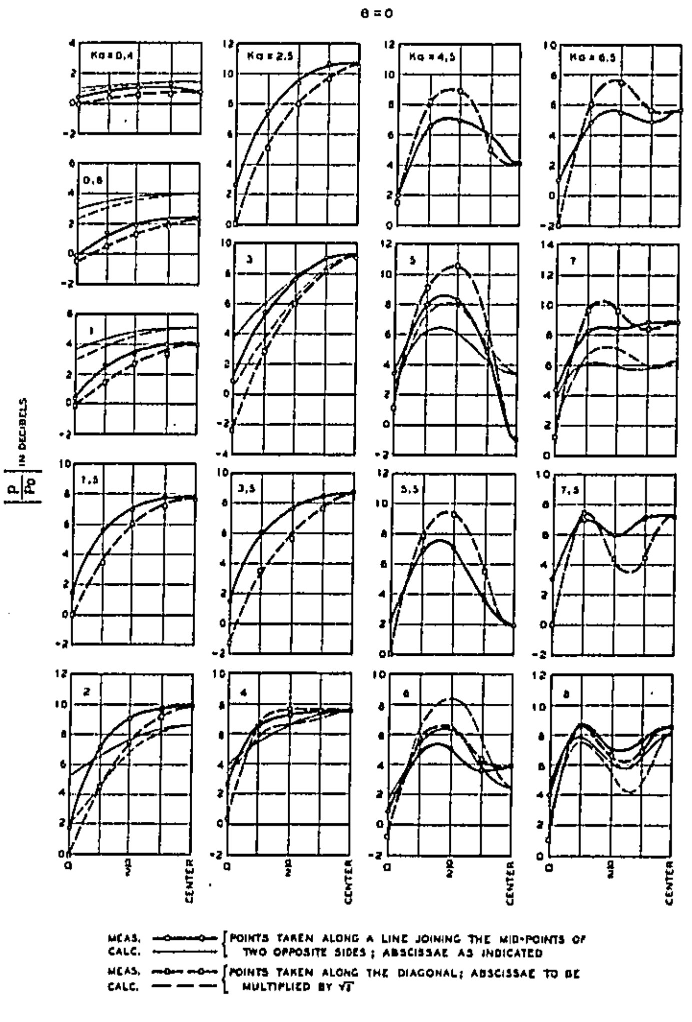

FIG. 12. Showing the ratio of the sound pressure p to the free-field pressure p_0 on the surface of a rigid square plate in the field of a plane wave of wave number k. The magnitude of this ratio, in decibels, is plotted along a line joining the mid-points of two opposite sides and along the diagonal as a function of ka for $\theta = 0$. The calculated values obtained from the approximate theory are indicated by the thin lines.

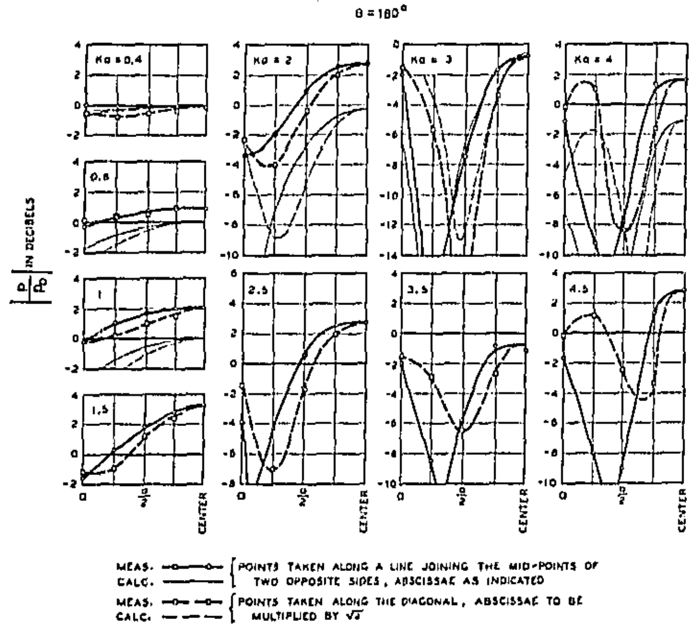

Fig. 13. Similar to Fig. 12 for $\theta = 180°$.

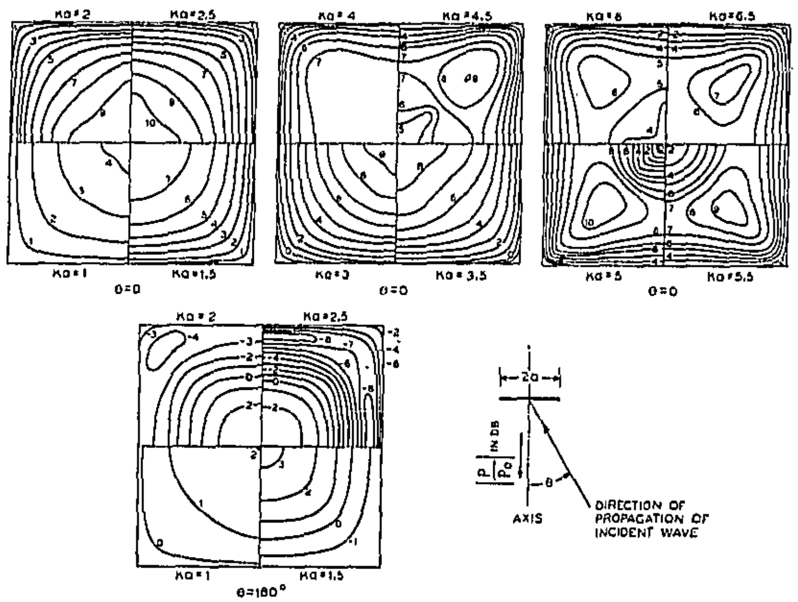

Fig. 14. Showing the approximate pressure distribution on the surface of the square plate obtained from the experimental data in Figs. 12 and 13 in the form of isobars $20 \log |p/p_0| = \text{const}$. Only one quarter of the pattern is shown. It may be completed by symmetry considerations.

Disk

Center

$$\theta=0, \quad |p/p_0|=[(2-\cos ka)^2+\sin^2 ka]^{\frac{1}{2}}$$
$$\theta=180°, \quad |p/p_0|=1$$

Edge

$$\theta=0, \quad |p/p_0|=1/2([3-J_0(2ka)]^2+S_0^2(2ka))^{\frac{1}{2}}$$
$$\theta=180°, \quad |p/p_0|=1/2([1+J_0(2ka)]^2+S_0^2(2ka))^{\frac{1}{2}}$$

where J_0 is the Bessel function of the first kind and S_0 is the Struve[16] function, both of order zero.

General Case

$$|p/p_0|=\left|1+\frac{\cos\theta}{|\cos\theta|}-(2\pi)^{-1}\cos\theta I\right|,$$

where

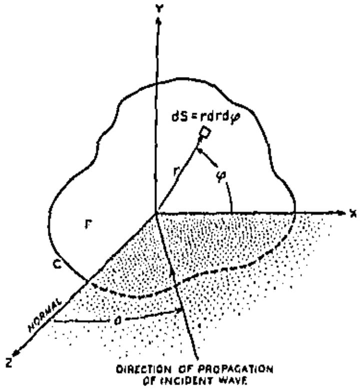

FIG. 15. Geometry pertaining to the approximate theory.

$$I\equiv\int_0^{2\pi}\frac{\exp(ika[\alpha\cos\varphi+\beta\sin\varphi+(1-\alpha^2\sin^2\varphi-\beta^2\cos^2\varphi+\alpha\beta\sin 2\varphi)^{\frac{1}{2}}][1-\sin\cos\varphi])}{1-\sin\theta\cos\varphi}d\varphi$$

where $\alpha^2+\beta^2\leq 1$.

Square

$$\theta=0, \quad |p/p_0|=|2-(2\pi)^{-1}I|$$
$$\theta=180°, \quad |p/p_0|=(2\pi)^{-1}|I|,$$

where

$$I\equiv\int_{-\tan^{-1}\frac{1-\beta}{1+\alpha}}^{\tan^{-1}\frac{1+\beta}{1+\alpha}}\exp[ika(1+\alpha)\sec\varphi]d\varphi$$
$$+\int_{\tan^{-1}\frac{1-\beta}{1+\alpha}-\frac{\pi}{2}}^{\frac{\pi}{2}-\tan^{-1}\frac{1+\beta}{1-\alpha}}\exp[ika(1+\beta)\sec\varphi]d\varphi$$
$$+\int_{-\tan^{-1}\frac{1-\beta}{1-\alpha}}^{-\tan^{-1}\frac{1-\beta}{1-\alpha}}\exp[ika(1-\alpha)\sec\varphi]d\varphi$$
$$+\int_{\tan^{-1}\frac{1-\beta}{1-\alpha}-\frac{\pi}{2}}^{\frac{\pi}{2}-\tan^{-1}\frac{1-\beta}{1-\alpha}}\exp[ika(1-\beta)\sec\varphi]d\varphi.$$

[16] G. N. Watson, *Theory of Bessel Functions* (The Macmillan Company, New York, 1944), p. 328.

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Theory of Ultrasonic Intensity Gain Due to Concave Reflectors*

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A concave reflector can be used to concentrate a beam of plane ultrasonic waves in the focal region, where the intensity I_f is much larger than the intensity I_i in the plane wave. When the sound wave-length is small compared to the dimensions of the beam and reflector, one can use the well-known Fraunhofer diffraction formulas to calculate the intensity gain, i.e., I_f/I_i. Expressions are derived for the maximum and average intensity gain in the zero-order image when the ultrasonic beam is circular or rectangular, together with formulas giving the total intensity falling upon circular or rectangular areas of arbitrary dimensions in the focal region.

INTRODUCTION

In this paper the well-known optical equations for Fraunhofer diffraction are applied to a system in which a beam of plane ultrasonic waves is concentrated into a small region with a concave reflector.

The conditions necessary for the use of Fraunhofer diffraction formulas may be stated as follows. A train of plane waves strikes an infinite screen in which there is a square or circular aperture provided with a thin lens of focal length f. One then can express the intensity distribution on a screen at a distance f from the aperture, and in a plane parallel to that containing the aperture, by the usual Fraunhofer formulas. The case in which a plane wave is normally incident upon a square or circular mirror of focal length f is strictly analogous, with the exception that the contribution of the incident wave before it strikes the mirror is neglected. One can think of the diffraction pattern superimposed on a background provided by the incoming plane wave. Again, if one makes the assumption that what strikes the mirror or lens is a "beam of plane waves" of circular or rectangular cross section, nothing is changed, except that now the dimensions of the beam replace those of the aperture in the first screen.

A sound beam radiating into a liquid from a plane piston in an infinite baffle has its own diffraction structure. Near the source one has only a fraction of the total energy fulfilling the plane wave assumption; this fraction gets larger as the wave-length gets smaller compared to the dimensions of the radiator. When the source is very large compared to the wave-length, and the reflector or lens is close to the source, the assumption of a well defined beam of plane waves is not only the simplest to handle mathematically, but it is also the one that most accurately describes the waves falling on the mirror. Thus, we assume that a beam of plane waves of wave-length λ, traveling along the principal axis z of a mirror or lens S, converges to the geometrical focus 0 at the origin of the coordinate system x, y, z, as shown in Fig. 1.

I. CIRCULAR SYMMETRY

Maximum Intensity Gain

We first take the case where the incoming beam has a circular cross section of radius R. In this case the Fraunhofer diffraction pattern in the xy plane has circular symmetry around the z axis. At a point P in the xy plane, the intensity is given by[1,2]

$$I = B\pi^2R^4[2J_1(z)/z]^2, \quad (1)$$

in which $J_1(z)$ is the Bessel function of the first order of argument z, and $z = 2\pi Rr/f\lambda$, in which r is the distance from the z axis and f is the focal length of the reflector or mirror. B is a normalizing constant which is to be adjusted so that the total intensity falling on the xy plane is equal to the total intensity in the incoming beam. Maximum and minimum values of I are given by

$$d/dz[J_1(z)/z] = 0 = J_2(z), \quad (2)$$

where $J_2(z)$ is the Bessel function of the second order. In particular, the zero-order maximum of intensity I_m occurs at $z=0$, where $z^{-1}J_1(z) = \frac{1}{2}$ and

$$I_m = B\pi^2R^4. \quad (3)$$

The total flux falling on a circle of radius r in the xy plane is

$$F = \int_0^r I2\pi r dr = B\pi R^2 f^2\lambda^2[1 - J_0^2(z) - J_1^2(z)]. \quad (4)$$

In particular, when $r=\infty$, the total flux is that in the incoming beam

$$F_\infty = B\pi R^2 f^2\lambda^2, \quad (5)$$

since $J_0(\infty) = 0 = J_1(\infty)$. But

$$F_\infty = \pi R^2 I_i, \quad (6)$$

* This research was aided by the ONR Contract N6 onr-255.

[1] A. Gray and G. B. Mathews, *Treatise on Bessel Functions* (MacMillan Company, Ltd., London, 1895).

[2] M. Born, *Optik* (Verlag. Julius Springer, Berlin, 1933), p. 157–160.

where I_i is the intensity of the incoming plane wave, so

$$B = I_i/(f\lambda)^2. \quad (7)$$

If one defines gain in intensity at any point P as

$$g = I_P/I_i, \quad (8)$$

where I_P is the intensity at the point P, then the maximum intensity gain g_m is

$$g_m = (\pi R^2/f\lambda)^2. \quad (9)$$

Average Intensity Gains

To obtain the average gain $\bar{g}$ for the intensity over an area in the focal region, one obtains from (4) and (7) the total flux passing through the circle in the xy plane of radius r as a multiple of I_i, and divides by πr^2 to obtain the average intensity. Thus,

$$\bar{g} = [1 - J_0^2(z) - J_1(z)](R/r)^2. \quad (10)$$

The first minimum of (1) occurs at $z = 3.8317$ or $r = 0.610f\lambda/R$, where $I_r = 0$, $J_1 = 0$, and $J_2 = 0.4028$, so that the average intensity gain in the zero-order diffraction image is

$$\bar{g} = 0.8378(R/r)^2 = 0.3116(R^2/f\lambda)^2. \quad (11)$$

II. RECTANGULAR SYMMETRY

Maximum Intensity Gain

When the beam of plane waves incident along the principal axis of the reflector has a rectangular cross section $\Delta x = a$, $\Delta y = b$, and the z axis is in the geometrical center of the beam, the intensity at a point $P_{(xy)}$ in the $z = 0$ plane is[2,3]

$$I_{xy} = Ba^2b^2[(\sin px)/px]^2[(\sin qy)/qy]^2, \quad (12)$$

where $p = \pi a/f\lambda$; $q = \pi b/f\lambda$. The first, and largest, maximum is at $x = y = 0$, where

$$I_m = Ba^2b^2. \quad (13)$$

The intensity is zero wherever px or qy is an integral multiple of π, and, in particular, the zero-order image is bounded by the lines of zero intensity

$$\pm x = f\lambda/a; \quad \pm y = f\lambda/b. \quad (14)$$

The parameters $f\lambda/a$ and $f\lambda/b$ are convenient natural scale units in which to express x and y.

For the total intensity falling on a rectangular area $\Delta x \Delta y$, bounded by the lines x_1, x_2, y_1, y_2, we have

$$F = Ba^2b^2 \int_{x_1}^{x_2}\int_{y_1}^{y_2} [(\sin px)/px]^2 \times [(\sin qy)/qy]^2 dy dx. \quad (15)$$

Using $2\sin^2\theta = 1 - \cos 2\theta$, writing $(2\pi ax/f\lambda) = k$,

[3] R. W. Wood, *Physical Optics* (The Macmillan Company, New York, 1923).

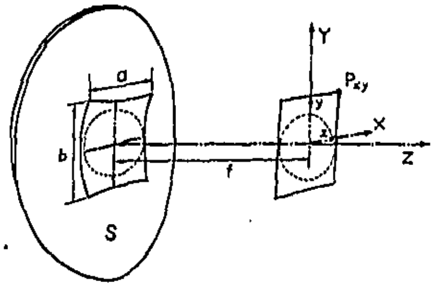

Fig. 1. Coordinate system for concave reflector or lens. Origin is at the geometrical focal point, with the z axis coincident with the principal axis of the reflector or lens. The effective aperture defines, or is defined by, the cross section of the incident beam of plane waves.

$(2\pi by/f\lambda) = m$, and integrating this becomes

$$F = \frac{Babf^2\lambda^2}{\pi^2}\left[\int_{k_1}^{k_2} \frac{\sin k dk}{k} - \left(\frac{1-\cos k}{k}\right)_{k_1}^{k_2}\right] \times \left[\int_{m_1}^{m_2} \frac{\sin m dm}{m} - \left(\frac{1-\cos m}{m}\right)_{m_1}^{m_2}\right], \quad (16)$$

where k is the value of k for $x = x_1$ etc. Since tables of the sine integral

$$Si\theta = \int_0^{\theta_1} \frac{\sin\theta}{\theta} d\theta$$

give the value of the integral from zero to θ_1 for the argument, it is useful to write

$$F = \frac{Babf^2\lambda^2}{\pi^2}\left[Si(k_2) - \frac{(1-\cos k_2)}{k_2} + \frac{1-\cos k_1}{k_1} - Si(k_1)\right]\left[Si(m_2) - \frac{(1-\cos m_2)}{m_2} + \frac{(1-\cos m_1)}{m_1} - Si(m_1)\right]. \quad (17)$$

We can use (17) at once to determine B in terms of I_i. Since $Si(\pm\infty) = \pm\pi/2$, we find for the total intensity

$$F_t = Babf^2\lambda^2 = I_i ab, \quad (18)$$

or

$$B = I_i/f^2\lambda^2.$$

For the maximum intensity and gain at the center of the zero-order image, we have from (13) and (18)

$$I_m = I_i(ab/f\lambda)^2 \quad \text{and} \quad g_m = (ab/f\lambda)^2. \quad (19)$$

TABLE I. Values of $K=[Si(k)-(1-\cos k)/k]/\pi$ for definite values of $k(=2\pi x')$.

k	K
0.1	0.01588
0.2	0.03181
0.3	0.04763
0.4	0.06339
0.6	0.09454
0.8	0.12509
1.0	0.15483
1.2	0.1836
1.4	0.2111
1.6	0.2374
1.8	0.2623
2.0	0.2856
2.5	0.3368
3.0	0.3773
π	0.3868
3.5	0.4074
4.0	0.4281
4.5	0.4409
5.0	0.4477
6.0	0.4514
2π	0.4514
7.0	0.4518
8.0	0.4555
9.0	0.4624
3π	0.4656
10.0	0.4693
11.0	0.4736
12.0	0.4749
4π	0.4750
13.0	0.4750
14.0	0.4757
15.0	0.4778
5π	0.4796
16.0	0.4803
20.0	0.4834
30.0	0.4898
40.0	0.4919
50.0	0.4937

Average Intensity Gain

The general expression (17) is greatly simplified under particular conditions. When $x_1=-x_2$ and $y_1=-y_2$, which is usually the case of greatest experimental interest,

$$F=\frac{4I_iab}{\pi^2}\left[Si(k_2)-\frac{(1-\cos k_2)}{k_2}\right]\times\left[Si(m_2)-\frac{(1-\cos m_2)}{m_2}\right]. \quad (20)$$

Since

$$\lim_{\theta\to\infty}\{Si\theta-[(1-\cos\theta)/\theta]\}=\pi/2,$$

it is useful to write (18) in the form

$$F=I_iab[4K(k_2)M(m_2)], \quad (21)$$

where

$$K=\frac{1}{\pi}\left[Si(k)-\frac{(1-\cos k)}{k}\right]$$

and

$$M=\frac{1}{\pi}\left[Si(m)-\frac{(1-\cos m)}{m}\right]. \quad (22)$$

K and M are even functions of the argument. Then we have for the average intensity and gain in the region bounded by $\pm x$, $\pm y$,

$$\bar{I}=(abKM/xy)I_i \quad \text{and} \quad \bar{g}=abKM/xy. \quad (23)$$

In the natural units defined by

$$x'=(a/f\lambda)x;\quad y'=(b/f\lambda)y;$$
$$\bar{I}=(ab/f\lambda)^2(KM/x'y')I_i; \quad (24)$$
$$\bar{g}=(ab/f\lambda)^2(KM/x'y').$$

The form of (23) indicates the physical nature of KM most clearly: of the total energy incident in the beam ab, the fraction KM falls on the surface bounded by $x=0$, $x=x_1$, $y=0$, $y=y_1$. The form (24) is useful for routine determinations of intensity gains, or of the functions K and M. Values of K as a function of x' (or M as a function of y') are given in Table I, and plotted in Fig. 2. One expresses x (or y) as x' (or y'), and finds K and M from the table or curve. These, together with a, b, f, and λ, determine the gain immediately. In particular, for the zero-order image, using (14) and (24),

$$\bar{g}=0.8151(ab/4xy)=0.2038(ab/f\lambda)^2. \quad (25)$$

III. SCREENING: LONG STRIPS AND STRAIGHT EDGE

It is useful to know the flux falling on a rectangular strip bounded by the lines $x=\pm\infty$, $y=y_1$ and $y=y_2$. Since $x=\pm\infty$, $K=\frac{1}{2}$

$$F=I_iab(M_2-M_1), \quad (26)$$

where M_1 and M_2 are the values of M for the argument corresponding to y_1 and y_2. When $y_1=-y_2$,

$$F=2I_iabM_1. \quad (27)$$

These expressions enable one to calculate at once the fraction of the total energy in the beam that

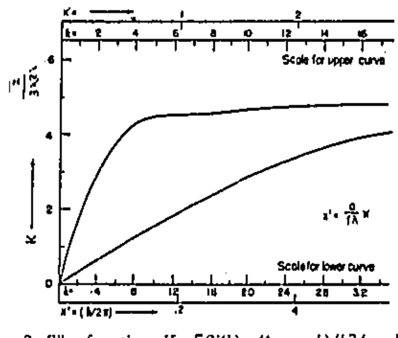

FIG. 2. The function $K=[Si(k)-(1-\cos k)/k]/\pi$ plotted as a function of the parameters $k(=2\pi x')$ and $x'(=xa/f\lambda)$, where the lengths x, a, f, and λ are measured in the same units. The curve is the same if one substitutes M, m, y', y, and b for K, k, x', x, and a.

strikes a given obstacle (long strip or cylinder) placed in the $z=0$ plane.

When $y_2=+\infty$, and y_1 is the position of the lower edge of a straight edge screen, the total flux that strikes the screen is

$$F=2I_i ab(\tfrac{1}{2}-M_1). \quad (28)$$

IV. FLUX FALLING ON CIRCULAR AREA DUE TO A RECTANGULAR BEAM

Although the expressions developed in I and II are not convenient for calculating the average intensity or intensity gain on a circular area in the $z=0$ plane, they can be used to get an approximate value when the center of the circular area is at the origin. For most experimental data, the following method is more accurate than the data. One writes

$$g=(ab/\pi r^2)[4K(k)M(m)], \quad (29)$$

where r is the radius of the circle in the $z=0$ plane. The arguments of the functions K and M are then obtained by setting $x=y=\frac{1}{2}(\pi r^2)^{\frac{1}{2}}$, and changing to the scaled variables $x'=(a/f\lambda)x$ and $y'=(b/f\lambda)y$. K and M are then determined for $k=2\pi x'$ and $m=2\pi y'$. Although the method yields values that are surprisingly accurate, it is not easy to estimate the actual error involved.

A more laborious method, but one which gives upper and lower limits to the calculated values, can be used to obtain a desired accuracy. The first quadrant of the circle is divided into i strips, each strip being Δy high and having minimum, maximum, and average lengths $(\underline{x}_i)$, $(\bar{x}_i)$, (x_i) as shown in Fig. 3. The total flux falling on the ith trapezoid is approximately

$$F_i=I_i ab[M(\bar{m}_i)-M(\underline{m}_i)](\tfrac{1}{2})[K(\underline{k}_i)+K(\bar{k}_i)], \quad (30)$$

where the $M(\underline{m}_i)$ and $M(\bar{m}_i)$ are the values corresponding to the lower and upper limits of Δy_i, and the $K(\underline{k}_i)$ and $K(\bar{k}_i)$ correspond to the ith trapezoid dimensions $\underline{x}_i$ and $\bar{x}_i$. The average intensity gain is

$$g=4ab\sum^i F_i/\pi r^2. \quad (31)$$

Obviously by taking Δy_i small enough, any desired accuracy can be obtained, although it will seldom be necessary to take more than $i=5$ or 10. An upper limit for g can be obtained, using not the arithmetical mean of the K's for the upper and lower value of x, i.e., $\frac{1}{2}[K(\bar{k}_i)+K(\underline{k}_i)]$, but only $K(\bar{k})$:

$$g=(4ab/\pi r^2)\sum^i[M(\bar{m}_i)-M(\underline{m}_i)]K(\bar{k}_i). \quad (32)$$

A lower limit is obtained by using $K(\underline{k})$:

$$g=(4ab/\pi r^2)\sum^i[M(\bar{m}_i)-M(\underline{m}_i)][K(\underline{k}_i)]. \quad (33)$$

From (32) and (33) one can obtain the maximum possible error in the intensity gain as calculated from either (29) or (31). For example, the average gain for a circular area ($r=0.781$ mm), when $f=34$ mm and $\lambda=0.353$ mm, was calculated from Eq. (29) to be 74. Equations (30), (31), and (33) were used then with the quadrant divided into 10 strips having values of $\bar{y}_i$ equal to 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.650, 0.70, 0.750, 0.781. From Eq. (30), g is 74.14; from Eq. (32), the upper limit g is 75.03; and from Eq. (33), the lower limit is 74.02.

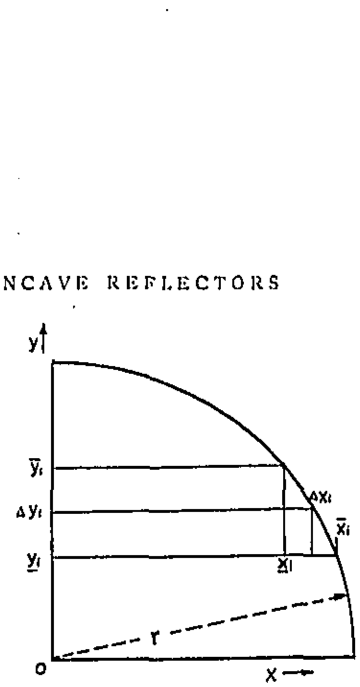

FIG. 3. Notation for division of circular quadrant into rectangular strips or trapezoids.

The authors wish to thank Professor K. F. Herzfeld for many helpful discussions and Mr. Stephen Malaker for assistance with the numerical calculations.

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Experimental Investigation of Ultrasonic Intensity Gain in Water Due to Concave Reflectors*

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This paper discusses a method of producing high intensity sound waves in liquids. A beam of ultrasonic waves (4.25 mc, 15×12 mm cross section, acoustic power≃2 watts) was focused with an ordinary watch glass (6.8 cm radius of curvature). The intensity in the focal region is large enough to raise an ultrasonic fountain 10 cm high accompanied by a spray of fog droplets. The distribution of intensity in the focal region was determined by measuring the screening effect of properly placed obstacles. The sound intensity in the focal region and in the plane wave was measured by the radiation pressure on beads of convenient size. The absolute intensity in the plane wave was also calculated from the driving potential and the measured mechanical Q of the crystal, and reasonable agreement was found with the direct measurement. A gain in intensity by a factor of about 70 was measured where simple diffraction theory predicts 74. For the highest voltages used the extrapolated negative peak pressure was 41 atmospheres. No cavitation was observed.

INTRODUCTION

THE need of an ultrasonic beam of known high intensity in liquids in many fields of research has stimulated investigation in the methods of producing such beams and measuring their intensity. In this paper we wish to present the results obtained by focusing high frequency sound waves in liquids with concave reflectors. Measurements of the acoustic energy density in the sound field by several methods are also discussed.

In much research concerned with the effect of high intensity sounds in liquids the total acoustic power supplied by the source is of secondary importance: the effects depend on the acoustic energy density at the point or region in which the phenomena are observed. Several methods are available for producing such regions of high energy density.

A simple approach to the problem would be to use a quartz crystal with a driving voltage which produces a resonant displacement amplitude just a little below the breaking strength. Difficulties arise because of the high voltages that are necessary to produce the theoretically maximum displacements. These might be avoided by using sources with a large mechanical sharpness of resonance (high Q). This could be done if the liquid into which the crystal radiates is itself a resonant column tuned to the same frequency as the sound source. The over-all mechanical Q of such a system can be made much higher[1] than that of a quartz crystal radiating into a semi-infinite liquid medium. For the latter a Q of 10 is typical while a Q of 5000 is not unusual for the double resonator combination. However, long before the elastic limit of the crystal is reached the solid surface pulls away from the liquid in the contraction part of the cycle. In that case the efficiency of the quartz as a sound source falls off rapidly as the particle displacement in the liquid is now considerably smaller than the displacement amplitude of the crystal surface. Thus in practice the limiting intensities[2] are determined by cavitation at the solid-liquid interface rather than by the electrical and mechanical properties of the crystal. An increase in hydrostatic pressure[3] can delay the onset of cavitation until somewhat higher energy densities are achieved.

The present work was undertaken to obtain a high intensity sound beam for investigating the cavitation process within a pure liquid. Since cavitation occurs at much lower intensities at boundary surfaces than within the liquid, the sound energy must cross the boundary surfaces at moderate intensity levels and then be concentrated by some means inside the liquid under investigation.

At frequencies above one megacycle there are two simple ways to concentrate the energy inside the liquid. In this frequency region the sound wavelength is of the order of magnitude of a millimeter so that one can use sound sources or reflectors that have dimensions much larger than the wave-length and familiar optical formulas can be applied to predict the behavior of the ultrasonic beam. This has recently been verified[4–7] for the case of a focusing quartz crystal radiator; however, Labaw[8] had concluded from his experimental work that "a slightly curved crystal gives a greater output (than a plane

* This research was aided by the ONR Contract N6 onr-255.
[1] F. E. Fox and George D. Rock, Proc. Inst. Radio Eng. 30, 29–33 (1942).
[2] Epstein, Andersen, and Harden, J. Acous. Soc. Am. 19, 248–253 (1947).
[3] Briggs, Johnson, and Mason, J. Acous. Soc. Am. 19, 664–677 (1947).
[4] G. W. Willard, J. Acous. Soc. Am. 19, 773 (1947).
[5] G. W. Willard, J. Acous. Soc. Am. 20, 589 (1948).
[6] J. F. Muller and G. W. Willard, J. Acous. Soc. Am. 20, 589 (1948).
[7] Louis Fein, J. Acous. Soc. Am. 20, 583 (1948).
[8] L. W. Labaw, J. Acous. Soc. Am. 16, 237–245 (1945).

one) . . . but that this advantage is owing primarily to increased amplitude of vibration for a given impressed voltage rather than to a confinement of the energy output within a smaller angle." Labaw used x cut quartz crystals of constant thickness, and it is probable that his crystals were poor radiators. In curved crystals those sections having a normal to the radiating surface that is not along the x axis have different elastic and piezoelectric moduli. These sections have, therefore, both frequency and radiation characteristics that differ from those of true x cut sections. By grinding the convex side so that the thickness is proportional to the square root of the elastic modulus in the direction of the interior normal,[5] one can arrange it so that all segments of the radiator have the same resonance frequency, but there is no way to compensate for the decreased efficiency due to the change in the piezoelectric modulus.

However, a beam of plane waves can be generated in, or transmitted into, the liquid at energy densities far below the cavitation level and a concave reflector can be used to concentrate most of the energy in a very small region near the focal point of the reflector. The interference of the incident and reflected waves produces energy densities at the reflector surface that are at most four times that in the incident beam so cavitation at the reflector surface can be avoided. In the focal region, however, the energy density may be several orders of magnitude higher than in the incident plane wave.

In the comparable optical case it is well known that 84 percent of the light collected by the circular objective of a telescope is concentrated in the zero-order diffraction image in the focal region. The radius of the zero-order image is given by the radius of the first diffraction minimum which falls at

$$r = 1.22 f\lambda/d, \qquad (1)$$

where f is the focal length, d is the diameter of the objective, and λ is the wave-length of the incident light. In the ultrasonic case d will usually be the diameter of the incident beam of plane waves which will ordinarily be smaller than the reflector. It is useful to define the intensity gain, g, of a focusing device as

$$g = I_f/I_i \qquad (2)$$

where I_f is the intensity at a point in the focal region, and I_i is the intensity of the plane wave. To give a numerical example, assume that an incident beam of plane ultrasonic waves in water has a radius of 1 cm, a frequency of 10 mc, and strikes a concave reflector of 3 cm focal length, being incident along the principal axis of the reflector. The diameter of the first diffraction ring is 0.55 mm, the ratio of the area of the incident beam to the area of the first diffraction image is 1300, and since 84 percent of the total energy is concentrated in the zero order, the average intensity gain in the zero-order image is 1100 if the coefficient of reflection is taken as 1.00. This is not the maximum intensity gain in the focal region since the intensity distribution has a maximum in the center of the spot. The maximum intensity gain[9] for a circular beam is $(\pi d^2/f\lambda)^2$, and in the case discussed here is close to 5000. To attempt to secure an equivalent gain with a curved crystal source one would have to use a crystal with a radius of curvature of 3 cm and a diameter of 2 cm so that the edge segments have normals inclined almost 20 degrees to the x axis and would be less efficient radiators than the central portions of the source.

Experimental measurements yield average values of the intensity, and average intensity gains depend on the intensity distribution in the region occupied by the measuring device. For this reason, simple diffraction theory was used to calculate the total energy passing through areas of selected size and shape at the focal point of the reflector in a plane parallel to the incident wave front when the incident beam is either circular or rectangular. The corresponding theoretical average and maximum gains in intensity are given elsewhere[9] together with other calculations that enable one to interpret experiments in which intensity distribution is measured instead of average intensities.

EXPERIMENTAL METHODS

In order to test the extent to which one may depend upon simple diffraction theory to predict the intensity gain obtainable by using curved reflectors for sound waves of high frequency, four

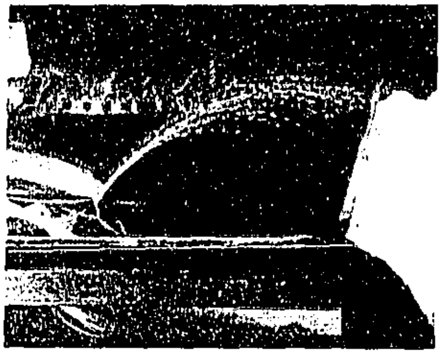


FIG. 1. The ultrasonic fountain. A stream of air is blowing the fog away from the camera. Note the "string of beads" effect. Exposure = 0.1 sec.; frequency = 4.25 mc. 1000 volts (peak) on crystal; focal length of reflector = 34 mm.

[9] Virginia Griffing and Francis E. Fox, J. Acous Soc. Am. 21, 348 (1949).

TABLE I. Ultrasonic fountain data: ejection velocity and remarks concerning fog formation.

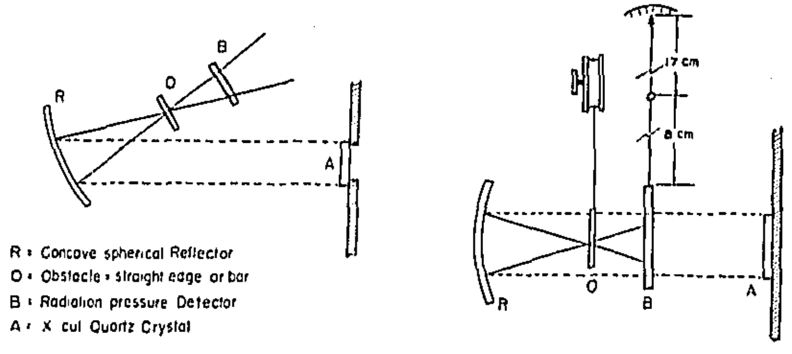
Distance from water surface to focus (mm)	Velocity of drops (cm/sec.)	Remarks concerning fog formation
4	88	none
2	139	slight
1	167	heavy
0	177	maximum

types of experiments were performed. The first consisted of observations made upon the "ultrasonic fountain" produced by a concentrated sound beam. These were largely qualitative but yield some idea of the gain experimentally obtainable. In the next two methods the sound waves were allowed to fall upon a radiation pressure indicator after diverging beyond the focal region; measurements were then made of the screening effect of various obstacles placed in the focal region. Finally the energy density of the sound in the focal region was measured directly by the radiation pressure on a small steel bead, and in the plane wave by pressure on a larger glass bead, so that the average intensity gains for the area covered by the small bead were measured directly.

All measurements were made with an x cut crystal 1 inch square having its fundamental resonant frequency at 4.25 megacycles. This was at one end of a small tank with the dimensions $15\times 8\times 8$ inches. One side of the crystal was in contact with the water in the tank, and the outer face of the crystal was exposed to the air. The high voltage electrode, a rectangle 15 mm long and 12 mm high, was on the latter side and did not cover the entire crystal. It is assumed that the radiating area in the tank is the same as the area of the high voltage electrode. The end of the tank facing the crystal was covered with a ½-inch thick slab of ρc rubber.*

I. The Ultrasonic Fountain

The tank was provided with a plane reflector, close to the crystal and tilted so that the reflected beam strikes the water surface approximately normally. Sufficient power was used to raise a small mound of water having approximately the area of the back electrode of the quartz, and about 1 mm high. The voltage on the crystal (1000 volts peak) was then maintained constant throughout the observations. A watch glass serving as a concave spherical reflector with a focal length f of 34 mm and diameter 36 mm (therefore, larger than the sound beam) was substituted for the plane reflector. The distance (y) between the air-water interface and the reflector was varied by changing the depth of the water in the tank. When y is approximately $2f$ the mound raised by radiation pressure is of the same general nature as that observed for the plane reflector, although somewhat elongated along the line in the plane containing the incident and reflected beam. As y is decreased, the mound height increases and the area decreases. As y approaches f the mound rises so high that the water is ejected in what first appears as a thick arched cylinder and finally as a very thin stream that rises 10 to 15 cm out of the water. At a critical level a cloud of fine fog is ejected together with the



(a) Top view. (b) Side view.
A—Sound source, X cut quartz crystal; B—Radiation pressure detector;
O—Obstacle, straight edge or rod; R—Concave spherical reflector.

FIG. 2. Experimental arrangement for screening measurements.

* At these frequencies the absorption in ρc rubber is very high so that all but a negligible fraction of a sound wave is absorbed in passing through the slab twice. Although the ρc match with water is not as good at these frequencies as it is in the 20-kc frequency region, the reflection is still small and is less than 5 percent at normal incidence for the frequency used in this work.

drops that form the arched stream. Wood and Loomis[10] have described similar phenomena. In their work, however, the power used was much greater than that used in the present work. They mention, for example, an applied potential of 50,000 volts at a frequency of 0.3 megacycle.

From observations upon the rate at which the fog droplets fall it was possible to estimate the droplet radius. All components of the cloud do not settle at the same rate but a fair fraction of the droplets can be observed to fall in air with limiting velocities less than 1 cm per sec. This gives a radius of less than 10^{-3} cm for the droplets.

A photograph of the phenomena observed near the critical value of y is reproduced in Fig. 1. The exposure time is 0.1 sec. A stream of air is blowing the fog away from the camera. The "string of beads" effect was at first thought to be due to drops ejected successively, but photographs made with different exposure times reveal that the apparently distinct drops in a chain are really photographs of the same drop in different positions. This is shown by the fact that the number of beads in each chain is proportional to the exposure time, and the length of each chain is approximately equal to the distance traveled in each exposure by a single drop. The velocity of the drop was obtained from measurements on the height and range of the stream of drops. Apparently, the drop is ejected as an ellipsoid with major axis along the stream. Surface tension then contracts the ellipsoid to a sphere (equilibrium shape), and the inertia of the mass from the ends of the ellipsoid curves it in beyond the spherical radius to form an oblate spheroid with its symmetry axis along the stream. The drop oscillates between these two extreme shapes as it moves in the stream, and the photographs show the drop in the positions where it has the shape which reflects the maximum amount of light back into the camera. Table I gives the velocity of the drops in the stream when the water surface is near the focus.

One can use two methods to get the approximate intensity gain: (a) By using the cross section of the stream when the beam is focused at the surface, (2 sq. mm), and comparing with the cross section of the parallel beam (180 sq. mm), one gets a gain of 90 if one assumes that all the energy of the original beam has been concentrated in the area over which the fountain is raised. From theoretical considerations[9] one finds an average intensity gain of 46 over the zero-order image. (b) The height of the mound raised by the unfocused beam (1 mm) is compared to that of the column at the critical focusing (100 mm). By assuming that the height

TABLE II. Screening efficiency of cylindrical rods (steel drills).

Drill	Diameter 2a mm	Deflection without obstacle mm	Min. deflection with obstacle mm	Screening ratio Experimental	Screening ratio Calculated
No. 53	1.51	13.2	3.0	77%	88%
No. 60	1.01	22.9	12.3	69%	77%

of the water raised is proportional to the radiation pressure one again finds a gain in intensity of about 100.

In an attempt to get absolute sound intensities, one determines first the intensity of the parallel beam when the potential applied to the quartz crystal is 1000 volts (peak): Two methods are available: radiation pressure measurements with the bead (Section III) and calculation from the potential and the Q of the crystal (Section IV). The former gives 1.8 and the latter 2.5 watts/cm², or a radiation pressure of (90 to 250) $(10^7/1.5\times10^5)$ $=(0.6 \text{ to } 1.7)\times10^4$ dynes/cm², if one uses 1.5×10^5 cm/sec, as the sound velocity in water.

The maximum velocity of the drops forming the fountain corresponds to a hydrostatic pressure of

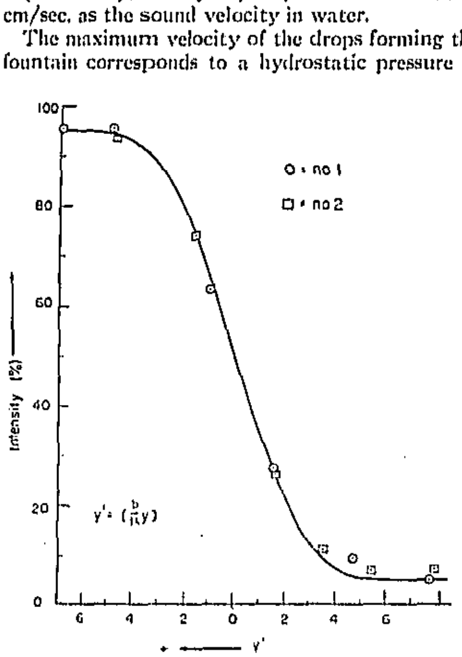


FIG. 3. Screening effect of a plate as a function of the vertical position of the front bottom straight edge. The geometrical focal point is at $y=0$ and the abscissas are given in units $y'=y(b/f\lambda)$ where y is vertical displacement in mm. The points give the deflection of the detector in percentage of the maximum deflection when the plate is removed. The solid line is the theoretical curve for the percentage of the total intensity that gets past the plate as its position is varied.

[10] R. W. Wood and L. Loomis, Phil. Mag. S7 4(2), 417–436 (1927).

1.5×10^4 dynes/cm², in good agreement with the preceding calculation from the intensity gain.**

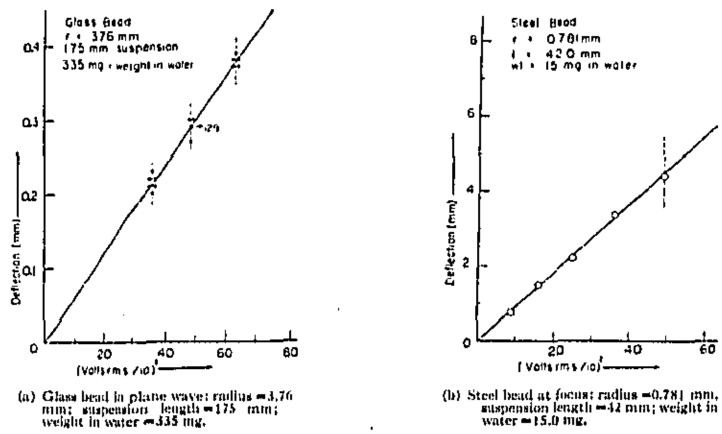
II. Intensity Distribution: Screening Methods

In order to obtain quantitative information on the distribution of intensity in the focal region of the mirror, the measurements were made of the screening efficiency of obstacles placed in the focal region. The experimental arrangement is shown in Fig. 2. The sound strikes the reflector, is focused in the neighborhood of 0, and falls on a radiation pressure detector (B) which is large enough to intercept all but a negligible fraction of the (diverging) sound beam. The radiation pressure detector was made of a slab of ρc rubber 3 cm square and about 4 mm thick on the end of a light stiff rod 25 cm long. This rod was weighted at the bottom and provided with a small cross bar 17 cm from the top which served as a fulcrum around which the rod could rotate. The deflection of the pointed top was read on a scale fixed to the supports on which the cross bar rotates. The sensitivity of the detector could be changed by varying the weights on the bottom of the rod. No attempt was made to calibrate this detector in terms of absolute intensity since it was used only to determine relative intensities. For the small angles used the deflection is proportional to the sound intensity.

In all measurements the detector was shielded from any pressure that might arise from a unidirectional mass flow of the liquid. An acoustically transparent screen of plastic material (commercially available as "Bub-o-loon"—for making plastic toy balloons) was prepared by blowing a balloon with as thin a wall as desired. After this had set, a piece of it was mounted on a wire frame and tested for transmission. Nearly all films thus prepared caused no measurable change in the maximum deflection of the detector after the mass motion had been eliminated by one such screen placed near the detector.

Consider a rectangular coordinate system with the origin at the principal focus of the mirror, the z axis in the direction of the principal axis of the reflector, and the y axis vertical. The obstacle was then attached to a double micrometer that permitted one to make small displacements in the x and y direction. The z position was varied by moving the stand to which the micrometer was attached. The obstacles used first were cylindrical rods (smooth shanks of steel drills). These were placed with the cylinder axis along the x direction and near the image of the sound source. The y and z positions were varied until the deflection of the detector was a minimum. The deflection was observed for each rod and compared with the deflection obtained when the rod was removed from the field between the reflector and the detector. The results are given in Table II.

In order to interpret these results we calculated[9] the fraction of the total intensity in the sound beam striking a strip in the xy plane at $z=0$ for which x varies from $+\infty$ to $-\infty$ and y varies from $+a$ to $-a$ (a is the radius of the rod). Since $2\pi a/\lambda$ is



(a) Glass bead in plane wave: radius =3.76 mm; suspension length =175 mm; weight in water =335 mg.

(b) Steel bead at focus: radius =0.781 mm, suspension length =42 mm; weight in water =15.0 mg.

FIG. 4. Variation of the radiation pressure on a spherical bead as a function of the square of the voltage on the sound source. The abscissa unit is (0.1 r.m.s. voltage)². The ordinates give the measured deflection (in mm) of the bead.

** The size of the fog droplet leads to an excess pressure within the droplet of 1.4×10^5 dynes/cm² because of surface tension. This is 10 times the value mentioned above. At first sight, one might have assumed that the radiation pressure must be at least as high as the excess pressure in the droplets.

large we neglect the forward scattering. The experimental and calculated values are given in columns 5 and 6 of Table II.

Another set of screening measurements were made in which a rectangular slab of ρc rubber 4 cm×3 cm cemented on a 1 mm brass plate of the same area was used as a screen. The deflections were observed as a function of the position of the bottom straight edge of the rubber slab. The front edge of the slab was placed directly above the focal region which had been previously located by adjusting a very small drill for maximum cut-off. The bottom of the straight edge was moved from a position where all the sound reaches the detector to one in which all the sound is cut off. In Fig. 3, the points correspond to deflections measured for various positions of the straight edge. Since the power was not the same in the two sets of readings, all readings are plotted as percentage of the maximum deflection observed when the straight edge is removed. The theoretical curve[9] showing the intensity getting past the straight edge as a function of the straight edge position is the solid line shown in Fig. 3.

III. Direct Intensity Measurements by Radiation Pressure

The most straightforward test of the diffraction theory is a direct measurement of the sound intensity in the focal region. This was done by measuring the radiation pressure*** on a small steel bead. To map the sound field accurately, the bead should be taken as small**** as possible. In practice, a bead with a diameter of 1.562 mm, weighing 17.0 mg in air, has excellent stability when mounted on a bifilar suspension 42 mm long made of single filaments of Nylon thread, especially if the whole suspension is under water so that surface tension effects are avoided. For these measurements the principal axis of the reflector coincided with the sound beam. A short focus telescope, mounted on a micrometer stage which could be moved in the z direction, was used to measure the deflection. The micrometer carrying the telescope was rigidly fastened to the same framework to which the double micrometer was attached. This double micrometer carried the bead suspension system. The position of the support which brought the deflected bead into the region of greatest sound intensity was found by trial and error.

For measurement of the much smaller intensity of the plane wave, a larger glass bead was used. It had a radius of 3.76 mm, a weight in water of 335 mg; the length of the suspension was 175 mm.

*** In all measurements of bead deflection described in the following, the screen mentioned in Section II was used to prevent any effect of mass flow.

**** The size of the zero-order image is 2.0×1.6 mm.

Measurements with the small bead were made between 30 and 70 volts (r.m.s.), on the large bead between 60 and 80 volts. In Figs. 4a and 4b, the deflections are plotted against the square of the voltage. One gets good straight lines which serves at the same time as a check on the standardization of the voltmeter. The values of the deflection given by the curves at 70 volts were used for the further calculations.

The connection between the force on the bead, $\bar{F}$, and the average density of the sound field $\bar{E}$ at the place of the bead is given[11] by

$$\bar{E} = \bar{F}/\pi r^2 Y, \qquad (3)$$

where Y is a complicated function of the densities of the bead and the medium and of the ratio of the bead radius r and the wave-length. It approximates unity for rigid spheres in plane progressive waves if $2\pi r/\lambda$ is large. In our case this quantity was larger than 10 which latter value would make $Y=0.95$.

Equation (3) actually applies to plane waves. It is, however, assumed that it holds in our case also for the average value over the cross section of the bead. A correction must be considered for the convergence of the beam. In the first approximation the force should be multiplied by the average value of the cosine between a ray and the axis. If $2\theta_0$ is the opening of the cone, this gives a factor

$$\cos\theta = \tfrac{1}{2}(1+\cos\theta_0) = \tfrac{1}{2}[1+(1+d^2/4f^2)^{-\frac{1}{2}}].$$

If the square root can be developed, (3) should be multiplied by $1+\frac{1}{4}(d/2f)^2$. Therefore, the intensity calculated from (3) for measurements with the small bead should be used with a factor $1/Y(\cos\theta)_{av} = 1.07$, while measurements with the large bead in the parallel beam involve the factor unity. In this manner, one finds that for 70 volts r.m.s., the intensity in the plane wave is 0.0184 watt/cm² and the average intensity in the focal region is 1.31 watts/cm², so that the average intensity gain is 71. The theoretical value for the gain in intensity is 74 computed for the energy passing through a circle† of radius 0.781 (that of the bead).

Accuracy of Measurements

The data in Fig. 4 allow at the most an uncertainty of 5 percent in choosing a line of best fit. This does not exclude a constant factor in the standardization of the voltmeter, or a systematic

[11] F. E. Fox, J. Acous. Soc. Am. 12, 147–149 (1940).

† See reference 9. The theoretical calculation neglects the contribution of the incident wave, which passes through the focal region before striking the concave mirror. The measured force on the small bead is the difference between the radiation pressure due to the plane incoming wave and that due to the intensity in the converging beam; in addition, a small fraction of the energy in the incident beam is scattered by the bead. Corrections for these two factors would reduce the theoretical intensity gain given here (74) by about 2.

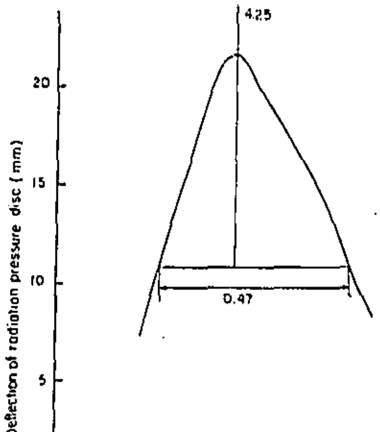


FIG. 5. Mechanical resonance curve of sound source.

error in the application of Eq. (3) to the convergent beam in the focal region. The agreement between calculated and measured values, however, leads us to the assumption that our errors are certainly less than 10 percent.

IV. Intensity Calculated from Applied Voltage

In the following, a method is described which permits a calculation of the plane wave intensity as generated at the crystal face from the voltage applied to the source. This can be done if the mechanical resonance curve of the source is known. It can be shown†† that $I = 0.0179(Qf_rV)^2$ erg/cm²/sec., where V is the r.m.s. voltage applied to an x cut crystal radiating into water at the response frequency of the crystal, f_r is the frequency in megacycles, and Q is the sharpness of resonance (mechanical) of the crystal. In general Q will vary from one sound source to another and will depend upon the acoustic radiation resistance of the fluid into which the crystal is radiating, the type of mounting of the crystal, etc. and should be determined for the sound source as actually used. This can be done by measuring the sound intensity for a constant V as the frequency is varied through resonance. Figure 5 shows the resonance curve of the crystal used in this work. In it the deflection of the ρc rubber radiation pressure indicator is plotted against the frequency in megacycles. The Q is given by $\Delta f/f_r$ where f_r is the frequency at which the deflection is maximum and Δf is the difference between the two frequencies for which the deflection is one-half the maximum. The value of $Q=9$ thus obtained can be used to compute the intensity. The calculated value of the sound intensity I_0 at the crystal face is found to be 0.0249 watt/cm² when the driving voltage is 70 volts r.m.s. The intensity calculated from the bead deflection was 0.0184 watt/cm².

†† See appendix.

CAVITATION

The ultimate purpose of the experiments described here is the development and study of equipment for the production of high negative pressure in liquids and to study the appearance of cavitation as a function of tension, frequency, and other variables.

If the highest voltage available (1200 peak volts) is used, the negative peak pressure, calculated from the extrapolated curve 4b and the intensity gain according to diffraction theory,††† is 41 atmospheres in the focal region. No cavitation was observed under these conditions (4.25 mc). For higher frequencies one should get a smaller focal region and even higher gains.

Systematic investigations of cavitation at high frequencies are planned.

ACKNOWLEDGMENTS

The authors wish to thank Professor K. F. Herzfeld for many helpful suggestions, and Miss Laura Cheng for assistance in the preparation of the drawings.

APPENDIX

Absolute Sound Intensities in Liquids as a Function of Applied Voltage and Mechanical Q of Source

Consider an x cut quartz crystal with thickness x small compared to other dimensions, having electrodes of negligible mass on the radiating faces, one of which is in contact with a liquid while the other is exposed to air. We use the following notation:

ρ, ρ_0 = density of quartz and liquid respectively,
c, c_0 = velocity of sound in quartz and liquid,
ξ = displacement amplitude of crystal face in contact with liquid,
x = thickness of x cut crystal (along the x axis),
$\omega = 2\pi f$, where f is the frequency of the applied potential V,
V = potential amplitude (peak) applied to electrode,
e_{11} = piezoelectric stress constant relating a field parallel to x to the compressional stress parallel to x,
d_{11} = piezoelectric strain constant,
q = stiffness constant of quartz = ρc^2.

Cady[11] has shown that if one assumes the energy carried away by the air is negligible compared to that radiated into

††† Intensity in the plane wave 2.7 watts/cm², a maximum gain in the center 213, maximum intensity 575 watts/cm².

[11] W. G. Cady, Report (declassified) No. 17, *Piezoelectric and Ultrasonic Phenomena in the Ultrasonic Trainer* (Massachusetts Institute of Technology, September 30, 1945), prepared under OSRD Contract OEMsr 262, sub-contract DIC 178188 with Radiation Laboratory, M.I.T.

the liquid

$$\xi^2=(e_{11}V/\omega x\rho c)^2(1-\cos\beta)^2/(1-(1-m^2)\cos^2\beta), \quad \text{(A.1)}$$

where $m=\rho_0c_0/\rho c$ and $\beta=2\pi x/\lambda=\omega x/c$. In what follows we consider the frequency to be varying in a small range about the resonant frequency ω_h of the h (odd) harmonic of the crystal. Following Cady one writes

$$\omega x/c=(\omega_h+\Delta\omega)x/c=h\pi+h\pi(\Delta\omega/\omega_h)=h\pi+r, \quad \text{(A.2)}$$

where

$$r=h\pi\Delta\omega/\omega_h, \text{ and } h=1, 3, 5 \cdots. \quad \text{(A.3)}$$

We write $\cos\beta=-\cos r$, expand $\cos r$, retain only the terms in r^2 since r is small near resonance, and find

$$\xi^2\simeq(e_{11}V/2\omega_h\rho cx)^2((4-r^2)^2/m^2+r^2), \quad \text{(A.4)}$$

while at resonance

$$\xi_m^2\simeq(2e_{11}V/\omega_h\rho cxm)^2. \quad \text{(A.5)}$$

One finds the value of r for which $\xi^2=\frac{1}{2}\xi_m^2$, after neglecting r^4 terms

$$(h\pi\Delta\omega/\omega_h)^2=r^2=m^2/(1-m^2). \quad \text{(A.6)}$$

Let $\omega_h/2\Delta\omega=Q_h$, the mechanical sharpness of resonance obtained by measuring the difference between the two values of ω that make $\xi^2=\frac{1}{2}\xi_m^2$. If energy leaves the crystal only by radiation into the liquid, as is assumed here, $(h\pi/Q_h)$ is independent of h so that Q_h is directly proportional to h.

From (A.6)

$$1/m^2=(1/r^2)-1=(4Q_h^2/h^2\pi^2)-1, \quad \text{(A.7)}$$

and we can write the resonance amplitude in (A.5) in terms of the measured Q_h. Thus

$$\xi_m^2\simeq\left(\frac{4e_{11}VQ_h}{\omega_h\rho cx\pi h}\right)^2\left(1-\frac{h^2\pi^2}{4Q_h^2}\right)\simeq\left(\frac{4d_{11}VQ_h}{\pi^2h^4}\right)\left(1-\frac{h^2\pi^2}{4Q_h^2}\right) \quad \text{(A.8)††††}$$

According to the theory, the value of Q is independent of the area of the quartz crystal providing the whole vibrating area radiates sound. If part of the front surface is masked, there is less radiation loss and the measured Q should be higher than the theoretical one.

On the other hand, the measured Q also contains the losses of the quartz due to its mounting, which cannot be measured independently in our case and result in a lower Q.

For this reason we have not used the theoretical formula $Q=h\pi/2m$ but have measured it using the width of the resonance curve. For the fundamental the measured value of $Q=9$ is appreciably below the theoretically calculated value of 16.

In most liquids Q for the fundamental resonance frequency is larger than 10 so that $(h^2\pi^2/4Q_h^2)$ is less than 0.03. Since the intensity measurements are seldom more accurate than 3 percent we omit this term in what follows.

The power radiated into the liquid is

$$I\simeq32\rho_0c_0(e_{11}/\rho c^2\pi h)^2(VQ_hf_h)^2, \quad \text{(A.9)}$$

where f_h is the frequency of the h harmonic.

For the fundamental‡ we have

$$I=12.2(10^{-20})\rho_0c_0Q^2f^2V^2 \text{ erg/cm}^2\text{/sec}, \quad \text{(A.10)}$$

In particular, for a crystal radiating at its fundamental resonance frequency into water

$$I=0.0179Q^2V^2f^2 \text{ erg/cm}^2\text{/sec}., \quad \text{(A.11)}$$

where f is in megacycles, V is the applied peak voltage, and Q is the measured sharpness of resonance for the fundamental.

†††† For the fundamental, Eq. (A.8) gives $\xi_m\simeq(4/\pi^2)d_{11}VQ\simeq(8/\pi^2)\xi_0Q$ where ξ_0 is the static deformation $(1/2)d_{11}V$ due to an applied voltage V. In a system with a single degree of freedom one would expect $\xi_m=\xi_0Q$ where Q is defined for the single resonant frequency. The factor $(8/\pi^2)$ appearing here is a consequence of the infinite number of degrees of freedom which permits an infinite number of resonance frequencies each with its associated Q.

‡ In the calculation of the numerical constants we have used the following values given by Cady (see reference 12): $e_{11}=5.2\times10^4$; $\rho c=1.54\times10^6$; $c=5.8\times10^5$. For c_0 we have however used 1.47×10^5. All values are in c.g.s. units. If one uses values of d_{11} found in the literature, e.g. in Cady (*Piezoelectricity*, p. 219, McGraw Hill, N. Y., 1946), the constants in Eqs. (10) and (11) may be increased by as much as 18 percent.

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Focusing Ultrasonic Radiators

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Piezoelectric ultrasonic radiators made in the form of a thin spherical shell radiate spherical sound waves which come to a focus at the center of curvature of the shell, thus enabling the production of much greater ultrasonic intensity in a small locality removed from the radiator than it is possible to obtain directly at the surface of a radiator. It is here shown by ultrasonic light diffraction pictures of the radiated sound field that the sharpness of focus is limited by wave diffraction in the manner well known in astronomical telescopes and may be calculated by optical diffraction formulas. By the same means the radiation efficiency of different areas of the curved surface is explored and the results compared with theory. The variation of efficiency is, of course, due to the variation of the effective elastic and piezoelectric constants of the differently oriented areas. Calculations are made of the radiation efficiency of a quartz radiator, and it is shown that a greatly improved focusing spherical radiator may be obtained by varying the thickness of the radiator to compensate for the varying frequency constant. Further, superior focusing cylindrical radiators may be obtained by special orientation or by thickness shaping or both.

I. INTRODUCTION

IN 1935 J. Greutzmacher[1] proposed a form of piezoelectric quartz ultrasonic radiator which by its own focusing action may produce an intensity of ultrasonic energy at its focus that is much greater than that at the radiator surface itself. Thus it is possible to obtain energy concentration without auxiliary lenses or reflectors and their concomitant energy losses. Since the region of high intensity is localized in the medium at a distance from the radiator it is easier to make use of the energy for destructive, exploratory or other purposes.

The form of the focusing radiator was that of a spherical shell, or more specifically, a concavo-convex lens of constant thickness, the two spherical surfaces having a common center of curvature. When made of piezoelectric quartz the radiator axis is preferably made to coincide with an x-crystallographic axis of the quartz. This may be designated as an x cut quartz focusing radiator (in tourmaline the z cut would be used). With electrode platings covering the two spherical surfaces and an applied alternating voltage of proper frequency the radiator thickness increases and decreases in phase all over the radiator, and hence radiates *spherical* sound waves which come to a focus at the center of curvature of the spherical surfaces. On the other hand a *plane* x cut quartz radiator, radiates *plane* sound waves which do not focus, except by the use of an auxiliary lens or curved reflector.

An element of area at the center of the focusing x cut radiator is truly x cut and hence has the same effective elastic and piezoelectric constants (or frequency constant and electro-mechanical coupling) as a plane x cut radiator. However, off-center areas, x′ cut, being of increasingly different orientation as they recede from the center have, in general, increasingly different constants than the center. In fact, if the radiator diameter were equal to its radius of curvature the x′ cut peripheral areas would be 30° off from x cut, and two such areas (diametrically opposite) would actually be true y cut surfaces, due to the trigonal symmetry of quartz. These regions would then radiate with zero amplitude due to lack of electromechanical coupling. Usually, however, radiators are made with half-angular-aperture considerably less than 30°, and reduction of coupling is not a major factor.

Actually a more serious reduction of efficiency for off-center regions is caused by the varying elastic constants and the correspondingly varying frequency constant. This applies to the usual usage wherein the energy is radiated into a non-metallic liquid (where the impedance mismatch between radiator and medium is about 10 to 1). Since the frequency constant varies over the surface of the radiator and the radiator is of constant thickness, the resonant frequencies of different regions are different. Thus when the radiator is operated at the resonant frequency of the center, outer regions of either higher or lower resonant frequency vibrate with reduced amplitude. This reduction of efficiency becomes serious beyond a half-angular-aperture of 15° when operating the radiator in the fundamental mode, and beyond even lesser apertures when operating in harmonic modes (as is commonly done).

As previously reported at meetings of the Acoustical Society,[2,3] the x cut quartz focusing radiator

[1] J. Greutzmacher, Zeits. f. Physik 96, 342 (1935). Or see pp. 25–26 of reference (4).

[2] G. W. Willard, J. Acous. Soc. Am. 19, 733(A) (1947); 20, 589(A) (1948).

[3] J. F. Muller and G. W. Willard, J. Acous. Soc. Am. 20, 589(A) (1948).

has been critically examined for radiation characteristics and concentrating power. When operated with water on the concave side only (air on the convex side), with an input energy of 14 watts/cm² on the effective area of 6.4 cm², the concentration of energy at the focus is such as to give an intensity of over 5 kw/cm² over a circular area less than one mm in diameter. Such high ultrasonic intensities give interesting heating, fog, and fountain effects. However no cavitation could be produced except by auxiliary means which involved the production of standing waves or the use of a circulating reverse water current, to cancel the normally induced circulation currents. These effects of high intensity, megacycle, ultrasonic energy are being further explored and will be described at a later date.

The present paper will give a detailed analysis of the radiation characteristics of the focusing radiator, as determined both experimentally and theoretically, and a discussion of means of obtaining improved focusing radiators.

II. EXPERIMENTAL SET-UP

A. Optical System for Recording Radiation Patterns

By making use of the well-known light-diffraction properties of ultrasonic waves[4] it is possible to obtain beautiful pictures of the radiation characteristics of ultrasonic radiators. The optical system thereof is shown in Fig. 1. An AH-4, 100-watt mercury lamp with a pair of condenser lenses illuminates the pinhole aperture S_1. The 4½-inch diameter, 11-inch focus lenses L_1 and L_2* first colimate the light through the tank and then refocus it onto the pinhead aperture S_2. It is in the plane of S_2 that the light diffraction spectra are obtained. For 5-Mc sound waves in water the angular separation of spectral order is about one-five hundredth of a radian, thus giving a suitable diameter for the S_1 and S_2 apertures of about 0.022 inch. Lens L_3 focuses the center plane of the cell onto the screen or film, thus forming a picture of the sound beam (in the present case in one-to-one size). Since the pinhead aperture S_2 stops all the undiffracted light (that not passing through a sound field) the picture background is dark, with the sound beam appearing bright. Use of a pinhole at S_2 would have given a negative rendition. This is not recommended, since the effective area of S_2 is then so reduced as to give insufficient picture resolution to obtain fine detail.

The same disadvantage applies also to a reversed positioning of the pinhole and pinhead apertures.

The open-top tank (10×2 inches in the horizontal plane) has cemented plate glass windows W with the radiator-mount gasketed in one end. A sound absorbing pad of compressed wool or of "Rho C" rubber in the opposite end prevents the reflection of sound waves. The tank may be moved, parallel to its length, through the optical systems in order to view the sound field at different distances from the radiator, and is rotatable by a small angle about a vertical axis through C_1 for alignment of sound wave fronts with the optical system.

The light intensities obtained were entirely sufficient for direct viewing with either a transmission screen** or a reflection screen. For taking pictures the light intensity was reduced 200-fold by using a double set of mercury green-line glass filters, permitting simple 5- to 10-second exposures on Kodak SS Ortho Portrait film. This also improved the optics. The writer's pictures in "Ultra-Sound Waves Made Visible"[5] were also obtained with the above arrangement.

B. Focusing Radiator and Mount

The x cut quartz focusing radiator was ground in the form of a concavo-convex lens with a concave radius of curvature of 63.5 mm (2½ inches), a constant thickness of 0.572 mm (22.5 mils) to operate at 5 Mc, a convex radius of curvature equal to the sum of the concave radius and the thickness, and a diameter of 38.1 mm (1½ inches), the axis of the radiator being aligned with an x crystallographic axis of the quartz. An axial cross section view of the radiator and mount is shown in Fig. 2, Q being the quartz radiator, and C the common center of the two spherical surfaces (as well as the theoretical focal point). The full *concave* surface of the radiator was metallized with gold by evaporation, to form the inner electrode. The radiator was cemented into the cylindrical mount with electrically con-

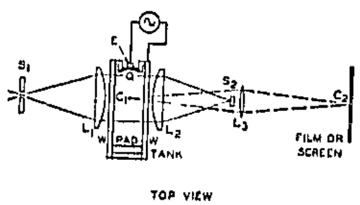

FIG. 1. The optical system for obtaining radiation patterns.

[4] L. Bergman-H. S. Hatfield, *Ultrasonics* (John Wiley and Sons, Inc., New York, 1939).

* These lenses were specially figured single achromats. A better lens for this purpose is the several element, Petzval type, as is often used for movie projectors. These lenses and the outside surfaces of the tank windows should be optically coated to cut down multiple reflections.

** An excellent screen may be made of thin plastic (say cellulose acetate, 15 to 25 mils thick) with a fine sand blast surface on both sides. The increased light diffusion from blasting both sides, gives better off-axis light intensities.

[5] G. W. Willard, Bell Lab. Record, XXV 5, 194–200 (1947).

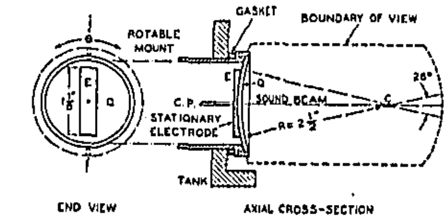

FIG. 2. The radiator mount, and field of view for radiation pattern figures.

ducting cement,*** thus making the inner electrode electrically continuous with the grounded tank. The radiator mount is gasketed into the end of the tank, with spring tension rings (not shown) on the outside for compressing the gasket. Radial holes in the rim of the mount permit it to be rotated with a spanner wrench.

The *convex* surface of the radiator is uncoated, the outside electrode being provided by lightly springing against this surface an appropriately shaped aluminum block. For the production of high energy sound beams, a 28.6 mm ($1\frac{1}{8}$ inch) diameter electrode is used, giving a half angular aperture of 13° for the sound field. For exploring the *nature* of the radiate sound field, the electrode is restricted to a rectangular area 28.6 mm×5.5 mm (end view of Fig. 2), and is held by guides (not shown) so that its length is always vertical. In either case, of course, the sound is radiated only from the area of the radiator which is covered by electrodes on both sides, i.e., the region covered by the external restricted electrode. Thus with the rotatable mount it is possible to radiate sound only from this strip area of the radiator which may be parallel to the XY plane of the radiator, the XZ plane, or any intermediate planes. The axis of the optical system is always normal to the length of the external electrode. The angular position of the mount, and hence of the crystallographic plane being explored, will be specified by the angle θ, which has the value zero for the XZ plane, ±90 degrees for the YZ plane, and intermediate values for intermediate XZ' planes (see Fig. 3). The sense of θ is such that θ is positive for a plane parallel to the crystallographic *minor cap face plane* ($\theta=+38°13'$). By this specification the major faces of the well known AT and CT quartz oscillators lie in positive planes. The boundary of the picture-views, as restricted by the top and bottom of the tank windows and by the edges of lenses L_1 and L_2, is shown in Fig. 2.

A word may be added about the width of the sound beam in the direction of the optical system. This is 5.5 mm at the radiator, becoming less toward the focus. As the author has shown,[6] the allowable width of plane waves for good light valving, and hence picture formation, may be as great as 36 mm for 5-Mc waves in water. The curvature of the present spherical waves may be shown to be sufficiently small for the narrow width used that fairly quantitative rendition of sound intensities should be obtained. When this radiator is operated at its third harmonic (with the same electrode) the valving is not as good, weaker pictures are obtained, and the intensity rendition should be less accurate. In any case, without extensive special controls and analysis it is impossible for the sound pictures to give a truly quantitative measure of the sound intensity all over the field. However, as will be seen in the accompanying pictures the intensity distribution is quite satisfactory for verifying the features discovered theoretically.

III. OBSERVED RADIATION CHARACTERISTICS

The ease of obtaining radiation pattern pictures under varying conditions of voltage, frequency and orientation encourages the accumulation of many more pictures than are necessary for good description of the radiator properties. Those shown here (Figs. 4 to 8) are selected to best show the special features of focusing quality and of radiation efficiency variations. Only two planes, $\theta=+38°$ and $\theta=-22°$ are included in the showing. These show typical effects in planes for which the effects are extreme. The over-all operation of a radiator with a full circular electrode is best determined by other means, because of the complications of optically analyzing a circular beam.

As previously noted the boundary of each view

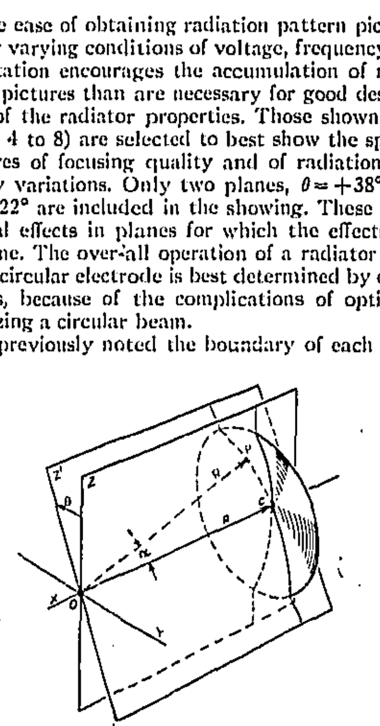

FIG. 3. The location of the point P on the surface of the radiator is defined by angles α and θ. X, Y, Z are crystallographic axes of the radiator.

*** For example: Conductive Silver Paint No. 4817, E. I. Du Pont de Nemours & Company, Electrochemical Department, Perth Amboy, New Jersey.

[6] G. W. Willard, J. Acous. Soc. Am. 21, 101 (Mar., 1949).

is as given in Fig. 2. The views, before reproduction, were 82 mm long in the horizontal direction and this was likewise the length of sound field covered. The center of the curved radiator was, of course, to the left of the left edge of the view (about 2 mm). The height of the external electrodes, and hence of the beam on leaving the radiator, was 28.6 mm, and the external diameter of the mount which shows in the views was 44.5 mm.

A. Sharpness of Focus

The radiation patterns of Figs. 4 and 5 have been chosen to show the degree of acoustic focusing and to show the effects of sound-wave diffraction on the sharpness of focus. Both figures are for the $\theta=-22°$ plane, which is approximately the plane of most uniform surface radiation. For Fig. 4 the radiator was operated near its fundamental frequency of 5 Mc, while for Fig. 5 the third-harmonic, 15-Mc mode was used. The lower view in each case corresponds to radiator excitation at a voltage sufficiently high to produce great enhancement of the weaker portions of the field. The radiator (which is to the left of the left edge of the figures) focuses at its center of curvature, within the accuracy of measurement. The strong core and weaker side-lobe diffraction pattern in the focal plane is similar to a cross section of the focal pattern of the astronomical telescope (strong core and successively weaker concentric rings). The sharpness of focus is thrice as sharp at 15 Mc as at 5 Mc, corresponding to increasing sharpness with decreasing wave-length in the optical case.

The sharpness of focus of the spherical radiator may be calculated from the optical formulas for diffraction of converging spherical waves passing through an aperture. In the present case, where the radiator has a *rectangular* electrode, the aperture is to be taken as rectangular of height h (and breadth b). If we consider the diffraction pattern only in the focal plane, parallel to the radiator and at a distance R therefrom, and only the axial cross section parallel to h, then the intensity distribution as given by standard optical texts is $I/I_0=(\sin\alpha/\alpha)^2$ where $\alpha=\pi hz/\lambda R$, and z is the distance from the axis for which the intensity is I.

Of particular interest are the distances for which $I/I_0=0$, which are given by $\alpha=\pi,\ 2\pi,\ldots n\pi$. For $h=28.6$ mm, $R=63.5$ mm, $\lambda=v/\text{frequency}$, and $v=1.5\times10^5$ cm/sec.,

$$2z_n=\frac{n(2\lambda R)}{(h)}=1.333n\ \text{(in mm)},\quad \text{for 5 Mc.}\qquad \text{(III.1)}$$

$$=0.444n\ \text{(in mm)},\quad \text{for 15 Mc.}$$

Measurement, in Fig. 4, of the separation $2z_n$ between the two first-order minima ($n=1$), or the second ($n=2$), etc., gives a slightly greater spacing than calculated. This is to be expected since as previously noted the radiation from the radiator surface is not strictly uniform (dropping slightly from center to edge), even for the $\theta=-22°$ plane.

Of more practical importance are the corresponding diffraction formulas for the spherical radiator used with a *circular* electrode of diameter d. The optical formulas in this case give $I/I_0=[2J_1(\alpha)/\alpha]^2$, where $J_1(\alpha)$ is the first-order Bessel function of $\alpha=\pi dz/\lambda R$. The intensity I is zero when $J_1(\alpha)=0$ (except when $\alpha=0$, then $I/I_0=1.0$), which occurs when $\alpha=3.832$, 7.016, 10.173, 13.328, etc. Thus, if $k_n=\alpha_n/\pi$, $d=h$, and λ, R, and h are as before, the

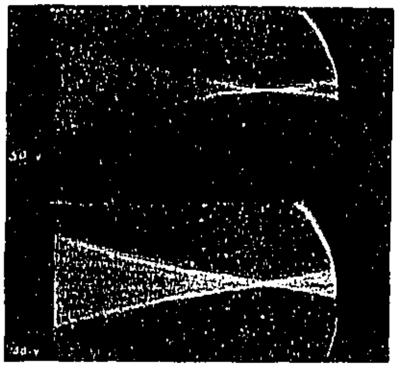

FIG. 4. Radiation in the $\theta=-22°$ plane for the 5-Mc mode (low intensity above and high intensity below). The radiator is at the left.

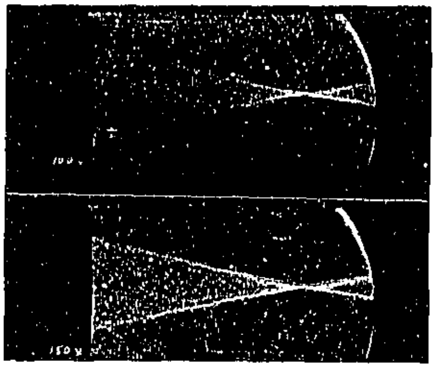

FIG. 5. Radiation in the $\theta=-22°$ plane for the 15-Mc mode (low intensity above and high intensity below). The radiator is at the left.

diameter $2z_n$ of the zero intensity rings is given by

$$2z_n = k_n\frac{(2\lambda R)}{d} = 1.333k_n \text{ (in mm), for 5 Mc} \quad \text{(III.2)}$$

$$= 0.444k_n \text{ (in mm), for 15 Mc,}$$

$$k_n = 1.22,\ 2.23,\ 3.24,\ 4.24\cdots(n+0.25).$$

The theoretical *average* intensity over the area of the core may be obtained from

$$I_{Av} = \frac{0.84W}{\pi z_1^2}, \quad \text{(III.3)}$$

where W is the total radiated power (assumed equal to the electrical power input V^2/R), πz_1^2 is the area of the core, and the constant 0.84 takes account of the fact that 84 percent of the energy passes through the core area (from optical diffraction theory). The theoretical *maximum* intensity, which occurs at the center of the core, is given by H. T. O'Neil[7] as

$$I_c = \frac{\pi k_1^2 W}{4z_1^2} = \frac{(\pi k_1/2)^2}{0.84} I_{Av} = 4.37 I_{Av}, \quad \text{(III.4)}$$

i.e., I_c is approximately 4.4 times the average intensity over the whole area of the core. Formula (III.3) is very useful in checking the sharpness of focus of a radiator, I_{Av} being easily measured (see following Section D). From formula (III.4), then, the maximum intensity at the center of the core is calculable from I_{Av}.

For the radiator here used, operated at 5 Mc, $z_1 = 0.081$ cm, and for W in watts, the theoretical I_{Av} and I_c are

$$I_{Av}(\text{watts/cm}^2) = 40W,$$
$$I_c(\text{watts/cm}^2) = 175W. \quad \text{(III.4)}$$

As shown in Section D, the experimentally determined value of I_{Av} is within 15 percent of the above theoretical value, if account is taken of the liquid attenuation from radiator to focal plane. Thus the present 5-Mc focusing radiator, with circular electrode, comes close to satisfying theoretical optical diffraction laws.

H. T. O'Neil[7] has made a mathematical analysis of the diffraction of ultrasonic waves from focusing (and plane) radiators and has derived a number of formulas which are very useful in designing focusing radiators, and has indicated under what ranges of frequency and radiator dimensions they apply. Deviations from optical theory become pronounced as the radiator frequency and angular aperture are reduced, until finally there is little semblance of geometrical focusing as known in optics, and the optical formulas are insufficient. The three most weakly focusing radiators (the 25-, 8-, and 7-cm radii radiators) of Labaw[8] and of Fein[9] appear to come in this classification. Their most strongly focusing radiator (4-cm radius) should approach optical focusing, but still should have only about one-sixteenth the concentrating power of the 5-Mc radiator here described.

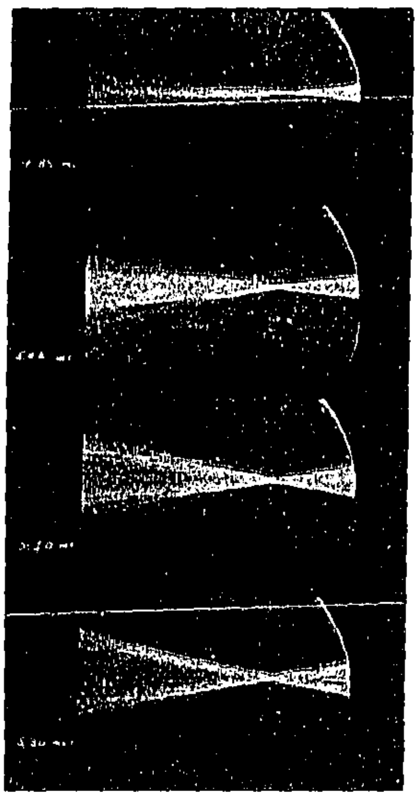

FIG. 6. Radiation in the $\theta = +38°$ plane for four frequencies near the 5-Mc mode.

B. Uniformity of Radiator Emission

The degree of uniformity or non-uniformity of radiation from different areas of the radiator surface is shown in Figs. 4 to 8. The two main reasons for this non-uniformity are non-uniformity of the effective piezoelectric constant and non-uniformity of the resonance frequency, over the area of the radiator. A third less important effect appears to be

[7] H. T. O'Neil, J. Acous. Soc. Am. 21, 60(A) (1949). Complete paper to appear later.
[8] L. W. Labaw, J. Acous. Soc. Am. 16, 237–245 (1945).
[9] Louis Fein, J. Acous. Soc. Am. 20, 583(A) (1948).

due to coupled modes of vibration, as will be finally described.

The first cause of non-uniformity, piezoelectric, results in a gradual drop off of radiation intensity in proceeding from center to edge of the radiator, more rapidly in the XY plane than in the XZ plane. For the radiator here used the peripheral portions are only 13° off from x cut**** so that this cause of radiation drop-off is small in all cases, and is not discernible in the views shown.

The effect of non-uniformity of resonant frequency, which is far more serious, is shown by Figs. 6 and 7 for the $\theta = +38°$ plane (near the plane of greatest non-uniformity), and at about 100 volts r.m.s. Figure 6 is for operation of the radiator at four different frequencies near to and including the fundamental frequency, 5 Mc, of the center of the radiator. It is clear that when the radiator is operated at 5 Mc the radiation is most intense from the central area of the radiator. When operated at lower frequencies the intensity is less and the drop-off from center to edge is faster. But when operated at higher frequencies there are two non-central regions of maximum intensity. Only these regions have resonant frequencies close to the applied frequency. Regions driven off-resonance radiate more weakly. Thus, the four views show that the lowest resonant frequency of the radiator, 5 Mc, occurs at the center and that the resonant frequency increases on receding from the center. In this $\theta = +38°$ plane the greatest total radiation is obtained by operating at a frequency of 5 Mc or slightly higher.

Figure 7 shows similar conditions for operation of the radiator at its *third harmonic*, 15 Mc. Here the localization of radiation region to the on-resonance area is thrice as sharp (as will be explained later). Thus the radiator is quite ineffective in this plane when operated in a harmonic mode.

A set of views (not shown) taken in the $\theta = -22°$ plane, but otherwise with conditions of varying frequency like that in Fig. 6, would have shown essentially *uniform* radiation intensity over the whole area (as in Fig. 4) for any frequency; the only variation being in the magnitude of this intensity, greatest for operation at 5 Mc, and decreasing for deviations therefrom either higher or lower. Thus in the $\theta = -22°$ plane the resonant frequency is uniform. When the radiator is operated near its third harmonic 15 Mc, in this same plane, the radiation should also be uniform as just described. Actually, it is found to be uniform except for a fine line structure, as described in the next section.

**** Zero intensity would just be reached at 30° out in the XY plane, and at 90° out in the XZ plane, for at these angles from the x axis in quartz the y and z axes occur, for which orientations there is no effective piezoelectric coupling.

When the radiator is examined in a plane near $\theta = -51°$ it is found that non-uniformity of radiation results from a frequency effect opposite to that shown in Figs. 6 and 7, but of lesser degree. That is, the resonant frequency *decreases* from center to edge.

As will be shown by the calculations to follow, the planes above examined ($\theta = +38°$, $-22°$, $-51°$) show the extreme effects and intermediate planes differ therefrom gradually as the radiator is shifted from one orientation to another. It is very interesting to follow this shift by eye as the radiator is being rotated (operating at otherwise constant conditions).

When the radiator is operated with a circular electrode the apparent non-uniformity of emission shown above is greatly reduced, since the optical arrangement then partially integrates effects for a wide range of θ planes. The narrow rectangular electrode is only used for analysis. For practical

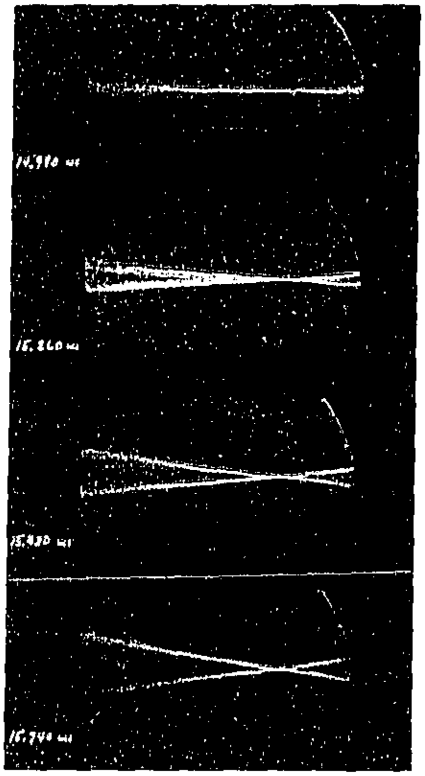

FIG. 7. Radiation in the $\theta = +38°$ plane for four frequencies near the 15-Mc mode.

use of the radiator in generating high intensity, localized energy the circular electrode is used.

C. Extraneous Coupling Effects

A rather minor non-uniformity of surface emission appears to be caused by coupling phenomena well-known in the design of high frequency crystal oscillator and filter elements. With such elements high harmonics of low frequency modes of vibration whose harmonic frequencies lie in the neighborhood of the desired high frequency mode, and which are elastically coupled thereto, effect the activity of vibration and also the actual frequency of vibration (which would normally be controlled only by the thickness dimension). Their deleterious effects may be reduced by various means: damping, dimensioning, shaping. With ultrasonic radiators which operate in contact with a liquid or solid medium, instead of air or in a vacuum, the damping is usually so great as to prevent observance of coupling effects. The writer has not experienced this effect in plane x cut quartz radiators. However, with lower frequency radiators where the greater thickness of the radiator (compared to its usually not-increased face dimensions) may decrease the ratio of cross dimension to thickness below say 20 to 1, coupling may become very pronounced. That it was observed in this relatively thin focusing radiator was a surprise, but may be related to its shape or to the variable resonant frequency.

This coupling effect was found to be most easily observed under the conditions used in procuring Fig. 8. Here the chosen plane of operation was $\theta = -22°$, where other non-uniformities are at a minimum. The frequency of excitation was near the third-harmonic 15 Mc, but lower than the lowest of Fig. 7 in order to best show the coupling effect. It is noted that at 14.500 Mc the emission is remarkably uniform, as is to be expected for any frequency in this plane. At a slightly higher frequency 14.51 Mc the simplest coupled mode pattern appears. At 14.58 Mc the second pronounced coupled mode pattern appears, at 14.64 the third, and at 14.70 the fourth. The frequency separation of these modes is about 0.06 Mc. As the frequency is further increased in 0.06 Mc steps similar patterns appear, for each step a new pair of striations developing, until the whole field is so closely packed that the striations fill the whole field and are not clear when operated at the frequency of 15 Mc. Further, through reaction on the driving circuit there is a tendency for the driving frequency,

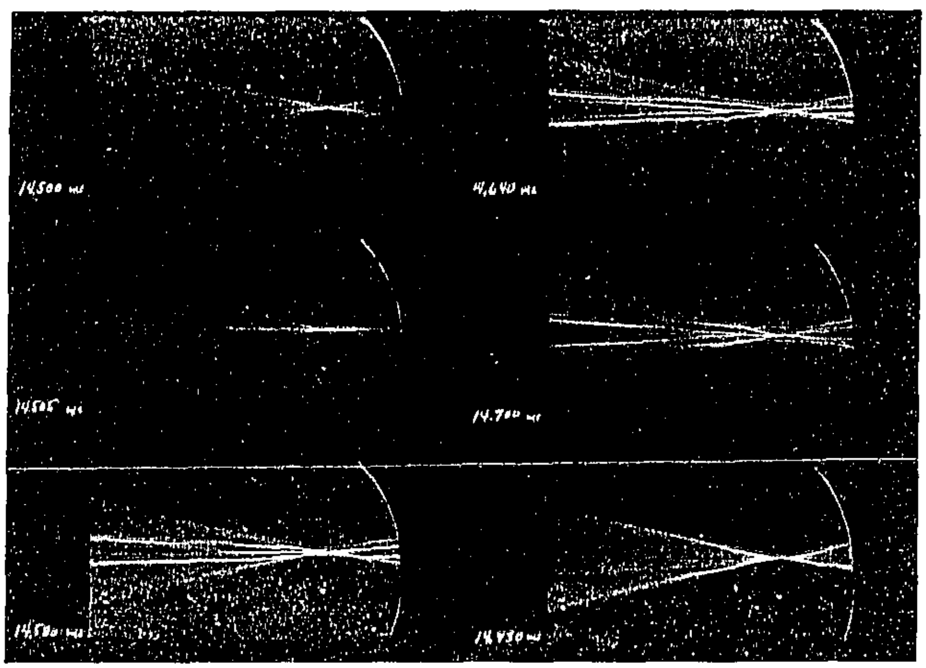

FIG. 8. Radiation in the $\theta = -22°$ plane for six frequencies near the 15-Mc mode, showing extraneous coupling effects.

though uniformly increased, to jump from one pattern frequency to the next.

However, with careful control it is possible to obtain a picture at an intermediate frequency. Such a pattern is shown for the frequency 14.73 Mc which is midway between the 14.70 Mc of the previous pattern and the next step to 14.76 Mc (not shown). This more uniform pattern is typical of that which is found between all steps. This changing order of striations and jumping frequency is characteristic of the well-known coupled-mode effect. Otherwise this coupled mode effect is not understood. It probably could be cured by dimensioning, or edge damping, or by thickness shaping. It is not apparent at 5 Mc the fundamental mode. Since its effect on radiation efficiency is so small under normal operating conditions, no attempts have been made to analyze it further.

D. Over-All Results with Circular Electrode

Though the above results are very useful in detailed analysis of the focusing radiator's behavior they do not give a clear picture of the effectiveness of the radiator when used with a full circular electrode, the normal manner of use. Also, as previously mentioned, the above optical method of observation is less suitable for use with circular sound beams. The following fountain, burning, and radiation pressure methods are more revealing.

For a qualitative study of the sharpness and location of focus one may form a fountain at the surface of the water. In the present case a steel block with 45° incline was used to reflect the beam vertically up to the water-air surface. By varying the water level the water surface may be moved through the focal point. With input voltage so adjusted as to just cause piercing of the surface tension film at best focus, 70 volts in the present case, small droplets are continually popped out, and slight changes in focus return the water surface to a small hill formation. At 100 volts a fountain is formed mainly of highly directed small drops with only a very short basal column of solid water, the drops reaching a height of 20 cm, and the column cross section being essentially circular of diameter about 1 mm. For increasing voltages the fountain rises higher. Starting at the lowest fountain levels (above 70 volts input) there is produced a cold fog which becomes very copious as the input is increased. Similar but smaller cross section fountains are produced at 15 Mc. It might be added that the fountain effects are not the result of the radiation-pressure induced circulation currents in the body of the water, for a membrane introduced just below the surface of the water does not materially effect the fountain.

The electrical power dissipated in the radiator can be figured from the known equivalent parallel resistance which is approximately 10^4 ohms. Thus 70 volts is equivalent to one-half watt, 100 volts to one watt, etc. The actual ultrasonic powers at the focus are somewhat reduced by attenuation in the water and loss by reflection, but less than 25 percent. However, this energy is mainly concentrated into an area of the order of one square millimeter thus giving the startling effects ordinarily associated with high power.

Heating and burning effects may also be used to study the focus. A highly attenuating, non-reflecting material placed at the focus will be strongly heated in a localized area in spite of the inherent, circulated water cooling (a thermocouple or thermometer records little rise of temperature because of the high acoustic reflectance and high thermal conduction). Highly absorbing materials such as rubber, phenol fiber, and methacrylate plastic (e.g. Lucite) will be strongly heated locally on the incident surface. In the first two materials little protuberances of melted material will appear, and on removal the material will have the characteristic odor of overheating. The size and shape of these melted areas, if not exposed too long, is a rough measure of the sharpness of focus. The phenol fiber does not melt but cracks out. With a much less absorbing plastic (e.g., polystyrene) localized internal heating can be produced in a thick piece placed so that the focus is internal. Here a short exposure produces temporary (longer exposure, permanent) changes in the material which by their optical effects show the conical focusing of the beam.[10] Over-exposure cracks the material by internal expansion. Focusing on the surface of polystyrene produces warts as for the more attenuating materials.

It might be added that when a person's finger is placed at the focus, an input of less than 100 volts (1 watt) produces a sensation of burning, though none of the normal burn characteristics (redness or blistering) have resulted. Since attenuation in flesh is fairly low and the reflection loss is negligible there is probably considerable danger of causing serious internal injury without the protective warning of discomfort.

With very simply performed radiation pressure measurements it is possible to obtain a fairly quantitative value of ultrasonic intensity over localized areas down to one millimeter in diameter. The side view of a radiation pressure measuring device is shown in Fig. 9. The axle of the balance rolls on two horizontal supports. Extended downward from the axle is an arm which mounts a sound receiving pad. Extended horizontally from the axle are two arms which mount an adjustable

[10] Melting in paraffin blocks have also been used to show focusing, see Lynn, Zwemmer, Chick, and Miller, J. Gen. Physiol. 26, 2 (1942).

counter weight CW and a balancing weight BW. Zero balance is indicated by a light-beam, mirror, and scale. In operation, the balancing weight BW is first removed and the counter weight CW adjusted to give a zero setting with no sound beam present. Then, with the sound beam turned on, the balancing weight BW is added and adjusted (together with voltage input adjustment if necessary) to reestablish the zero setting. Thus all readings are taken with the pad surface in a definite predetermined location. The radiation force on the pad is then given by the product of the balancing weight BW and the ratio of its distance to the axle, to the distance of the sound beam center to the axle. For the comparison of two nearly equal energy sound beams it is convenient to leave the balancing weight fixed and vary the input voltage, the ratio of voltages squared giving the ratio of forces or powers. A diaphragm D with hole therein may be used to select a small localized region of an extended sound beam for measurement. In this manner the small focal region may be selected for measurement, and the power therein compared with that in the whole beam, obtained with the diaphragm removed.

The material of the pad and the diaphragm should be acoustically non-reflecting and sufficiently attenuating to prevent through transmission. At frequencies of 5 Mc and above in water it is not difficult to select one of the rubbers which will be satisfactory. (Nice holes may be bored with sharp metal drills if the rubber is first frozen stiff.)

The effect on the balance of radiation pressure induced circulating currents cannot normally be greater than that due to the small energy losses creating them, that is the energy which is attenuated between radiator and pad. At 5 Mc and below this loss is small in water, and may be calculated or measured. Or, the circulation may be eliminated by placing in front of the pad a thin stationarily mounted film of acoustically transparent material (polystyrene may be rolled out to less than 20 microns and introduces little loss).

The relation between radiation force F (in grams/cm²) and sound energy W (in watts/cm²) may be obtained from the well known formulas $S=J/v$, where J is the sound intensity in ergs/cm²/sec., v is the velocity of sound in cm/sec. (which is 1.5×10^5 in water at room temperature), and S is the radiation pressure in dynes/cm². Since $10^7W=J$ and $S=980F$,

$$W(\text{watts})=1.5\times10^{-2}S(\text{dynes})=14.7F(\text{grams}), \quad (\text{III}.5)$$

which may be taken as the integrated watts and grams over the area of beam being measured. This of course applies to a beam which is totally absorbed and not reflected (the force would be double for a beam perfectly reflected back on itself).

The radiation force balance of Fig. 9 measures only the components of force which are normal to the axle of the balance and the arm supporting the pad (i.e., force components parallel to the radiator axis). This applies whether the pad is normal to this direction as shown or otherwise oriented. The error caused by the angular distribution of the beam is negligible in the present case for the edge rays are only 13° off-axis (98.5 percent effective). That the orientation of the pad itself is immaterial may be taken advantage of to eliminate possible pad-reflection contributions to the force, by orienting the pad at 45° to the above position and preventing the reflected beam from returning to the pad, by further reflection and attenuation.

The radiation force balance has been used to check the ultrasonic power radiated from the curved radiator surface, the power arriving at a plane beyond the focus, and the power passing through a 1.63-mm diameter hole in the diaphragm placed in the focal plane. This was carried out at moderate input powers of about 10 watts giving a radiation force around 0.7 gram (much higher powers gave unsteady balance due to fluctuating circulation currents). The radiated sound power checked the input electrical power, and the attenuation loss down the beam checked the calculated attenuation (7 percent), both within about 5 percent. It was found that about 75 percent of the power in the focal plane passed through a hole of diameter of 1.63 mm (calculated diameter of first diffraction minimum, formula (III.2)). That this value is less than the theoretical 84 percent, given by optical diffraction formulas, is not surprising since the sound emission from the radiator is non-uniform thus violating the assumptions of the diffraction theory.

Thus when the radiator is operated at 1000 volts (100 watts input on 6.4 cm² area) there is about 70 watts of sound passing through the 1.63-mm diameter core of the beam. This gives an *average* intensity over the core of $I_M=3.4$ kw/cm². Now assuming only that the *intensity distribution* over the core area in the actual experimental case is the same as that given by diffraction theory, formula (III.4) indicates that the maximum intensity at the center of the core would be 15 kw/cm² (instead of 5 kw/cm², as previously reported[3]). This high intensity corresponds to a particle acceleration amplitude of ±45 million times the acceleration of gravity, and to a hydrostatic pressure amplitude of ±210 times atmospheric pressure. Certainly the maximum core intensity and amplitudes could not be less than one-half of these calculated values.

Even with such intense ultrasonic agitation it has not been found possible to produce visible cavitation in the water, except by special auxiliary means. As reported at an Acoustical Society meeting, but not discovered in time to be included in the abstract,[3] visible cavitation could however be produced by two special means, each of which involved counteracting the normally high speed water circulation through the core. In the first case, by using a reflector to return the beam upon itself, cavitation could be produced with one-fourth the above power. In the other case a reverse current stream of water from a small rubber tube, directed through the core, gave like results. Thus the lack of observable cavitation with the unobstructed sound beam may be due to the water circulating through the high intensity core region so fast that the cavities cannot grow to observable size, or maybe not even be formed.

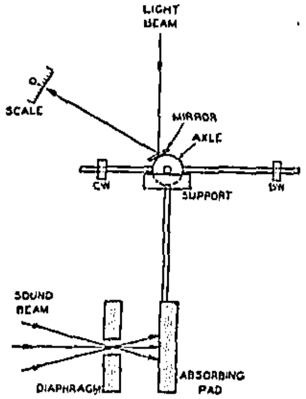

FIG. 9. Radiation pressure balance.

IV. CALCULATION OF EMISSION *VS.* LOCATION (α, θ)

When the x cut curved radiator is driven at a resonant frequency of its central area, that central area will emit or radiate most strongly while areas removed from the center will radiate more weakly. As has been mentioned this change of emission results from the changing piezoelectric and elastic properties with orientation in the crystal. The electrical and acoustical characteristics of all non-central x′ cut locations (later defined by the angles α and θ) will be given in terms of the characteristics of the center location, i.e., in terms of the standard x cut radiator.

Of the various possible ways of treating the problem the following method appears to have advantages in that the results derived may be readily checked from either the electrical input side or the acoustic output side. This method involves finding first the resonance frequency and the equivalent parallel resistance and reactance of a standard, flat x cut radiator when driven at its resonant frequency. Then the variation of resistance with off-resonant frequency shift is obtained. The product of this frequency function and the resonant resistance gives the effective parallel resistance for the actual driving frequency. The only power dissipated in the radiator is given by the quotient of the applied voltage squared and the resistance, V^2/R, and (assuming perfect conversion) this is the ultrasonic power radiated.

A. Parallel Resistance and Reactance at Resonance

For a thickness, longitudinal mode piezoelectric radiator of thickness t (cm) and area A (cm²), driven at one of its resonant frequencies Nf_r $(N=1, 3, 5, \cdots)$ and radiating from *one-side* only, the equivalent parallel resistance is

$$R_r=\frac{(\rho v)_M t^2}{4e^2A}(\text{e.s.u.})=0.225\times10^{12}\times\frac{(\rho v)_M t^2}{e^2A}(\text{ohms}),\qquad \text{(IV.1)}$$

where $(\rho v)_M$ is the (density×velocity) product of the medium into which the radiator radiates, and e is the ratio $P_t/(\Delta t/t)$ of the piezoelectric polarization in the thickness direction to the compressional strain in the same direction, when all other strains are zero. Similarly the parallel reactance is given by

$$-X=\frac{1}{N\omega C_0}=\frac{2t}{Nf_rKA}(\text{e.s.u.})=\frac{1.8\times10^{12}t}{Nf_rKA}(\text{ohms}),\qquad \text{(IV.2)}$$

where K is the dielectric constant in the thickness direction. The resonant frequencies are given by

$$Nf_r=\frac{N(c/4\rho)^{\frac{1}{2}}}{t}(\text{c.p.s.}),\qquad \text{(IV.3)}$$

where ρ is the density of the radiator and c is the ratio $T_T/\Delta t/t)$ of the compressional stress to strain in the t direction when all other strains are zero as before. The *fundamental* resonant frequency is of course f_r, while the other possible frequencies are odd harmonics thereof.

For the special case of x cut quartz, under the above conditions, $e=e_{11}=5.2\times10^4$ c.s.u., $(f_r$

$=(c_{11}/4\rho)^{\frac{1}{2}}=2.86\times10^5$ cm/sec., $K=K_{11}=4.55$, and,

$$R_r=\frac{83.3(\rho v)_M t^2}{A}\text{(ohms)}$$

$$=\frac{.68\times10^{13}(\rho v)_M}{f_r^2 A}\text{(ohms)},\quad\text{(IV.4)}$$

$$-X_r=\frac{0.396\times10^{12}t}{Nf_rA}\text{(ohms)}$$

$$=\frac{0.113\times10^{18}}{Nf_r^2A}\text{(ohms)},\quad\text{(IV.5)}$$

$$f_r=\frac{2.86\times10^5}{t}\text{(c.p.s.)}.\quad\text{(IV.6)}$$

For the further specialization, that of x cut quartz working into *water*, still on *one-side* only, $(\rho v)_M=1.5\times10^5$ g/cm² sec.

$$R_r\doteq\frac{10^{18}}{f_r^2A}\text{(ohms)},$$

and

$$-X_r\doteq\frac{0.11\times10^{18}}{Nf_r^2A}\text{(ohms)}.\quad\text{(VI.7)}$$

For a radiator working into the same $(\rho v)_M$ medium on *two sides*, all above values of R_r are to be doubled, while the values of X_r and f_r remain the same.

Thus, for example, a 5-Mc, x cut quartz radiator of electrode area one square centimeter, radiating into water on one side only (negligible radiation into air on the other side) would have a parallel resistance of $R_r=40{,}000$ ohms and reactance of $X_r=4400$ ohms when driven at the fundamental resonant frequency of 5 Mc. The electrical power dissipated in the radiator (and hence the radiated sound power), for an applied voltage of $V=200$ volts, r.m.s., would be $W_r=V^2/R_r=1$ watt. When the same radiator is operated at its third harmonic 15 Mc, the values of R_r and W_r are the same, but X_r is one-third as much. If operated with water on both sides R_r is doubled, W_r is halved and X_r is the same. The values of R_r and X_r are also of interest in arranging the coupling to the driving circuit, and can be measured on a Q meter.

B. Off-Resonance Effect

All above values of parallel resistance R_r and reactance X_r apply when the radiator is operated at one of its resonance frequencies Nf_r. When operated off-resonance different values will prevail, which, however, may be obtained by multiplying the resonance values by a suitable frequency function. Only that for the resistance is of special interest here.

The *off-resonance* parallel resistance R is given in terms of the *resonant* parallel resistance R_r and the frequency function Ω by

$$R=R_r\cdot\Omega.\quad\text{(IV.8)}$$

The exact value of Ω for *one-side* radiation is

$$\Omega=1+(4M^2-2+C^2)C^2,\quad\text{(IV.9)}$$

$$M=\frac{(\rho v)_R}{(\rho v)_M},\frac{\text{(radiator)}}{\text{(medium)}},\quad C=\cot\left(\frac{\pi}{2}\cdot\frac{f}{f_r}\right),$$

where f is the operating frequency, and f_r is the fundamental resonant frequency of the radiator as given by (IV.3) or (IV.6). The Ω-function reduces to unity for $f=Nf_r(N=1,3,5,\cdots)$ but becomes increasingly large on receding from any resonant frequency Nf_r. The Ω-function is greatly simplified under the restrictions that $M>5$ and that the operating frequency f differs from some resonant frequency Nf_r by less than 20 percent of the fundamental resonant frequency f_r. The first restriction is probably met for all piezoelectric materials radiating into non-metallic liquids (for quartz to water $M=10$). The latter restriction covers as wide a range of off-resonance operation as is usually of practical interest.

For *one-side* radiation and the restrictions described above and recorded below

$$\Omega=1+4M^2C^2,\quad\text{(IV.10)}$$

within one percent if,

$$C^2=\cot^2\left(\frac{\pi}{2}\frac{f}{f_r}\right)=\tan^2\left(\pm\frac{\pi}{2}\frac{\Delta f}{f_r}\right),$$

$$M=\frac{(\rho v)_R}{(\rho v)_M}>5,\quad\pm\frac{\Delta f}{f_r}=\frac{f-Nf_r}{f_r}<0.2.$$

(For two-side radiation, without restrictions, $\Omega=1+MC^2$, with M, C, f, and Δf as above.)

For x cut quartz radiating from one side only into water, $M=(\rho v)_q/(\rho v)_w=(2.65\times5.72\times10^5)/(1.00\times1.50\times10^5)\doteq10$, $4M^2-2\doteq400$ and approximately

$$\Omega\doteq1+400\cot^2\left(\frac{\pi}{2}\cdot\frac{f}{f_r}\right)$$

$$=1+400\tan^2\left(\pm\frac{\pi}{2}\cdot\frac{\Delta f}{f_r}\right),\quad\text{(IV.11)}$$

$$\Delta f=f-Nf_r<0.2f_r.$$

The solid line curve in Fig. 10 is a plot of $1/\Omega$ *vs.* f/f_r and $\Delta f/f_r$. Since the power radiated at an off-resonance operating frequency f is given by $W=V^2/R$, and at a resonance frequency Nf_r by

$W_r = V^2/R_r$, and since $R = R_r \cdot \Omega$,

$$\frac{W}{W_r} = \frac{R_r}{R} = \frac{1}{\Omega}. \qquad \text{(IV.12)}$$

Thus Fig. 10 also gives the ratio of the power that would be radiated off-resonance to that on-resonance. For the example noted in the last paragraph of Section A above, a 5 Mc x cut quartz radiator of area 1 cm² radiating into water on one side only, it is seen that if the radiator is driven only 0.5-Mc off-resonance (at 4.5, 5.5, 14.5, 15.5, etc.) the parallel resistance is more than eleven fold, and its power output cut to one-eleventh, of the values holding at resonance.

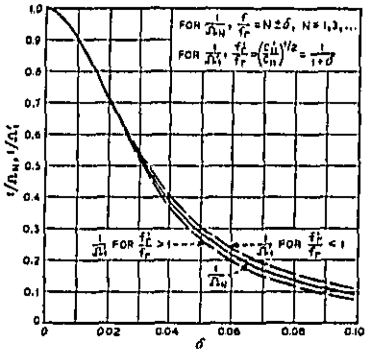

FIG. 10. A plot of the frequency dependent functions, Ω and Ω′, of formulas (IV.11) and (IV.23).

C. The Focusing Radiator

From the preceding section we can now determine the electrical and radiation characteristics of any given elemental area of the *focusing* x cut quartz radiator. The elemental area under consideration will be considered to lie at the point P, Fig. 3, on the radiator surface. Corresponding to the radiator surface being x cut at the center C, it is x′ cut at any other point P. Corresponding to the previous use of c_{11} and e_{11} for an x cut surface we will now also use c_{11}' and e_{11}' for an x′ cut surface. Similarly there will be unprimed and primed terms f_r, $(\rho v)_q$, R_v, R, etc.

The (α, θ) designation of the x′ cut, P location is according to Fig. 3. The angle α is measured between the X and X' directions, i.e., between OC and OP. The angle θ is measured between the XZ' plane, in which plane P lies and the XZ plane (X and Z refer to crystallographic axes, and Z' to an axis normal to X but at the angle θ to Z). The sense of θ is chosen so that a *minor cap face* plane of quartz is parallel to the $\theta = +38°\ 13'$ XZ' plane.

The value of α for a point P on the periphery of the utilized area of the radiator (the area covered by electrodes on both sides) is denoted by α_p and is called the *half-angular-aperture* of the radiator. A full hemispherical radiator, $\alpha_p = 90°$, would include: three truly x cut areas at $(0°, \theta)$ and $(60°, \pm 90°)$; four y cut areas at $(30°, \pm 90°)$ and $(90° \pm 90°)$; and two z cut areas at $(90°, 0°)$ and $(90°, 180°)$. Practically, a radiator would normally be made with a half-angular aperture smaller than $\alpha_p = 30°$, thus eliminating the inactive y and z cut areas, and the outer x cut areas which are out of phase with the remaining central x cut area at $(0°, \theta)$. However, the sespecific orientations are especially useful in checking the formulas to be developed for c_{11}' and e_{11}' *vs.* (α, θ). Likewise points in the XY plane $(\alpha, \pm 90°)$ and in the XZ plane $(\alpha, 0°)$ or $(\alpha, 180°)$ are useful, since formulas for these planes are well-known and recorded in the literature.[11]

We will now derive formulas for e_{11}'/e_{11} and c_{11}'/c_{11}, where e_{11} and c_{11} refer to the x direction as before, and e_{11}' and c_{11}' commonly make use of a different angular description than above described, namely: X' makes the angles α, β, and γ with the X, Y, and Z crystallographic axes and have the direction cosines l, m, and n, respectively. Now α has the same meaning in either system, $l = \cos\alpha$, and it can be shown that $m = k \sin\theta$ and $n = k \cos\theta$, where $k = \sin\alpha$, $(l^2 + m^2 + n^2 = 1)$. Using these substitutions for l, m, and n in standard formulas for quartz[12] we have

$$\frac{e_{11}'}{e_{11}} = l^3 - 3lk^2s^2, \qquad \text{(IV.18)}$$

$$\frac{c_{11}'}{c_{11}} = l^4 + k^2l^2P + k^4Q,$$

where

$$P = 2s^2 + \left(\frac{4c_{44} + 2c_{13}}{c_{11}}\right)c^2 + 6\left(\frac{c_{14}}{c_{11}}\right)(2sc),$$

$$Q = s^4 + \left(\frac{c_{33}}{c_{11}}\right)c^4 + \left(\frac{c_{44} + c_{13}/2}{c_{11}}\right)(2sc)^2 - 2\left(\frac{c_{14}}{c_{11}}\right)s^2(2sc)$$

$$k = \sin\alpha, \quad l = \cos\alpha, \quad s = \sin\theta, \quad c = \cos\theta.$$

[11] W. G. Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946), see Chaps. IV, VI, VIII.

[12] See e.g., reference (11). The formula for e_{11}' is given by (194), upon insertion of e_{ij} for quartz from p. 191, Class 18 and c_{11}' is given by (28) p. 70, upon insertion of c_{ij} for quartz from p. 55, Group VIII.

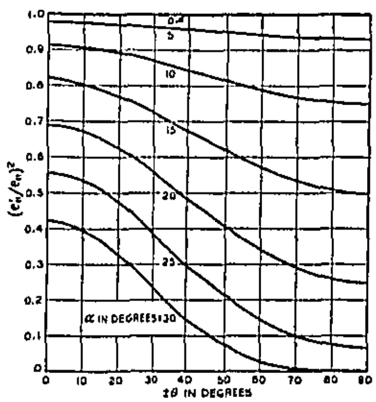

FIG. 11. A plot of the piezoelectric variations with orientation (α, θ), formula (IV.18).

Calculation of c_{11}'/c_{11} is simplified by having all functions of θ separated into the P and Q formulas.

Using W. P. Mason's 1943 values for the c constants,[13]

$$c_{11}=86.1 \quad c_{33}=107.1 \quad c_{44}=58.6$$
$$c_{12}=5.1 \quad c_{13}=10.5 \quad c_{14}=18.2$$

all times 10^{10} dynes/cm², we have

$$P=2s^2+(2.97)c^2+(1.27)(2sc), \qquad \text{(IV.19)}$$
$$Q=s^4+(1.244)c^4+(0.742)(2sc)-(0.423)s^2(2sc).$$

Figure 11 is a plot of $(e_{11}'/e_{11})^2$ and Fig. 12 of $(c_{11}'/c_{11})^{\frac{1}{2}}$, both *vs.* (α, θ) and for quartz. It will be found that e_{11}'/e_{11} and c_{11}'/c_{11} each reduce to unity at $\alpha=0$, and may be further easily checked for proper values for the specific axes and planes noted in a preceding paragraph. (The variation of the dielectric constant K_{11}' from the x cut value K_{11} is small, $K_{11}'=K_{11}\cos^2\alpha+K_{33}\sin^2\alpha$, and does not enter into the following formulas anyway.)

The parallel resistance at resonance R' for the (α, θ) location is given from (IV.1) as $R_r'=(\rho v)_M t^2/4(e_{11}')^2 A$, or in terms of R_r for x cut quartz, formulas (IV.4) or (IV.7), as

$$R_r'/R_r=(e_{11}/e_{11}')^2, \text{ and } W_r'/W_r=(e_{11}'/e_{11})^2. \qquad \text{(IV.20)}$$

Similarly, the fundamental resonant frequency f_r' for the (α, θ) location is given in terms of f_r, formula (IV.6), by

$$f_r'/f_r=(c_{11}'/c_{11})^{\frac{1}{2}}, \qquad \text{(IV.21)}$$

for a radiator in which the thickness is everywhere the same. Actually, of course, R_r' is not obtained unless the exciting frequency is Nf_r', i.e., a resonant frequency for this location.

In practice the whole radiator will be excited at a single resonant frequency f (later chosen to be f_r or Nf_r), and the off-resonance resistance at the (α, θ) location is given by $R'=\Omega' R_r'$. Taking account of (IV.20) and noting that the input, or radiated, power from the (α, θ) location is $W'=V^2/R'$, the resistance and power for the (α, θ) location are each given in terms of the resonant resistance R_r and power W_r for a standard x cut radiator by

$$R'/R_r=(e_{11}/e_{11}')^2\Omega'$$

and

$$W'/W_r=(e_{11}'/e_{11})^2 1/\Omega', \qquad \text{(IV.22)}$$

where $W_r=V^2/R_r$ and R_r is given by (IV.4) or (IV.7), e_{11}'/e_{11} is given by (IV.18), and Ω' is to be determined.

The frequency function Ω' may be determined from formula (IV.10) by proper substitution. The impedance ratio $M'=(\rho v')_Q/(\rho v)_M$ changes with orientation (α, θ) since $v'=(c_{11}'/\rho)^{\frac{1}{2}}$, but may be given in terms of M for x cut quartz by $M'=(c_{11}'/c_{11})^{\frac{1}{2}}M$. In practice the whole radiator is excited at a resonant frequency of its center $f=nf_r$,† this being the value to use for f in (IV.10). On the other hand f_r of (IV.10) is to be replaced by f_r' the resonant frequency in the (α, θ) location. Finally, taking account of (IV.21)

$$\Omega'=1+4M'^2(f_r'/f_r)^2\cot^2[N(\pi/2)(f_r/f_r')], \qquad \text{(IV.23)}$$
$$=1+4M^2(c_{11}'/c_{11})\cot^2[N(\pi/2)(c_{11}/c_{11}')^{\frac{1}{2}}],$$

where $M=(\rho v)_Q/(\rho v)_M$, subscript Q standing for x cut quartz and M for the liquid medium. For quartz-to-water, as with (IV.11), $[4M^2(c_{11}'/c_{11})-2]\doteq[400(c_{11}'/c_{11})]$.

The dashed curves of Fig. 10 are a plot of $1/\Omega'$ *vs.* (f_r/f_r') for quartz-to-water, the upper curve apply-

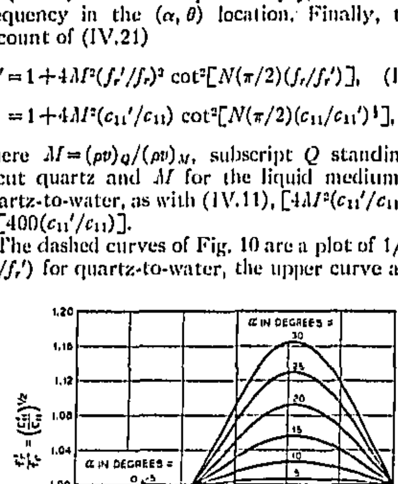

FIG. 12. A plot of the elastic or frequency constant variations with orientation (α, θ), formulas (IV.18), (IV.19), and (IV.21).

[13] W. P. Mason, Bell Sys. Tech. J. 22, 178–223 (1943); or reference (11), p. 135.

† A small gain in output may be obtained by operating at a slightly higher frequency, but this will not be developed here.

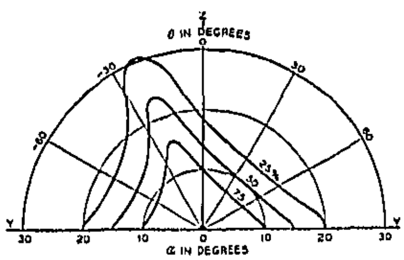

FIG. 13. Radiation efficiency *versus* (α, θ) for a *constant-thickness* radiator operated in the *fundamental* mode.

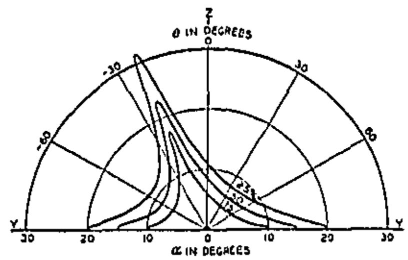

FIG. 14. Radiation efficiency *versus* (α, θ) for a *constant-thickness* radiator operated in the *third harmonic* mode.

ing when the fundamental resonant frequency for the (α, θ) location f_r' is less than that of center f_r, and the lower curve when $f_r' > f_r$.

D. Final Theoretical Results and Suggested Improvements

The final results of the above theory are given in Figs. 13–16. To facilitate use of the theoretical data the α and θ coordinates have been recorded in polar form and the functions plotted *versus* (α, θ) are recorded as contours of constant values. This limited set of figures, derived for preceding formulas and curves, explains not only the action of standard, constant-thickness, quartz, focusing radiators radiating into water on one side only, but also shows how to design superior, *variable-thickness* radiators and explains their action. Both spherical and cylindrical focusing radiators will be covered.

Figure 13 is a plot of what is termed the *radiation efficiency* in percent *versus* (α, θ) location *for a constant-thickness radiator* operated in the *fundamental mode.* The radiator is assumed to be operated at the fundamental resonant frequency of its center $\alpha =$ zero, and to radiate a sound intensity of 100 units at this location. At any (α, θ) location falling on the curve labeled 50 (e.g., $\alpha \doteq 15°$, $\theta = \pm 90°$) the radiated intensity will then be 50 percent of that at the center. For any (α, θ) location on the curve labeled 25 (e.g., $\alpha = 20°$, $\theta = \pm 90°$) the radiated sound intensity will be down to 25 percent of that at the center. It is to be noted that for a radiator with the periphery of its effective area at $\alpha = 30°$ there would be over one-half of the effective area radiating with an intensity less than 25 percent of that at the center.†† Thus there is little advantage in constructing a radiator with a half-angular-aperture greater than 15 or 20 degrees.

Another important feature of Fig. 13 indicates a preferred design for a cylindrical focusing radiator (that is a constant-thickness, cylindrical shell, for obtaining a *line* focus). Note that the radiation efficiency drops off, from center to edge, most rapidly in the $\theta \doteq +35°$ plane and least rapidly in the $\theta \doteq -23°$ plane (as was also shown by the radiation patterns (Figs. 4 and 6, respectively)). Thus it is seen that a constant-thickness cylindrical radiator should be made with its curvature in the $\theta \doteq -23°$ plane.

The above figure was derived from a plot (not shown) of $W'/W_r = (e_{11}'/e_{11})^2/\Omega'$ *vs.* (α, θ), formula (IV.22), where (e_{11}'/e_{11}) may be obtained from (IV.18) or Fig. 11, and Ω' from a substitution of $(f_r'/f_r) = (c_{11}'/c_{11})^{\frac{1}{2}}$ values from (IV.18) and (IV.19) or Fig. 12 in formula (IV.23) or Fig. 10, dashed curves. The major cause for loss of efficiency on receding from the center is due to the off-resonance effect, as given by $1/\Omega'$ (compare Fig. 13 with Fig. 16, for which $W'/W_r = (e_{11}'/e_{11})^2$ alone).

Figure 14 is a plot of the *radiation efficiency versus* (α, θ) *for a constant-thickness radiator* operated in the *third harmonic mode.* The description of this figure is like that for the preceding Fig. 13, fundamental operation, except that here the drop-off in efficiency on receding from the center is markedly greater. For third harmonic operation alone there is a little advantage in using a half-angular-aperture greater than $\alpha = 10$ to 15 degrees. (As the order of harmonic is raised beyond the third this restric-

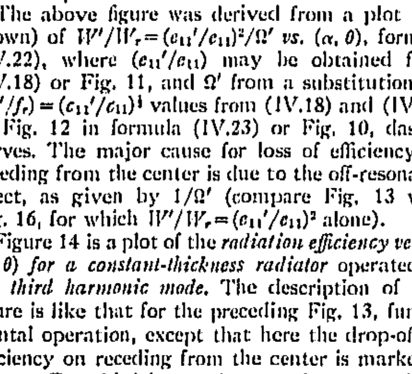

FIG. 15. Frequency constant and corrective thickness-shaping *vs.* (α, θ).

†† The integrated radiation efficiency of the 5-Mc radiator herein described (i.e., compared to a flat x cut radiator of the same area), was about 50 percent when operated in the fundamental mode.

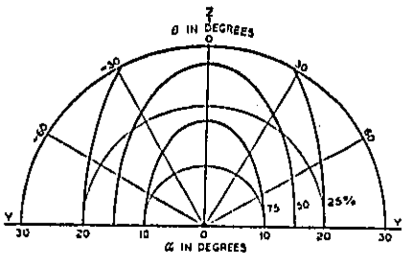

FIG. 16. Radiation efficiency *vs.* (α, θ) for a *thickness-shaped* radiator, any harmonic mode of operation.

tion becomes still more pronounced. The very marked superiority of the $\theta \doteq -23°$ plane over the $\theta \doteq +35°$ plane is also shown in the radiation patterns of Figs. 5 and 7, respectively. As noted above the major cause of off-center loss of efficiency is due to the off-resonance effect. Thus the shape (and values) of the efficiency plots of Figs. 13 and 14 are largely dictated by the frequency *versus* (α, θ) location curves as will be seen from the description of the following figure.

Figure 15 is a plot of *frequency constant* and of *corrective thickness-shaping vs.* (α, θ). In the first use of this figure, the center of the radiator is assumed to have a resonant frequency of unity at the center, and at any other location (α, θ) the resonant frequency is given by the contour passing through this location (or by extrapolation). Thus at the location $(15°, +35°)$ the frequency is 1.06 times that at the center. It is clear that the resonant frequency is equal to that at the center for all values of α in the planes $\theta \doteq -23°$ and $\theta = \pm 90°$, and varies most rapidly in the $\theta \doteq +35°$ plane, as is also shown in the last case by the radiation patterns of Figs. 6 and 7. The values recorded are, of course, independent of the order of harmonic operation. Since the whole radiator must be driven at a single frequency there are necessarily large areas which are driven off-resonance and hence more weakly than if driven at resonance. This explains the major losses of efficiency recorded in Figs. 13 and 14.

The above figure is derived directly from Fig. 12 which is a plot of $f_r'/f_r = (c_{11}'/c_{11})^{\frac{1}{2}}$ *vs.* (α, θ), obtained from formulas (IV.18) and (IV.19). Now the large loss of efficiency due to off-resonance operation of the constant-thickness radiator can be eliminated if the radiator is *thickness-shaped* in such a manner that the resonant frequency is everywhere the same. Since $f_r = (c_{11}/4\rho)^{\frac{1}{2}}/t$ at the center and $f_r' = (c_{11}'/4\rho)^{\frac{1}{2}}/t'$ at an (α, θ) location, then if $f_r' = f_r$, $t'/t = (c_{11}'/c_{11})^{\frac{1}{2}}$. Hence Fig. 15 is also a plot of the required t'/t *vs.* (α, θ) to obtain a radiator with a single value of resonant frequency all over.

Thus Fig. 15 also shows the corrective thickness-shaping required to obtain a superior radiator. In all areas where curve is labeled 1.00 the thickness is to have the proper value of t to give the desired frequency $f_r = (c_{11}/4\rho)^{\frac{1}{2}}/t$. For (α, θ) locations on the curve labeled 1.04 the radiator is to be made thicker, $t' = 1.04t$, and along the curve 0.96 thinner $t' = 0.96t$. The accuracy of this adjustment need not be great for, as seen from Fig. 10, an error of 1 percent in thickness will result in a loss of only about 10 percent in radiation efficiency. The improved efficiency of a properly thickness-shaped, *constant-frequency* radiator is shown by the following Fig. 16. J. F. Muller of these Laboratories has adjusted a 1-Mc radiator with a half-angular-aperture of $\alpha = 14°$), and with a very moderate effort has obtained about one-half of the expected improvement.

Figure 16 is a plot of the *radiation efficiency vs.* (α, θ) for a *thickness-shaped radiator, any mode* of harmonic operation. The off-center loss of efficiency here results only from the drop-off of the effective piezoelectric constant, since the whole radiator now has a uniform resonant frequency. The plot is obtained directly from Fig. 11, or formulas (IV.20) and (IV.18) (the small change of R_r with t, formula (IV.1)), has been neglected. The great gain in efficiency of outer regions is apparent, compare with Figs. 13 and 14, especially for harmonic mode operation.

It is clear that a superior cylindrical radiator would have its curvature is in the $\theta \doteq$ zero-degree plane (i.e., the XZ plane), providing it is appropriately thickness-shaped according to Fig. 15. In this cylindrical case approximately correct shaping is easily obtained by using truly circular curves for both concave and convex sides, the convex radius of curvature being somewhat greater than the sum of the central thickness and the concave radius, thus making the radiator thicker at the edges than the center (like a diverging concave-convex cylindrical lens).

At the beginning of this section it was noted that the results to be given were for quartz radiators radiating into *water* on one side only. For radiation into other liquids (or into water on both sides) some of the above results will be different. Since only Ω' is affected by the radiation medium through M, formula (IV.23), only Figs. 13 and 14 will be changed. Further since M does not vary greatly among non-metallic liquids, even these figures will be approximately true for most cases.

It might be noted in closing that the principles and general formulas above applied to quartz focusing radiators may also be applied to radiators of other crystalline materials. In the case of tourmaline, for example, the standard, unshaped radiator would be a z cut, the axis of the radiator being parallel to a z crystallographic axis. The characteristics of the center of the radiator would be given by the e_{33} and c_{33} constants, and at (α, θ) locations by e_{33}' and c_{33}' formulas (which are, of course, different from those for quartz). While the frequency constant varies less in tourmaline than in quartz, the effect of driving off-resonance is greater so that thickness-shaping is still worth while. Orientations other than truly z cut are also indicated.

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Ultrasonic Lenses of Plastic Materials

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The properties of certain plastic substances have been examined with the idea of using them to construct solid lenses for focussing ultrasonic radiation. Some experiments are described which illustrate the advantages offered by such lenses. The use of a plano-cylindrical or a plano-spherical lens permits a reduction to 1/10 or 1/100 respectively of the energy which must be emitted by a quartz crystal to produce a given intensity of ultrasonic radiation over a given region.

INTRODUCTION

It is undoubtedly useful to have available ultrasonic waves of high intensity, even if the region in which they can be propagated is relatively small. It would be advantageous, therefore, to achieve a concentration of the radiation sent out by the source by means of a dioptric system similar to those used with light.

Hopwood[1] suggested some possible methods of constructing dioptric acoustic systems, but the first experimental system, using aluminum lenses, was made by Bez-Bardilli.[2]

Giacomini[3] and afterwards, Pohlman,[4] employed lenses formed by a thin envelope of solid material, filled with a liquid. Ernst[5] observed recently that modern plastic materials have physical properties which allow the construction of promising dioptric systems.

This paper deals with experiments with plastic lenses. The concentration of the energy was studied when the lenses were either in the path of the radiation in the liquid, or in contact with a quartz generator. The advantages of these plastic lenses were extended to liquids that dissolve the material, using arrangements such as doublets, formed by a solid lens and a liquid one.

CHARACTERISTICS OF LENS MATERIALS

The material must possess the following: a velocity of propagation as different as possible from that of the liquid medium in which the lens is to be used; a characteristic acoustic impedance as close as possible to that of the surrounding liquid; and finally, a low coefficient of absorption. The velocity referred to is that of longitudinal waves since the propagation in the solid, under the circumstances of interest to us, occurs mainly by longitudinal waves.

There are certain plastics that satisfy the required conditions. Ernst has suggested polystyrene and polymethylmethacrylate. We have found it convenient to use polymethylmethacrylate (Plexiglas), which appears to be the best lens material among those studied.[6]

It is necessary to determine the velocity of propagation of longitudinal waves in the lens material. We made a measurement based on the refraction produced by a prism with one face normal to the incident ultrasonic beam. Under such conditions, the propagation within the prism occurs essentially by longitudinal waves having a velocity: $c=(K/\rho)^{\frac{1}{2}}$, where K is the bulk modulus.

Figure 1 shows the ultrasonic field rendered visible by the striation method. The arrow indicates the direction of motion of the waves emitted by a quartz vibrator with a frequency of 8 Mc/sec. The angle of incidence on the exit surface of the prism

[1] F. L. Hopwood, Some Properties of Inaudible Sound, Nature **128**, 748 (1931), London; Ultrasonic Lenses and Prisms, J. Sci. Inst. **23**, 63 (1946).
[2] W. Bez-Bardilli, Über ein Ultraschall Totalreflectometer zur Messung von Schallgeschwindigkeiten sowie der elastischen Konstanten fester Körper, Zeits. f. Physik **96**, 761 (1935).
[3] A. Giacomini, Alcuni Esperimenti di Ottica degli Ultrasuoni, Alta Frequenza **7**, 660 (1938).
[4] R. Pohlman, Über die Möglichkeit einer akustichen Abbildung in Analogie zur Optischen, Zeits. f. Physik **113**, 697 (1938).
[5] P. Ernst, Ultrasonic Lenses and Transmission Plates, J. Sci. Inst. **22**, 238 (1945).
[6] This material, available under the name of Plexiglas, is manufactured in Italy by Soc. "Plexiglas," Milano.

is 45 degrees. The liquid is distilled water at a temperature of 15°C. Applying the refraction law to the transition from Plexiglas to water, we obtained for the velocity in Plexiglas: 2820 m/sec. The characteristic acoustic resistance, ρc, referring to the propagation of longitudinal waves is, therefore, 3.3×10^5 g/cm² sec., the density being 1.18 g/cm³.

Special attention should be drawn to the chemical properties of this plastic. Generally, aqueous solutions of inorganic salts do not alter the material, but the hydrocarbons and many other organic liquids dissolve it. We observed, however, that carbon disulphide does not affect the Plexiglas.

SOLID ACOUSTIC LENSES

The characteristics of the materials that we have considered above make them suitable for focussing acoustic radiation according to a method analogous to that used in optics. It is necessary, however, to point out that the analogy is not perfect. It must be remembered that even in the case of ultrasonic waves of high frequency, the wave-lengths are much greater than those of light; furthermore, the propagation of elastic waves in a solid is not perfectly analogous to the propagation of visible radiation.

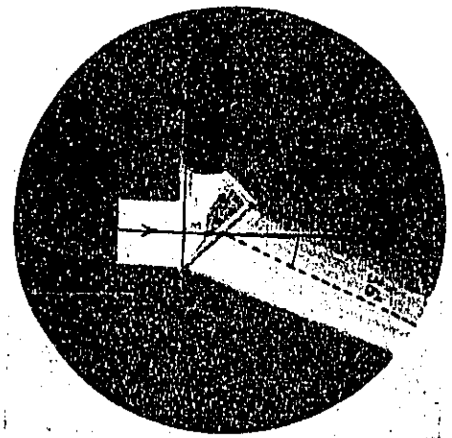

FIG. 1. Deflection of an ultrasonic beam through a Plexiglas prism.

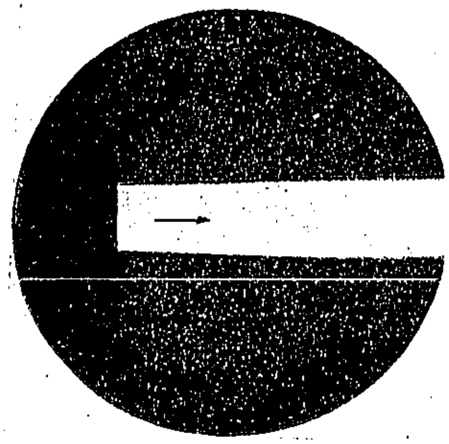

FIG. 2. Ultrasonic beam in water, by a square quartz, 16×16 mm; the frequency is 8 Mc/sec.

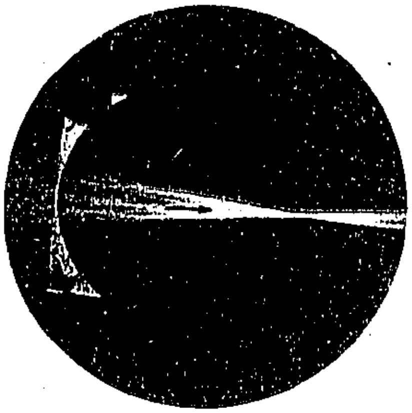

FIG. 3. Plexiglas lens ($r=25$ mm) in water.

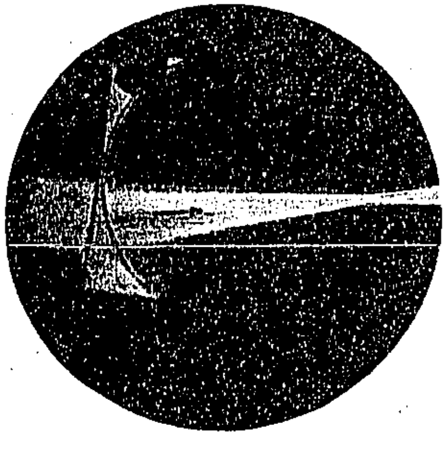

FIG. 4. Plexiglas lens ($r=30$ mm) in water.

We have found it desirable to have the radiation strike a lens at normal incidence to a plane surface since the energy that succeeds in passing through a plane cylindrical lens diminishes greatly if the surface of entry is the cylindrical surface instead of being the plane one. It is preferable, furthermore, that the thickness of the lens be as small as possible in view of the noticeable absorption.

For a plano-cylindrical or plano-spherical lens, arranged as above, it is easy to establish a relationship which approximately describes its behavior. If the radiation falls normally on the plane surface of the lens, it continues to propagate in the solid in the same direction in the form of longitudinal waves. On the spherical or cylindrical face, the incident longitudinal wave gives rise to a refracted wave in the liquid and two reflected waves in the solid, one of them being longitudinal, the other transverse. If we take into consideration the absorption of the material and neglect the contribution to the transmission which is made by the two waves arising in the solid at the curved surface, the behavior of the lens is determined solely by the refraction at the spherical or cylindrical surface.

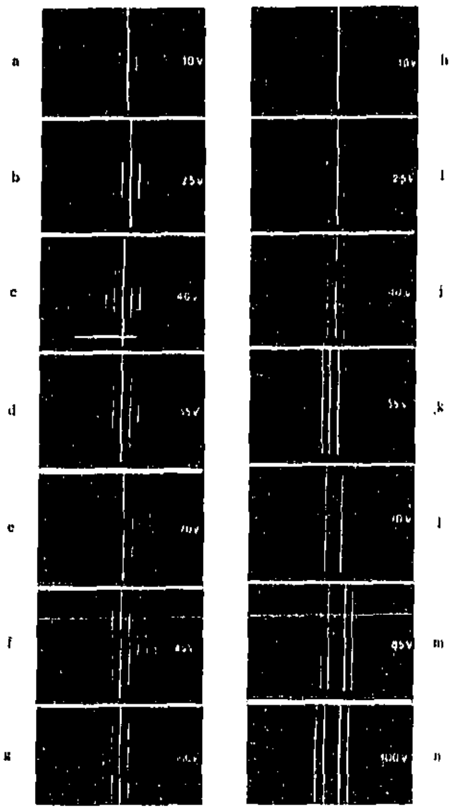

FIG. 5. Diffraction patterns: a–g with a lens ($r=25$ mm); h–p, without a lens.

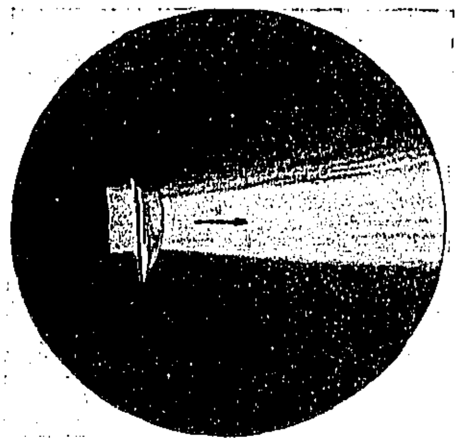

FIG. 6. Diverging Plexiglas lens ($r=25$ mm) in water.

Even taking into account the difference in the wave-lengths in the acoustic case and in that of visible radiation, one can validly extend the formula for optics to this case; the focussing effect is approximately,[7]

$$f=\frac{r}{1-V_l/V_s}=\frac{r}{1-(1/n_{sl})}. \qquad (1)$$

In this relationship f is the focal distance, r is the radius of the spherical or cylindrical surface, and $V_s/V_l=n_{sl}$ is the index of acoustic refraction for transmission from the solid to the liquid medium.

The velocity in the solid, V_s, is greater than that in the liquid, V_l, and therefore the denominator of Eq. (1) is always positive, that is, the lens is convergent or divergent depending on whether it is concave or convex. Furthermore, we can see that the focal length of a lens varies considerably with the velocity of the liquid in which it is immersed.

SOME ACOUSTIC SOLID LENSES USED

Figure 2 shows, by the striation method, an ultrasonic beam directed into water from a vibrating quartz crystal at a frequency of 8 Mc/sec., while Fig. 3 shows how the beam is concentrated when a Plexiglas plano-cylindrical lens with a radius of

[7] P. Ernst, Measurement and Specification of Ultrasonic Lenses, J. Acous. Soc. Am. 19, 474 (1947); in this paper, the ratio V_s/V_l appears, instead of V_l/V_s.

curvature of 25 mm is interposed normal to the direction of propagation. Similar results were obtained when a lens with a radius of curvature of 20 mm was used.

The validity of Eq. (1) is well confirmed experimentally. For example, in the experiments to which Fig. 3 relates, the temperature of the water was 16.7°C and therefore the velocity of propagation was 1476 m/sec. According to Eq. (1), the focal length should be 52.5 mm, and the measured focal length checked this value within experimental error.

Figure 4 shows an ultrasonic beam incident off the center of a plano-cylindrical lens ($r=30$ mm). As can be seen from the photograph, the lens still focusses reasonably well.

In the construction of lenses, it is desirable to reduce their thickness to a minimum. The lens of Fig. 3 is 1 mm thick along the focal axis.

It is interesting to observe in Fig. 3 the increased luminosity near the focus, caused by the concentration of energy. In order to obtain a better picture of the focussing action, we have directed into the liquid a parallel beam of monochromatic light through a small rectangular zone, centered on the focus of the acoustic lens. The diffraction patterns produced by the ultrasonic waves are photographed with a single lens. Figure 5 reproduces these photo-

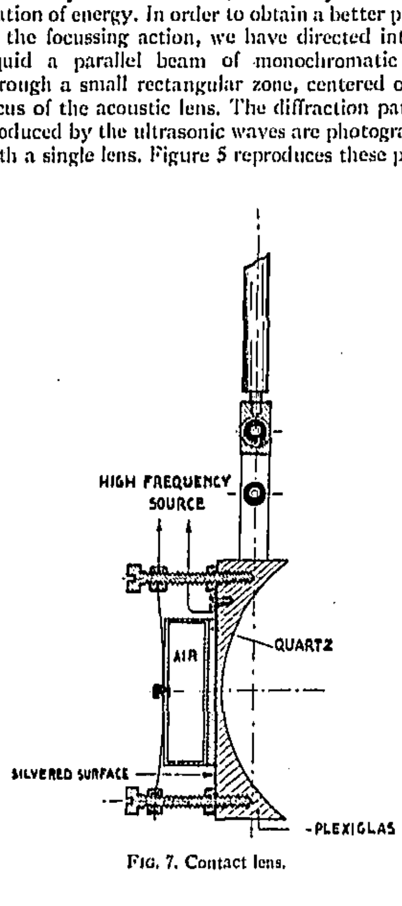

Fig. 7. Contact lens.

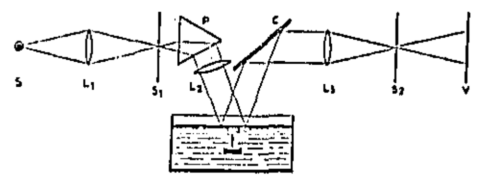

Fig. 8. Optical arrangement for observation of the liquid surface receiving the ultrasonic beam.

graphs (a–g), obtained by varying the voltage across the quartz plate, between 10 and 100 volts at intervals of 15 volts. It is apparent that the number of the observable diffraction images increases with the voltage and that the greatest intensities are observed on the axis. The other diffraction patterns (h–n) refer to the same zone of the ultrasonic field when examined after removal of the lens. Comparing photographs obtained with the same voltage across the quartz, e.g. 100 volts, one can observe the great advantage resulting from the use of the lens in spite of the absorption in the solid material. For the first appearance of the third order pattern, the quartz plate needs 85 volts, while the use of the lens reduces the supply voltage needed to 25 volts. *To obtain the same intensity*, the potential required with the same lens is reduced by a factor of $3\frac{1}{3}$ and therefore the energy emitted by the quartz is reduced by a factor of 11. Figure 6 shows the effect on the beam of Fig. 2, produced by a divergent lens ($r=25$ mm).

CONTACT LENSES

A better performance and a more practicable system can be obtained by placing the Plexiglas lenses directly in contact with the quartz emitter; in this way the energy is transmitted directly from the quartz to the Plexiglas, and from the latter to the liquid. Since the characteristics of the Plexiglas are intermediate between those of the quartz and those of the liquid, the presence of the lens facilitates the transmission of energy from the quartz to the liquid.

Figure 7 shows some contact lenses that we have constructed. The plane surface of the lens is silvered and forms one of the electrodes between which the quartz is held. The other electrode consists of a small cylinder filled with air, having very thin walls. This ensures that the ultrasonic radiation occurs essentially from one side only of the quartz, thus allowing a better performance. We have made two lenses of this type, one plano-cylindrical and one plano-spherical, both having the same radius of curvature ($r=25$ mm) and therefore the same focal length, if used in the same liquid. In order to show their performance, we have examined the ultrasonic beams emergent from these lenses when used with the same quartz. For comparison, we have tried a third case in which the quartz, arranged in a manner identical with that of Fig. 7, is placed behind a disk of Plexiglas of thickness equal to that of the lenses along their focal axes (1 mm).

In all three cases the quartz has been placed horizontally to give an upward radiation and at the same depth below the surface of the liquid. The ultrasonic radiation which reaches the surface of the liquid causes an increase in curvature with an increase in intensity. It is possible in this manner to determine the cross-section of the beam. Observations of the surface of the liquid are made by the method suggested by Toepler, illustrated in the diagram of Fig. 8. The source and the condenser L_1 form a secondary point source at the circular window S_1. The luminous rays from this secondary source are deviated by a prism and made parallel by a lens L_2. The beam is reflected from the surface of the liquid, and then by a mirror. The lens L_3

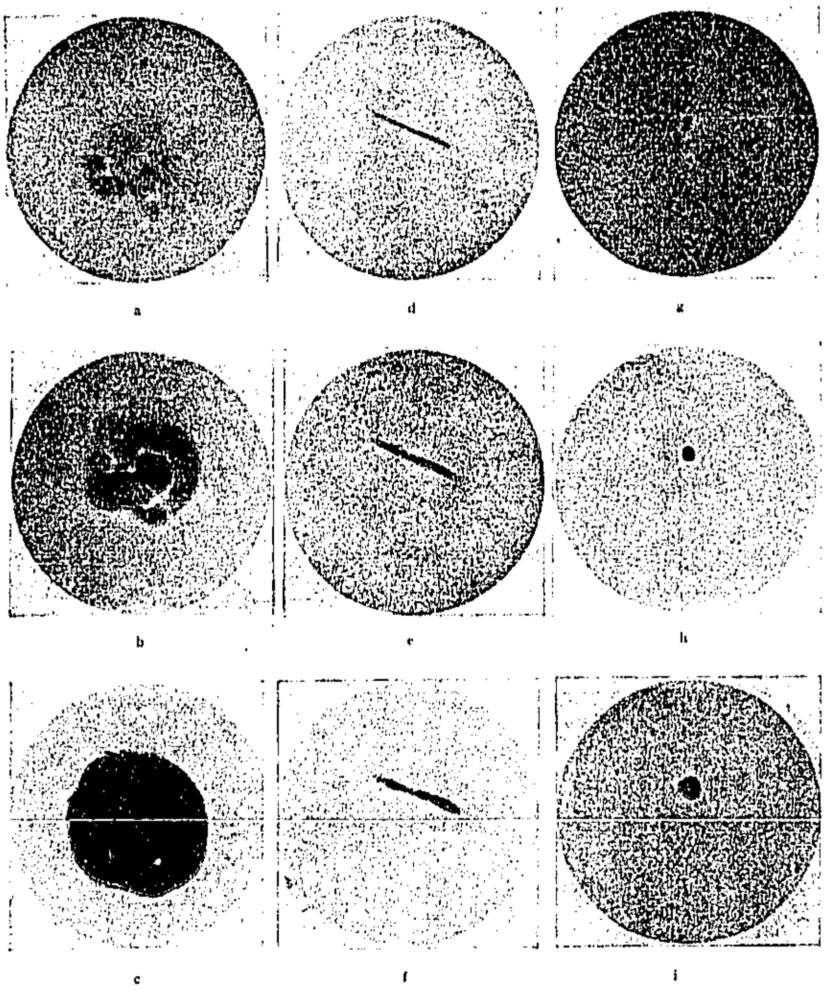

FIG. 9. Image of the liquid surface when it is struck by an ultrasonic beam. In a, b, c, a Plexiglas disk is put in front of the quartz and the supplying voltages are 75, 150 and 250 volts; in d, e, f, the Plexiglas disk is substituted by a cylindrical lens ($r=25$ mm) and the voltages are 30, 40 and 70 volts; in g, h, i, the quartz is back to a spherical lens ($r=25$ mm) and the voltages are 15, 20 and 30 volts. Frequency: 4.2 Mc/sec. Liquid: paraffin oil.

focuses the beam in the plane of the screen S_2, giving an image of the source and an image of the surface of the liquid on the ground glass screen, V. The screen S_2 has a circular hole that will pass only rays reflected from the horizontal surface of the liquid. In the curved zone of the liquid surface, the rays are reflected in different directions and are stopped by the screen S_2. On the ground glass screen, the image of the surface appears uniformly illuminated if the liquid is at rest, and appears dark where ultrasonic radiation strikes the surface.

By varying the depth at which the lens is immersed, it is possible to place the focus at the surface of the liquid, and so to measure the focal length.

The photographs of Fig. 9 show the results of experiments in paraffin oil, for several input voltages under the following conditions:

A. disk of Plexiglas in contact with the quartz (a, b, c),
B. plano-cylindrical lens (d, e, f),
C. plano-spherical lens (g, h, i).

The frequency is 4.2 megacycles, the depth of immersion about 50 mm. It is obvious from these photographs that excellent focussing can be obtained with such lenses.

The method of evaluation adopted, which makes use of the surface tension of the liquid, does not

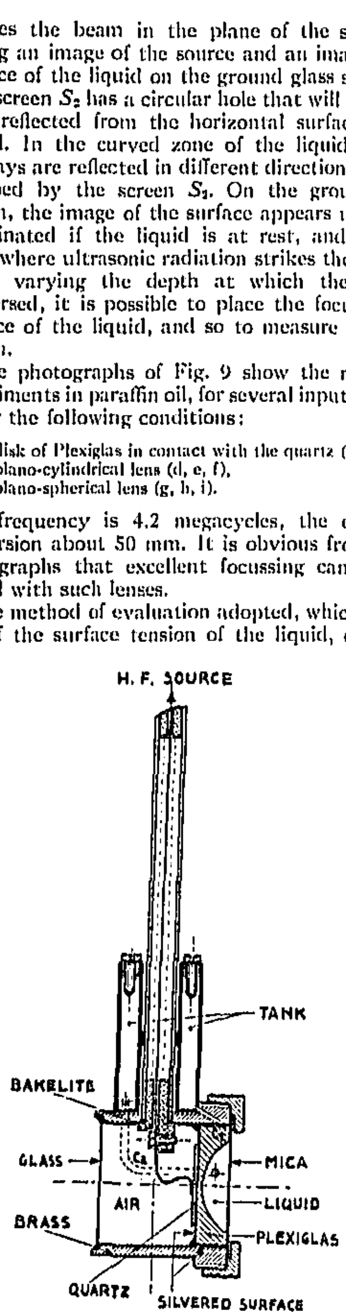

FIG. 10. Double lens, for liquid dissolving the Plexiglas.

permit of a quantitative measurement. As a qualitative indication, the potentials necessary to obtain the first slight shade in the image are: for the disk, 38 volts; for the cylindrical lens, 13 volts; for the spherical lens, 3 volts. This is a rough indication, but it agrees fairly well with the results of the section on some acoustic solid lenses used, and demonstrates the advantages which may be obtained.

LENS FOR LIQUIDS THAT DISSOLVE PLEXIGLAS

In the experiments described above, many common organic liquids could not have been used because they dissolve Plexiglas. We have adopted the arrangements shown in Fig. 10 to eliminate contact between the Plexiglas and the external liquid. The quartz in this case is in contact with the plane surface of the Plexiglas lens, but the latter is not immersed in the external liquid. The Plexiglas is, instead, in contact with a liquid which has no effect on it. This liquid, that fills the cavity formed by the Plexiglas lens and a mica window, is introduced through two channels that communicate with two tanks.

The refractive element interposed between the quartz and the external liquid is composed of a solid contact lens and a liquid lens formed by the liquid that fills the space between the solid lens and the mica window. The optical formula can be applied, the focal length of the doublet being given by

$$1/f = 1/f_1 + 1/f_2. \qquad (2)$$

The focal length of the solid lens (f_1) is positive because the lens is concave, while the focal length of the liquid lens (f_2) can be either positive or negative, depending on the values of the velocities in the liquid of the lens and in the external liquid. For this liquid lens we can find a relationship similar to Eq. (1):

$$f_2 = -\frac{r}{1-(V_2/V_1)}, \qquad (3)$$

V_2 being the velocity in the external liquid, and V_1 that in the liquid of the lens.

The selection among those liquids which do not attack Plexiglas must be made also from the point of view of the velocity, remembering that it is desirable for it to have a characteristic acoustic impedance intermediate between that of Plexiglas and that of the external liquid. Furthermore, the coefficient of absorption ought to be as low as possible. Water and carbon disulphide lend themselves very well in most cases.

Figure 11 shows the operation of such a system. The cylindrical surface has a radius of 25 mm, the liquid medium of the lens is water, while the external liquid is xylol.

CONCLUSIONS

In contrast to other systems which have been suggested in the past for the construction of acoustic lenses, plastic materials allow a more simple construction because of the ease with which they can be worked mechanically. Moreover, the chemical properties of such materials do not impose any great limitations, since it is easy to avoid contact between the plastic materials and any liquid in which they dissolve. The concentration of energy produced by such systems is very satisfactory. It can, in fact, be said that the use of a plano-cylindrical lens permits a reduction of the energy emitted by piezoelectric quartz to one tenth to obtain the same intensity in a certain region of the ultrasonic field. Finally, a plano-spherical lens under the same conditions reduces the energy to one hundredth.

Acoustic lenses constructed from such plastic materials appear to be useful when it is necessary to produce high intensity ultrasonic radiation over a small area.

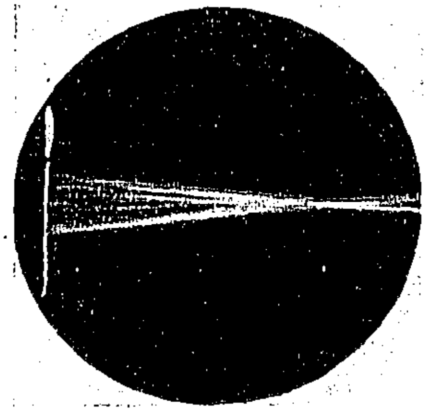

FIG. 11. Double lens, containing a liquid lens (water), in xylene.

The author wishes to thank Professor A. Giacomini for having suggested the subject of this research and for the advice he has given while it was being carried out.

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 21, NUMBER 4 JULY, 1949

A Low "Q" Directional Magnetostrictive Electroacoustic Transducer

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(Received April 30, 1949)

The description of a lamination design for the magnetostrictive motors of a directional transducer array. The design makes possible the efficient operation of the transducer with a "Q" of 6 under a full water load. Array patterns are presented to show that the laminated motors radiate as plane pistons into the medium.

I. INTRODUCTION

DIRECTIONAL magnetostrictive transducers for use as underwater sound projectors and receivers may consist of arrays of longitudinal mechanical vibrators bonded to a common diaphragm or sound window. The individual vibrators are bar-like structures, laminated to decrease eddy current losses. One end of each bar presents a radiating surface transmitting sound through the window to the medium. These surfaces are spaced so as to cover about 80 percent of the window area, and the vibration of each may be controlled in amplitude and phase to produce the desired sound pattern.

The longitudinal vibrators are mechanically resonant systems having an optimum efficiency of energy conversion in the vicinity of the resonant frequency. For example, a transducer in a constant pressure sound field will develop a voltage across its terminals which varies with frequency in a manner illustrated by Fig. 4. The band width of the response curve may be described in terms of its "Q" given by the quotient of the band width between the 3 db down points into the frequency of the peak response. This quantity is determined by the mechanical "Q" of the resonant system where

$$Q=\bar{M}\omega_0/R. \qquad (1)$$

$\bar{M}$ is the equivalent mass of the resonator, ω_0 its resonant angular velocity, and R is the dissipative load imposed upon it.

One advantage of the laminated type of magnetostrictive transducer is the possibility it offers for wide variations in band widths through proper design. A recent paper[1] indicated a practical range of Q from 6 to 40 for this particular style of lamination. As the possibility of the Q of 6 has been questioned, its design was undertaken at the Ordnance Research Laboratory of the Pennsylvania State College, and simple arrays were constructed to test its behavior.

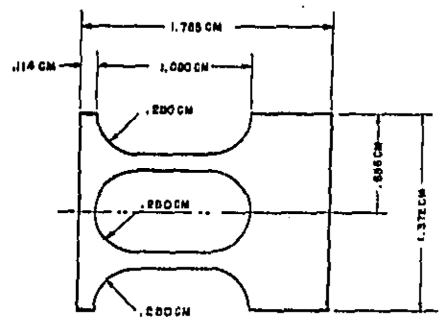

FIG. 1. 84-kc lamination.

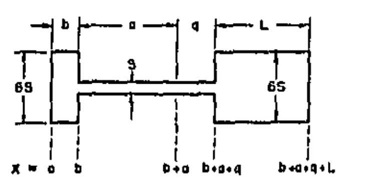

FIG. 2. Simplified form of lamination.

II. LAMINATION DESIGN

The lamination is shown in Fig. 1. To calculate the mechanical "Q" of this system according to Eq. (1), its equivalent mass must be determined. By equivalent mass is meant the mass which a simple spring and mass system would have if this mass and spring, moving with the same frequency and amplitude as the radiating end of the lamination of Fig. 1, possessed the same amount of energy as that of the lamination. Figure 2 shows a simplified form of Fig. 1. When the system of Fig. 2 is vibrating longitudinally in its fundamental mode, the velocity amplitude $\dot{\xi}$ may be expressed as the following functions of x:

$$0<x<b;\quad \dot{\xi}=A\cos kx$$

$$b<x<b+a+q;\quad \dot{\xi}=A\frac{\cos kb}{\sin ka}\sin k(b+a-x)$$

$$b+a+q<x<b+a+q+L; \qquad (2)$$

$$\dot{\xi}=-A\frac{\cos kb\,\sin kq}{\sin ka\,\cos kL}\cos k[x-(b+a+q+L)].$$

[1] L. Camp, J. Acous. Soc. Am. 20, 616–19 (1948).

Let units be chosen such that the area of the radiating face of a stack of the laminations is unity. At a time when all of the energy is kinetic:

$$\frac{1}{2}\bar{M}A^2=\frac{\rho A}{2}\left[\int_0^b \cos^2 kx\,dx+\frac{1}{6}\frac{\cos^2 kb}{\sin^2 ka}\right.$$
$$\times\int_b^{b+a+q}\sin^2 k(b+a-x)\,dx+\frac{\cos^2 kb\,\sin^2 kq}{\sin^2 ka\,\cos^2 kL}$$
$$\left.\times\int_{b+a+q}^{b+a+q+L}\cos^2 k[x-(b+a+q+L)]\,dx\right]$$

or

$$\bar{M}=\frac{\rho}{2}\left[b+(q+a)\frac{\sin 2kb}{\sin 2ka}+L\frac{\sin 2kb\,\sin 2kq}{\sin 2ka\,\sin 2kL}\right].$$

III. EXPERIMENTAL DATA AND CONCLUSION

Stacks were made, using the laminations of Fig. 1, from 0.004-in. 2V Permendur with Cycleweld C-3 cement serving as insulation and bonding agent between laminations. This structure has a density of 6.2 and the velocity of sound in it is 5.25×10^5 cm/sec. The other quantities are: radiating area $=1.7$ cm², $k=2\pi/\lambda=1.00$ radians/sec., $a=0.742$ cm, $b=0.175$ cm, $q=0.216$ cm, $L=0.633$ cm, $f_0=84$ kc, $M=1.85$ g per unit radiating area. In addition to the radiation load, there are losses associated with the unloaded oscillator which may

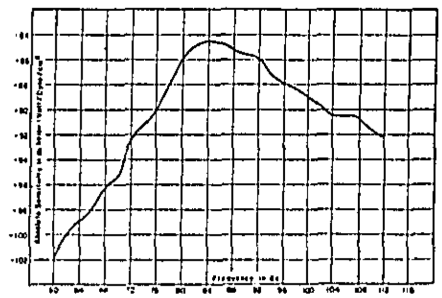

FIG. 4. Open circuit voltage response.

be determined from the Q of the motional impedance circle shown in Fig. 3. If R_i is the loss associated with unit radiating area and the total area is A,

$$Q=\bar{M}\omega_0/R_iA=41 \text{ from impedance circle,}$$

$$R_i=\frac{1.85\times2\pi\times84000}{41\times1.7}=14000 \text{ ohms.}$$

With the additional radiation resistance of ρC into a water load, the Q of this transducer should be

$$Q=(1.85\times2\pi\times84000)/(1.64\times10^5)=6,$$

which is the value shown by the response curve of Fig. 4.

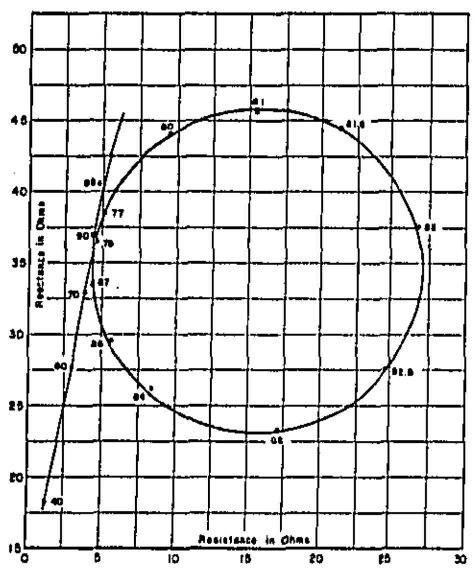

FIG. 3. Motional impedance of unloaded stack.

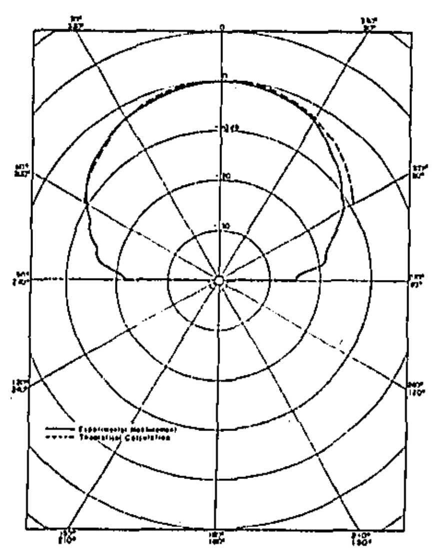

FIG. 5. Radiation pattern of a single stack.

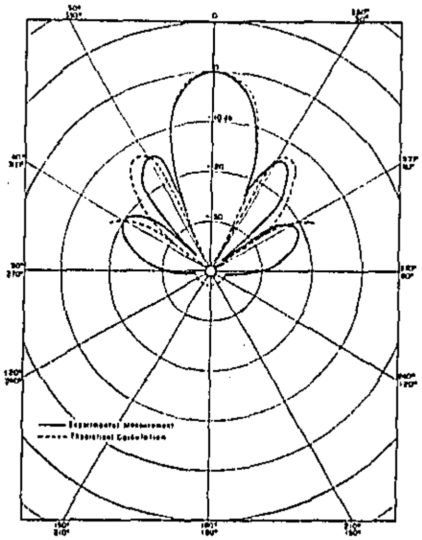
FIG. 6. Radiation pattern of nine-stack square array.

Three other questions to which the experimental data provide answers are: (1) Does the radiating face of a stack of the laminations behave like a piston when driving into water? (2) What is the efficiency of a transducer made of these stacks? (3) What is its power handling capacity?

Figure 5 is a measured pattern in the plane of the laminations for a single stack compared with the theoretical pattern of a plane piston. Figure 6 is a similar comparison of the pattern of a nine-stack square array, all stacks driven uniformly. These patterns show that the stacks do radiate as pistons into water. The directivity ratio of the array is 0.01, its input impedance at 84 kc is $100+j394$ ohms. Using these data and the open circuit voltage response, the efficiency of the transducer is −3 db. This transducer operates on the remanent magnetization of half-hard Permendur. From the impedance, the number of turns in the exciting coils, and the stack dimensions, it can be shown that the stack can be safely driven with 20 watts r.m.s. power. The power handling capacity may be greatly increased by using a soft material and some means of polarization.

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Direct Reading Microdisplacement Meter

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(Received March 7, 1949)

Equipment has been built for measuring vibratory displacements of very small mechanical elements such as phonograph styli and piezoelectric crystals. It employs a probe of small dimensions so that virtually point measurements may be made. The probe does not contact the point under measurement and therefore imposes no mechanical load. The variation in capacitance between probe and vibrating surface is used to measure the displacement. Through the use of a built-in calibrator, the sensitivity may be adjusted electrically for direct meter reading of vibratory displacement without resorting to precise adjustment of condenser plate spacing. Displacement amplitudes of less than 10^{-8} cm may be measured. The output signal corresponds accurately to the displacement both in magnitude and in phase over a wide frequency range so that complex vibrations are portrayed accurately on a cathode-ray oscilloscope. The equipment has been calibrated by four independent methods, including a reciprocity method, with close agreement.

INTRODUCTION

MANY convenient and accurate instruments are available for measuring impedance, voltage and current in electronic circuits. Unfortunately, in the development of mechanical vibrating systems, such as microphones and phonograph pick-ups, just the opposite situation exists. Analogous instruments for measuring mechanical impedance, force, and velocity are not generally available, although many specialized instruments have been built to meet particular requirements. Thus, several years ago when it became necessary to measure the vibratory displacements of very small areas on piezoelectric crystals and to do so without imposing any mechanical load on the crystals, it was necessary to develop a special instrument for the purpose.

DESCRIPTION OF EQUIPMENT

The model to be described has been in constant use since early 1946 and incorporates the results of extensive experience with two predecessor models. It employs the well-known principle of imparting to a variable condenser the vibrations to be measured and including that condenser in the frequency determining network of an oscillator so that the oscillator is frequency modulated by the vibrations. The output of an f-m receiver tuned to the oscillator thus corresponds to the vibrations. A number of novel features are incorporated which permit reduction of the condenser plates to unusually small dimensions and which permit direct meter reading of displacement or alternatively the determination of transducer sensitivity by a null method.

Figure 1 is a schematic diagram. At 1 is a piezoelectric "Bimorph" transducer element whose sensitivity is to be measured. A metal rod 2, having a diameter on the order of 0.04 inch, is supported with its end very close to the vibrating face of the crystal. The capacity between the end of the rod and the crystal electrode forms a part of the frequency determining circuit of oscillator 3, which operates at about 100 megacycles. The rod 2, called a probe, is supported for endwise vibration on two leaf springs and can be excited in this mode of vibration by an *ADP* "Bimorph" element 5. A voice coil 6 partakes of the same vibration as rod 2, and hence the voltage induced in the coil is proportional to the velocity of the probe. A Miller type integrating amplifier 7 provides a voltage proportional to the vibratory displacement of the coil and probe. For convenience, the gain of the amplifier is preset to produce an output of 1 volt for 10^{-4} cm or 10^{-3} cm vibratory displacement of the probe, depending on the position of a 10:1 attenuator. The adjustable phase shifter is designed to have negligible effect on the amplitude of the signal transmitted through it. Its purpose will appear later.

The instrument may be used in either of two ways, depending on the type of information desired. One is a balance or null method, and the other is a direct meter reading method.

BALANCE METHOD

In the balance method, the two crystals are connected to a common oscillator through separate potentiometers for individually adjusting the driving voltages. The circuit to the driving crystal 5 also includes an adjustable phase shifter. In operation, one potentiometer is adjusted to apply approximately the desired voltage to the test crystal and then the other potentiometer and the phase control are adjusted so that the test crystal and probe vibrate with equal amplitudes and exactly in phase. When this adjustment is accomplished there is no relative motion between the probe and the test crystal, and accordingly there is no frequency modulation of the oscillator. A null detector at the output of the f-m receiver indicates this condition.

Next the driving voltage on the test crystal is sampled by calibrated attenuator 8, and compared with the output of the voice coil amplifier in a null

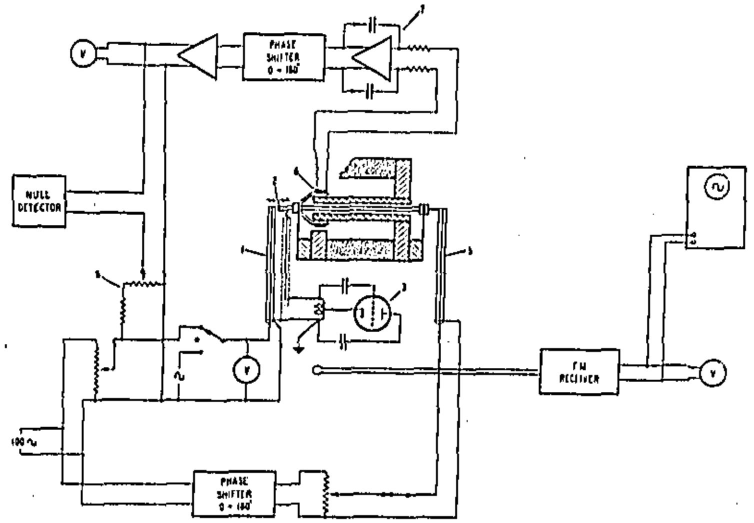

FIG. 1. Schematic diagram of microdisplacement meter.

detector. The amplitude of the voltage sample is adjusted by means of the attenuator to equal the output of the voice coil amplifier, and the phase shifter in the voice coil circuit is adjusted to bring the two voltages into phase opposition to achieve a balance.

With the two voltages equal, all necessary information is available for determining the sensitivity of the test crystal. Let:

- E_{vc} = voltage output of voice coil amplifier,
- D_{vc} = displacement of voice coil,
- E_x = driving voltage on crystal 1,
- D_x = displacement of crystal 1,
- E_a = voltage output of attenuator 8,
- S_{vc} = sensitivity of voice coil system = E_{vc}/D_{vc},
- S_x = sensitivity of crystal = D_x/E_x, and
- a = attenuator (8) ratio = E_a/E_x.

Now by balancing the crystal and probe displacements we obtain the relation

$$D_{vc}=D_x. \quad (1)$$

From the above definitions

$$D_{vc}=E_{vc}/S_{vc}, \quad (2)$$

$$D_x=S_xE_x. \quad (3)$$

Substituting (2) and (3) in (1) we obtain

$$S_x=E_{vc}/S_{vc}E_x. \quad (4)$$

By balancing the attenuator and amplified voice coil voltages we obtain the relation

$$E_a=E_{vc}. \quad (5)$$

From the definition of a:

$$E_a=aE_x. \quad (6)$$

Substituting (6) in (5):

$$E_{vc}=aE_x. \quad (7)$$

Substituting (7) in (4):

$$S_x=aE_x/S_{vc}E_x=a/S_{vc}. \quad (8)$$

Thus the crystal sensitivity in cm/volt is merely the reciprocal of the voice coil system sensitivity (10^3 volts/cm or 10^4 volts/cm) multiplied by the ratio a of the attenuator.

If the relationship between crystal displacement and driving current is required, attenuator 8 is replaced by a calibrated variable resistor in series with the crystal and adjusted to make the voltage drop across it equal to the output of the voice coil amplifier. Alternatively, the relationship between displacement and driving charge may be determined by placing a variable calibrated condenser in series with the crystal and adjusting it so that the voltage across it equals the voice coil amplifier output.

Furthermore, input-velocity relationships of transducers may be determined directly by this balance method by eliminating the integrating amplifier so that the output of the voice coil amplifier is proportional to velocity rather than displacement. The balance method is rapid and convenient to use, it eliminates meter errors, and it reduces the computation of transducer sensitivity to a simple operation such as multiplying the attenuator ratio by 10^{-3} or 10^{-4}. Furthermore, reliable measurements can be made at amplitudes below the noise level of the equipment with the use of a tuned null detector to improve the precision of balance adjustment. Use of the method is limited however, to relatively low frequencies because of the rather large mass of the probe-coil-driving crystal system making it difficult to obtain sufficient amplitude of vibration of the probe at frequencies much above the resonance frequency of about 300 cycles. The equipment, however, is not limited to low frequency operation as the modulated oscillator f-m receiver combination has uniform sensitivity over a wide frequency range, and by using a simple calibrating adjustment, direct meter readings of vibratory displacement may be made.

DIRECT READING METHOD

For direct reading wide frequency range applications, it is necessary to calibrate the equipment for each setup as the sensitivity depends on the spacing of the probe from the vibrating object and the required spacing is too small for accurate preadjustment. With the test transducer in place but not excited, crystal 5 is excited at 100 cycles to drive the probe with a desired amplitude as indicated by the voltmeter at the output of the voice coil amplifier. Since the gain of that amplifier has been preset to provide 1 volt output for 10^{-3} or 10^{-4} cm, the meter reading in volts$\times 10^{-3}$ or 10^{-4} as the case may be equals the displacement in centimeters. The audio gain control in the f-m receiver then is set so that the volt meter at the receiver output has the same reading. Then it also reads directly the amplitude of vibration displacement of the probe, provided that the test transducer remains without excitation. Now it should be recalled that the f-m receiver output is proportional to the relative displacement of probe and test crystal. Consequently, if the probe excitation is shut off and the test crystal excited, the f-m receiver meter reads the displacement amplitude of the test crystal. Since the response of the equipment is flat, calibration at 100 cycles serves for the whole audio frequency range. The vibration wave form of the test crystal may be observed on the screen of the cathode-ray oscillograph at the receiver output.

MECHANICAL LOADING

One of the primary reasons for using a variable capacity as the measuring element was the fact that mechanical loading of the device under test could be reduced to negligible value even for vibrating systems of very low mechanical impedance. If the vibrating object under examination has a conductive surface as in the case of a flexing piezoelectric crystal element, that surface may be used as the vibrating condenser plate making it unnecessary to make any mechanical connection to the vibrating device. If the vibrating object is a good high frequency dielectric material, measurement also can be made without the addition of a condenser plate, as will be shown later. In some cases however, it may be desirable to provide a thin conductive surface by applying metal foil or conductive paint. Thus for most purposes, the equipment measures vibratory displacement without imposing any loading and may therefore be called a zero impedance displacement meter.

PROBE

In order to measure the vibratory displacement of a very small object such as a phonograph stylus, or to explore small vibrating surfaces such as headphone diaphragms or flexing type piezoelectric elements, it was necessary to reduce the probe electrode area to a minimum; and in order to confine the measurement to a small area immediately opposite the probe, a shield was installed around the remainder of the probe. Figure 2 shows the probe construction. The metal rod 1 forming the probe electrode is pressed into a polystyrene rod 2. The base end of the rod then is mounted in a brass support 3 and the active end is turned down to a cone. Next a shield layer of conductive paint 4 is applied to the whole assembly including the exposed end of the rod 1 and overlapping the base 3. After the paint has dried, the end of the probe is turned off flat until a very slight insulating ring margin appears around the electrode separating it from the shield. The probe shield is "grounded" through the leaf spring suspension. The active probe connection is brought out through an opening in the conducting film shield by a small rod 5. A very thin wire 6 connects this to a rigid conductor 7 leading to the oscillator through a shield 8. Interchangeable probes of various active areas are provided, the smallest having an active area of about 0.005 square inches although the sensitivity is adequate for use of much smaller areas.

When no external object is located in the vicinity of the probe electrode, the oscillator frequency is determined in part by capacity due to electrostatic flux lines from the probe face through the air to

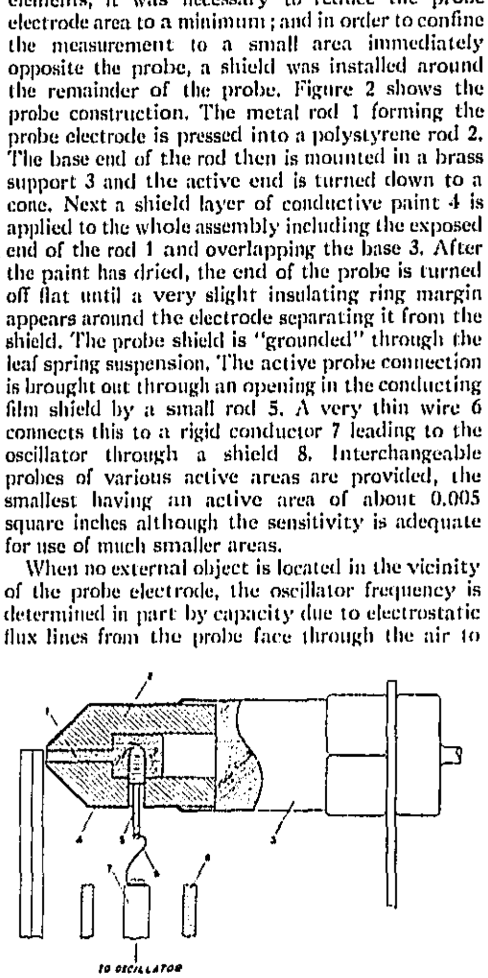

FIG. 2. Probe details. The thickness of the shield is exaggerated.

adjacent areas of the probe shield. Now if a dielectric material having a dielectric constant differing from that of air is brought up close to the probe, the flux density is modified by the dielectric with consequent change in oscillator frequency. Thus, vibrations of a dielectric material can be measured without adding a conductive coating. In this case the flux concentration is not as good as in the case of measurements of a conductive element and for this reason the application of a conductive surface sometimes is desirable.

The matter of flux concentration is one of considerable importance, as very often it is desired to measure the vibration of a small area approaching a point. Since the electrostatic flux lines tend to spread out at the periphery of the electrode, the actual area over which measurement is made is somewhat larger than the probe area. This error decreases as the spacing between electrode and vibrating surface is made a smaller fraction of the probe diameter. The smallest probe that has been used has an active diameter of 0.040 inch. The spacing can readily be reduced to 0.002 inch or less so that the fringing effect is not serious as long as small amplitudes are involved permitting such close spacing.

Further importance of close spacing and its limitations may be observed from the following approximate analysis which assumes that the frequency deviation is very small compared with the mean frequency, a condition which always obtains in practice when using such a small vibrating capacity. The frequency deviation Δf resulting from a small change in capacitance ΔC in an LC oscillator is given by

$$\Delta f = f_0 \Delta C / 2C_0.$$

Where f_0 is the frequency of the unmodulated oscillator and C_0 is the unmodulated capacitance of the oscillator circuit. Neglecting edge effects, the change in capacitance of a parallel plate condenser having plate area A and plate spacing t is given by

$$\Delta C = KA\Delta t/(t^2 - t\Delta t).$$

Where Δt is the change in plate spacing. Substi-

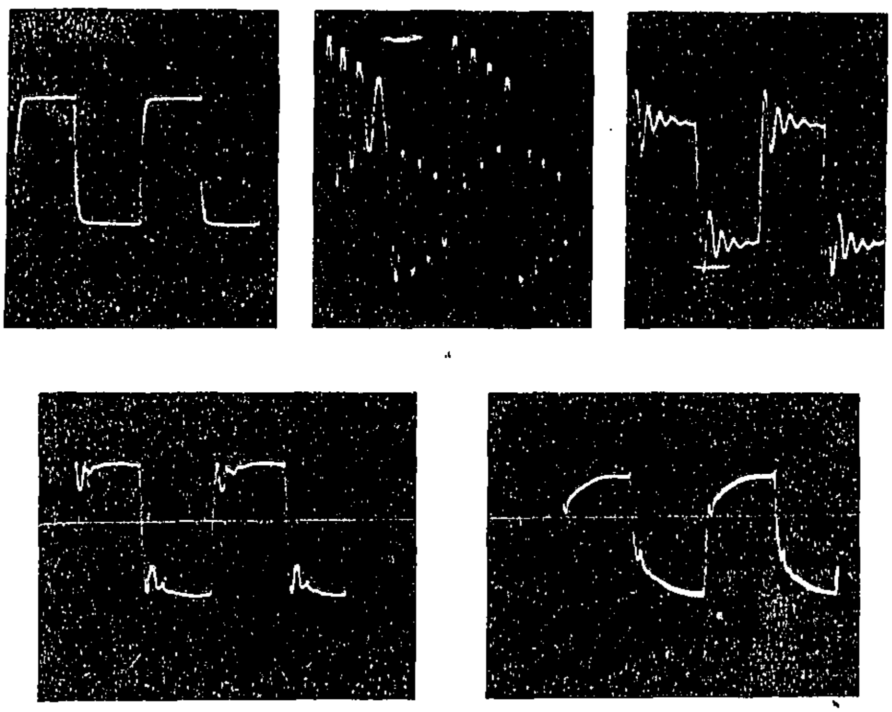

FIG. 3. Cathode-ray oscilloscope traces of driving voltage, and wave form of crystal displacement with various degrees of damping.

tuting this in the expression for frequency we have

$$\Delta f = f_0 K A \Delta l / 2C_0(l^2 - l\Delta l).$$

The above expression for Δf shows that the frequency deviation and hence the f-m receiver output is proportional to the displacement Δl to be measured only if the mean plate spacing l is large compared with the vibratory displacement Δl. If the spacing is too close, the wave form of the f-m receiver output will be distorted. On the other hand, for a given vibration amplitude Δl, the deviation Δf increases as the plate spacing l is decreased, making close spacing desirable for high sensitivity. Thus for measurement of very small vibratory displacements, very close spacing of probe and vibrating surface is desirable for sensitivity and can be employed without distortion. For large amplitudes larger spacing is required to preserve linearity but this is permissible because less sensitivity is required. Of course, the considerations of linearity apply only to the direct meter reading method of using the microdisplacement meter.

SENSITIVITY

This relationship together with the fact that the noise output due to circuit disturbances is not influenced by probe position, permits measurements over an extremely wide amplitude range. Within broad limits, provided that the probe spacing is carefully adjusted for the vibration amplitude involved, the signal to noise ratio is virtually independent of vibration amplitude. Using the balance method, the maximum amplitude that can be measured is about 10^{-3} cm as that is the maximum amplitude available from the crystal driving the probe. In direct reading applications somewhat larger displacement amplitudes may be measured. The noise level corresponds to an amplitude of about 2.5×10^{-7} cm for the smallest probe so that in direct reading applications, measurements can be made with reasonable accuracy at amplitudes of 2.5×10^{-6} cm while for the null method using a tuned null indicator to sharpen the null, amplitudes as small as 10^{-7} cm may be measured.

OSCILLATOR-RECEIVER

It is well known that the signal to noise ratio in an f-m system increases as the frequency deviation increases. Referring to the expression for deviation Δf it is obvious that a high oscillator frequency and low oscillator circuit capacitance are desirable. Fortunately these two are quite compatable. A frequency of approximately 100 megacycles was chosen as it made possible the use of a commercially available f-m receiver with only minor modifications and because using a much higher frequency would impose severe limitations on the geometry of the modulated oscillator circuit.

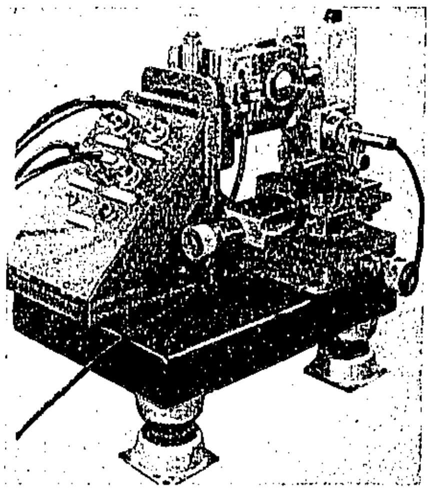

FIG. 4. Photograph of microdisplacement meter.

In f-m broadcasting it is standard practice to emphasize the high frequencies in the transmitter and to de-emphasize them correspondingly in the receiver. This made it necessary to modify the audiofrequency portion of the receiver to achieve flat frequency response. The de-emphasis network was removed and the entire audio amplifier was replaced by a resistance capacity coupled amplifier designed to reduce phase shift errors to a minimum.

RESPONSE

The frequency response and phase shift of the system were determined by "measuring" the sensitivity of a short expander bar of ADP crystal having a resonance frequency of about 50 kc. From theoretical considerations the displacement of the end of such a bar should be in phase with and proportional to the driving voltage for all frequencies up to at least 10 kc. Thus the phase shift was determined by measuring the phase angle between the crystal driving voltage and the f-m receiver output, and the frequency response was determined from the ratio of receiver output to crystal voltage. Over the audiofrequency range the sensitivity is independent of frequency within the limits of error of measurement, about ±1 percent, and the phase shift increases approximately proportional to frequency, indicating a time delay of about 10 micro seconds. Thus complex vibratory wave forms are portrayed accurately in the cathode-ray oscillograph at the receiver output.

The use of the microdisplacement meter for examining complex vibration wave forms is illustrated in Fig. 3 which shows the displacement-time relationship of a "Bimorph" crystal with various degrees of mechanical damping applied at the drive point for "square wave" excitation. The resonance frequency of the mounted "Bimorph" was about 700 cycles and the fundamental frequency of the "square wave" driving voltage was 100 cycles. In the upper row the left hand trace shows the "square wave" driving voltage, the central trace shows the vibratory displacement for no external damping, and the right-hand trace shows the displacement for damping somewhat less than critical. In the lower figures the left-hand trace shows the displacement for approximately critical damping and the right-hand trace shows over damping. It appears that some secondary resonance has not been completely suppressed by the damping.

CONSTRUCTION

Figure 4 is a photograph of the microdisplacement meter. The magnetic circuit for the voice coil is mounted on a lathe cross slide arranged to provide vertical adjustment of the axis of the probe. The 100 megacycle oscillator is contained in the aluminum box carried by the magnetic structure, and the probe driving crystal is contained in a box to the rear of the magnetic system. Additional lathe parts provide a table, adjustable in the horizontal plane, on which may be mounted the device to be measured. A large twister "Bimorph" crystal is shown mounted in position in a ball-point type holder permitting free vibration of all four crystal corners, the upper corner being in position in front of the probe for displacement measurements. The various parts of the displacement meter are mounted on a surface plate having "Lord" mounting feet for vibration isolation.

Figure 5 is a photograph of the electronic equipment rack. The upper panel is a null detector. Next below is a two channel amplifier with phase shifter for driving the probe crystal and the test transducer. Near the center is the voice coil integrator-amplifier with phase shifter and below that is the f-m receiver. The tuning shaft of the receiver is belted to a servo motor through a gear reduction contained behind the panel just below the receiver. The motor is actuated by any positive or negative unbalance voltage out of the receiver discriminator to retune the receiver to center frequency. This automatic tuning system was built to keep the receiver tuned to the oscillator primarily during long time temperature runs on transducer sensitivity but has proved to be a great convenience in daily use of the instrument as it permits readjustment of probe spacing without the necessity for retuning.

FIG. 5. Photograph of electronic equipment rack.

CALIBRATION

Both the null method and the direct reading method of using the microdisplacement meter depend on calibration of the voice coil for information concerning the vibratory displacement or velocity of the probe.

The voice coil calibration has been determined by four independent methods with excellent agreement.

One method called the static method, is based on the fact that in a moving coil system the ratio of voltage induced in the coil to the velocity of the coil causing that voltage to be induced is the same as the ratio of the blocked mechanical force developed by the coil to the current passed through the coil to create that force:

$$e/v = f/i.$$

The ratio e/v is the voice coil sensitivity figure desired. It was determined by measuring f/i. The probe driving crystal was removed, and a fixed condenser plate was positioned close to the end of the probe. The f-m receiver with tuning motor disabled was tuned to the high frequency oscillator and the reading of the receiver tuning meter noted. A known force then was applied axially to the probe-voice coil assembly thus displacing it from its normal position and detuning the receiver, and then a metered direct current was passed through the coil and adjusted to restore the probe and voice coil to the original position as indicated by restoration of the f-m receiver tuning meter to its initial position. With this adjustment the suspension springs were not flexed so the externally applied force just equaled the force developed by the current in the coil. The ratio of this applied force to the current read on the meter is the sensitivity.

The temperature coefficient of the microdisplacement meter calibration was measured by this method. The microdisplacement meter was installed in an oven and arrangements were made to apply and remove the known force by remote control. Over the range of 0 to 40°C the sensitivity varies less than $\pm\frac{1}{2}$ percent.

Another calibration method involved "measuring" the sensitivity of a large ADP crystal plate whose sensitivity also was calculated from the dimensions of the plate and basic piezoelectric constant for the crystal which are known with considerable accuracy. Two such plates have been mounted in holders adapted for easy installation in measuring position and are used for frequent checks of the meter calibration.

A third method is illustrated in Fig. 6. The probe was replaced by a pair of parallel, insulated condenser plates. A grounded shield plate was located between the two and arranged to be positioned by a micrometer screw so that either vibration of the pair of plates or adjustment of the shield plate by the micrometer changed the capacity between the two vibratory plates. The capacity between the plates formed one arm of a capacity bridge driven by a 100-kc oscillator. The capacities between the shield and each plate are indicated in dotted lines. One is across one fixed arm of the bridge and is swamped out by the large capacity of that arm. The other is across the bridge output and likewise is swamped out by the other capacities across the output. The bridge is adjusted nearly to balance and then a curve is plotted of micrometer screw adjustment *vs.* d.c. voltage developed by the rectifier due to bridge unbalance. The rectifier circuit was designed to have equal d.c. and a.c. loads so that the slope of the static curve gives the sensitivity to relative motion of the condenser plates for vibratory motion as well as "static" displacement. Thus the slope of the curve together with the a.c. voltage output of the rectifier produced by an unknown amplitude of vibration of the voice coil may be combined to determine the amplitude of vibration.

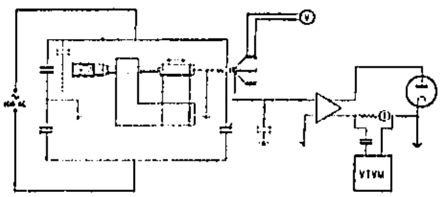

FIG. 6. Schematic diagram of "Static" a-m calibration setup.

The voice coil voltage is measured at the same time and the ratio of this to the vibratory displacement is the desired sensitivity figure. Actually, rather than measure separately the rectifier a.c. output and the integrated voice coil output, the two are balanced by means of the phase shifter and a calibrated attenuator in a manner analogous to the balance method of using the equipment. This method of calibration was the first one used and has been repeated a number of times throughout the life of the microdisplacement meter. There is some indication of a gradual increase in sensitivity but the extremes differ by less than 2 percent.

The fourth method of calibration was the reciprocity method, following the technique of Trent.[1]

The four methods of calibration agree within ± 2 percent of the average value.

[1] H. M. Trent, "The Absolute Calibration of Electromechanical Pickups," J. App. Mechanics **15**, 49–52 (1948).

Auditory Masking of Multiple Tones by Random Noise

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One, two, four, and eight simple tones were presented to listeners against a background of thermal noise. The masked thresholds for the single tones and the various combinations were determined, for different spacings of the tones. In the case of two tones, the improvement in threshold with respect to a single tone was slight or negligible unless the tones were within one critical band, when the improvement increased as the spacing decreased. In the case of four or eight tones all separated by more than a critical band, the improvement was slight (less than 3 db) or negligible, apparently depending on the combination of frequencies.

INTRODUCTION

THE fact that aural detection of weak underwater sounds is ordinarily limited by masking has led the Psychophysics Section of the Navy Electronics Laboratory to conduct a continuing study in this field. Earlier work[1] by this group has shown that the masked threshold of most signals encountered in underwater listening can be predicted from the spectra of the signal and the background noise, provided that the noise has a continuous spectrum, with no very steep slopes; i.e., no negative slopes steeper than about 20 db/octave.

To predict the masked threshold one must know the signal and noise levels in suitably defined "critical bands." The critical band widths have been determined experimentally by H. Fletcher of the Bell Telephone Laboratories in masked-threshold experiments using bands of flat random noise as masking background and a single pure tone in the middle of the band as signal.[2,3]

As the band width of the noise is increased from a few c.p.s., the threshold intensity (the intensity at which half of the signals presented are detected) increases proportionally, up to a certain "critical band width," then remains constant. The critical band widths determined in this way at various frequencies are shown by the small circles in Fig. 1 (right-hand scale). Let us call the critical band width determined in this way $W_1(f)$.

For bands wider than approximately $W_1(f)$, it is found that the signal is heard half of the time when its intensity is equal to the intensity of the noise in a band of width $W_2(f)$, regardless of the band width of the background noise. $W_2(f)$ has been called the kappa-band since it is defined by $\kappa = 10 \log_{10} W_2$. Values for κ have been published in several BTL papers[4] and have been used to give the values for W_2 shown in Fig. 1 (black dots). According to Fig. 1, $W_1(f)$ and $W_2(f)$ are experimentally almost equal. We will, therefore, henceforth use values for the critical band widths determined by the W_2 data which is more precise. Furthermore, we will not distinguish between the bands W_1 or W_2 but will use the term critical band to refer to both.

No statistical evidence of a significant discrepancy was presented by Fletcher. If it is assumed that they are in fact equal, it follows that not only is noise outside the critical band ineffective in masking a tone centered in the band, but the *intensity* of the just audible tone is equal to the intensity of the noise in the critical band,[5] provided that the amount of masking lies in the range from about 15 to 80 db. Since the noise spectrum is constant, the critical band width in c.p.s. is equal to the ratio of the signal intensity, for 50 percent detection, to the intensity of the noise in a 1-c.p.s. band. The corresponding differences in level, on the db scale, are given by the left-hand scale in Fig. 1.

Once the critical band width has been determined as a function of frequency, the masked threshold of a pure tone in noise of continuous but non-uniform spectrum can be predicted. The criterion is that the signal intensity must equal the intensity of the noise in the critical band centered at the signal frequency. In complex underwater signals, one single-frequency component is usually so much stronger than the rest that the critical band criterion applied to this component gives a clear and unambiguous prediction of the masked threshold of the whole signal. The question arises, however, is there an improvement in recognition when two or more components of the signal just satisfy the criterion separately? In particular, is there an improvement in recognition (a) when two pure tones are within the same critical band, (b) when two, four, or eight pure tones are all in different critical bands? It is these questions that the experiments reported here were designed to answer, over a limited range of frequency, intensity, and frequency relations.

[1] R. S. Gales, "Auditory masking in sonar listening systems" (to be published).
[2] H. Fletcher, Rev. Mod. Phys. 12, 47–65 (1940).
[3] H. Fletcher, J. Acous. Soc. Am. 9, 283 (1938).
[4] H. Fletcher and W. A. Munson, J. Acous. Soc. Am. 9, 10 (1937), Fig. 16.

[5] This definition is implied by the usage of N. R. French and J. C. Steinberg, J. Acous. Soc. Am. 19, 96 (1947), Fig. 7.

THE EXPERIMENTS

Five observers were seated in separate booths provided with headphones and a voting key. Before each test they were given a chance to become accustomed to the masking noise. They were then given a practice period, in which they were instructed to vote whenever and as long as they heard the signal. As soon as the experimenter was satisfied that they were voting on the correct signal, the observers were told that the test was about to begin and that they should continue voting as instructed. During the test, the background noise was on continuously, and the signal was normally presented for 3 seconds and removed for 2 seconds. The only exception was the 5-second presentation period used to reduce sampling error when the frequencies were only 1 c.p.s. apart. On one-third of the presentation periods, no signal was presented at all. The observers were aware that there were blank presentation periods but did not know when they were to occur. Errors of commission occurred at an average rate of less than one per observer per test, which indicates that guessing was not a serious factor. At the same time, competition between observers helped to keep them doing their best. The signals were presented at eight different levels, separated in most tests by 2-db steps, but in some by 1-db steps.

To insure that all component tones would reach their respective masked thresholds at the same setting of the variable attenuator, preliminary tests were given to determine their separate thresholds. In 2-, 4-, and 8-tone tests, the levels of the signals were adjusted in accordance with the thresholds determined in the preliminary tests, then the signals were mixed electrically and the mixture was varied in level by known amounts to constitute the variable test stimulus. The tone mixture was then mixed with the background noise and presented to the listeners.

The order of presentation of the different levels seemed perfectly random to the observers, but was not completely random. It was thought desirable to make the same number of presentations at each level and also all possible successions of one level by another with no duplications, to avoid possible bias due to a preponderance of loud or soft items preceding items near threshold.[6] To do this it was necessary to adopt a systematic pattern. However, the levels were assigned to the elements of the pattern at random. No tendency to memorize the test sequences or to anticipate the following level from an item was noticed by any of the observers.

Figure 2 shows a block diagram of the apparatus.

[6] T. H. Schafer, "Influence of the preceding item in measurements of the noise-masked threshold by a modified constant method" (to be published in J. Exper. Psychol.).

A closed loop of random noise on film was played on a film reproducer, amplified, equalized and passed through a fine-adjustment attenuator and an isolating attenuator. Pure tones from eight Hewlett-Packard oscillators were mixed at controllable levels, then passed through a random condition selector, in which a punched tape controls the level of the presentation. A Conn chromatic stroboscope was used to set the frequencies accurately to a precision of approximately 0.1 percent. The signal from the random condition selector passed through a signal-timing switch controlled by a closed timing loop on a second film reproducer. After passing through an isolating attenuator, the signal was mixed with the noise and the combination was passed through a 400–1600 c.p.s. band-pass filter, amplified, and fed to the headphones of the listeners and experimenter. The volume indicator across the output of the power amplifier was used to calibrate the signal and the noise before and after each test.

Each test was 6 minutes long and included six presentations at each of the eight levels and 24 blanks. Only one type of signal was used in a test. The tests were given in groups designed to contrast, under as nearly the same conditions as possible, the response of the observers to different stimuli. The order of tests was random in a given replication of a group of tests, but the order was different each time and when the number of different signals was not too great, all orders were exhausted. The headphones were matched carefully, but to minimize errors from this source each observer was assigned at random to every booth at some time during a series of replications of a test group. Five or six replications was the usual number.

Figure 3 shows the spectrum of the noise in terms of level in the critical bands shown by dots in Fig. 1 plotted in db above the same origin as the average measured absolute auditory threshold of the listeners. The signals used ranged from 600 to 1500 c.p.s. In this range the noise spectrum is flat. The slight rise at the high end in Fig. 3 is due to the widening of the critical band above 1000 c.p.s.

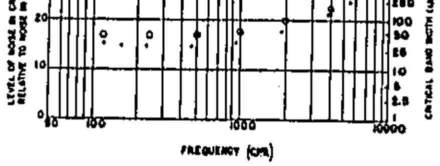

FIG. 1. The width of the critical band as a function of frequency, and the level of the noise in the critical band in db relative to the spectrum level of the noise

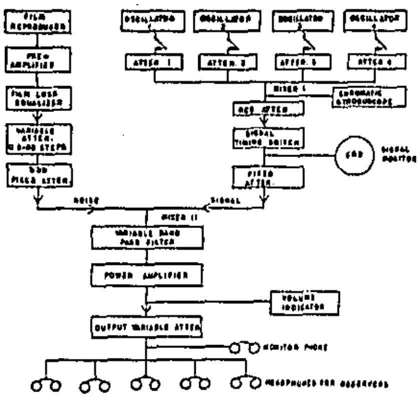

FIG. 2. Block diagram of the apparatus.

Some of the first tests were run with approximately 55 db of masking, some with approximately 15 db. There was no systematic difference in either the masked thresholds or the improvement in masked threshold with multiple tones, and extraneous noise made the tests less precise, so the low level tests were dropped part way through the program.

Figure 4 shows some cathode-ray oscillograms of representative signals, increasing in complexity from the sine wave in the upper left to the complicated but almost periodic resultant of four non-integrally related sine waves.

Figure 5 shows a transition curve (recognition probability as a function of level) for an 800-c.p.s. tone and another for four tones between 600 and 1100 c.p.s. These are averaged over 5 observers and six tests, hence are very much smoother than is usually observed with a single observer's transition curve on a single test. However, all the transition curves are of this form, with all very loud signals being heard, and fewer and fewer being heard as the signal gets weaker. The 800-c.p.s. curve given here checks Fletcher's work very closely; notice that 50 percent recognition takes place when the signal is approximately equal to the noise in a 40-c.p.s. band, the critical band at 800 c.p.s. determined by Fletcher (Fig. 1, black dots). Notice also that the 50 percent level for the 4-tone test is 3 db lower, indicating that for the group of 4 tones chosen, 603, 751, 851, and 1103 c.p.s., the threshold intensity is not independent of the number of tones. Nor is it proportional to the number; for in that case, the 4-tone curve would be 6 db to the left of the single-tone curve, as the power in the resultant of 4 waves of the same amplitude is 4 times that in a single wave.

DISCUSSION OF RESULTS FOR TWO TONES

Figure 6 shows the mean difference between an observer's threshold on a 1-tone test and a 2-tone test taken within an hour. This will be called the mean improvement in threshold. The vertical bars cover the 98 percent confidence interval of the mean. That is, if the measurements could be repeated under the same conditions, the mean would lie between these limits in 98 percent of the samples. The 98 percent confidence limits are obtained as follows. Let $\bar{X}$ be the mean improvement in threshold in a sample of N measurements, μ the mean improvement in threshold in the population of measurements represented by the sample, s the standard deviation of the improvement in threshold in the sample, and N the number of data in the sample. Then $\bar{X}$ is distributed approximately normally about μ, with standard error $\sigma_m = s/(N-1)^{\frac{1}{2}}$.

From a table of deviations of the normal curve, it can be seen that for large samples ($N \geqq 30$), the probability P that the sample mean $\bar{X}$ differs from the population mean μ by less than 2.33 standard errors is 98 percent. That is, $P(|\bar{X}-\mu| \leq 2.33\sigma_m) = 0.98$. The 98 percent confidence interval of the true mean is thus an interval $4.66\sigma_m$ wide and centered at the sample mean.

For two tones 500 c.p.s., or about 10 critical bands, apart, the threshold is not lowered significantly (Fig. 6). That is, the data are consistent with the hypothesis that the improvement in threshold is zero. For two tones 414 c.p.s. apart, the threshold is lowered 1 to 2 db. It happens that the two tones used (603 and 1017 c.p.s.) make a good major sixth, which is a smooth and easily recognizable musical interval; but not enough different intervals were studied to enable us to say that the musical character of the interval improves recognition. This is an interesting problem for future research. For two tones 100 c.p.s. apart, that is, with one critical band intervening, the threshold is not lowered significantly. In view of the probably significant improvement for two tones 414 c.p.s. apart, it seems safest to conclude that the mean improvement with two tones separated by more than one critical band is from 0 to 2 db.

When the two tones are within one critical band, the threshold drops as the frequency difference decreases. The improvement appears to approach a maximum of 6 db as the beat frequency approaches zero and the maximum amplitude approaches twice the amplitude of one of the components.

These observations can all be correlated by assuming that (1) the ear analyzes the sound spectrum into a series of adjacent bands by some means analogous to a series of band pass filters and (2) the ear responds to the "output" of one of the "filters" like a rectifier followed by a low pass filter. Neither assumption is novel; (1) is the outcome of Fletcher's work on the masked threshold[2,3] and (2) has been proposed by Nyquist[7] and Munson[8] to account for the growth of auditory sensation.

There seems to be little theoretical reason for preferring any particular circuit for performing the functions of analysis, rectification and measurement; the tuned-secondary diode detector shown in Fig. 7 will serve. At the left, two e.m.f.'s differing in frequency by Δf but having the same amplitude E are fed to a transformer whose secondary is tuned to the mean frequency f. The secondary is shown shunted by the equivalent parallel resistance of all the factors causing dissipation in the transformer. The maximum amplitude of the primary e.m.f. is $2E$, occurring when the two signals are in phase. The maximum amplitude of the secondary e.m.f. is $2Ex(a)$, where $a=Q\Delta f/2f$ is a function of the Q of the tuned circuit and the fractional detuning of either of the signals. The factor Q is defined as $f/(n-m)$ where m and n are the frequencies at which the response is down 3 db. The factor $x(a)$ is the response of the tuned circuit relative to maximum, for different Qs and different amounts of detuning. Its graph is called the universal resonance curve.[9] The output of the detector is the product of the applied e.m.f., $2Ex(a)$, the detection efficiency, D (a constant depending on the ratio of load resistance to plate resistance), and a factor $y(RC, \Delta f)$ depending on the time constant RC of the low pass filter and the separation of the signal frequencies, Δf.

To test the analogy, the signal frequency pairs used were put through a rectifier and an RC filter. The output was measured for each pair and several different time constants. The results are given in Table I, expressed as the number of db increases over the response to a single tone, read to the nearest 0.5 db. The results for different frequency spacings at a fixed time constant are obtained from reading down the columns. The results for $RC=200$ milliseconds are also plotted on Fig. 6 as a series of small circles. They fit the data fairly well at 1, 3 and 10 c.p.s. separation but remain too high at greater separations. The difference of 0.8 db at 25 c.p.s. between the mean masking data and the analog data can be explained as a loss at the signal frequencies on the sloping sides of the critical band filter. This interpretation makes it possible to compute the Q of the critical band filter, assuming a simple resonant circuit. For a loss of 0.8 db, corresponding to $x(a)=0.91$, $a=0.21=12.5Q/800$. Hence $Q=14$.

A simple resonant circuit having a given Q will have a pass band to the half power points given by f/Q which, at 800 c.p.s. and for a Q of 14, will equal $800/14=57$ c.p.s. This value is in fair agreement with Fletcher's determination of 40 or 50 c.p.s. for the critical band width in the vicinity of 800 c.p.s. That two tones separated by 100 c.p.s. will be resolved by the critical band filters is readily checked by computing the response of a resonant circuit of $Q=14$. If the circuit is considered tuned to f_1 or f_2 it will respond fully to its resonant frequency and attenuate the other 10 db. If considered tuned to the frequency midway between f_1 and f_2 each tone will be attenuated 5.5 db below the level of the single tone. In this case, even with the 6-db increase at the peak of the beat cycle the maximum level will only be 0.5 db above that of a single tone.

No data are available at frequencies between 25 and 100 c.p.s. Hence, it is not possible at present to check the value of 14 computed for Q. However, with further work along this line it may be possible not only to determine a Q for the critical band filter on the assumption that it has the characteristic of a simple resonant circuit, but to determine the actual filter characteristic. The shape of the filter characteristic deduced will depend to some extent on the time constant chosen for the integrating circuit, however.

RESULTS FOR MANY TONES

The improvement in threshold observed with more than 2 tones depends on the sets of tones used, so the sets used will be described briefly. The frequencies in each set are given in Fig. 8. In all sets all of the tones are in separate critical bands. A is a set of 4 tones used in the 2-tone tests. The set forms a very pleasing and easily recognized chord, but one which is foreign to our system of harmony.

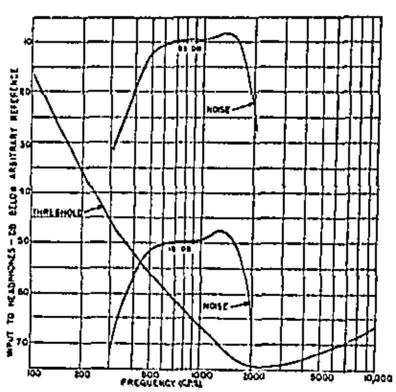

FIG. 3. The critical band spectrum of the noise in relation to the observers' average absolute auditory threshold.

[7] H. Nyquist, "Telegraph theory—Electrical equivalent of the ear as a receiver," NDRC File 36680-3(V) (January 13, 1942).

[8] W. A. Munson, J. Acous. Soc. Am. 19, 584, 591 (1947).

[9] F. E. Terman, *Radio Engineers' Handbook* (McGraw-Hill Book Company, Inc., New York, 1943), p. 137.

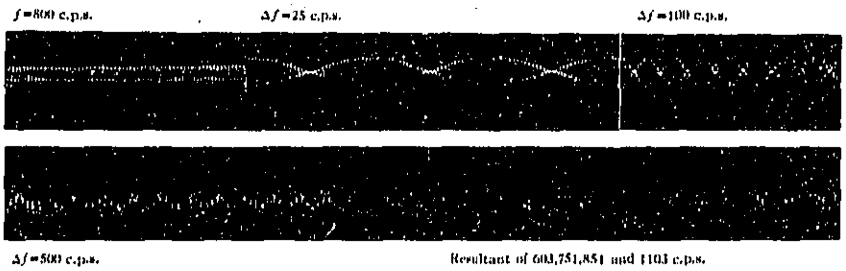

FIG. 4. Oscillograms of some test signals.

B is a diminished-seventh chord in 12-tone equal temperament, chosen as a typical smooth, inoffensive chord. C is a major-seventh chord, quite dissonant even with the pure tones used in this experiment. The tones used in D are separated successively by 200 c.p.s., giving a rough 200-c.p.s. beat note in the headphones. F is a group of 8 tones separated successively by 225 cents, an arbitrary non-musical interval; and E is composed of alternate tones of F. Thus the sets of tones used form a rather fair sample of what could be constructed in the range of frequency studied.

In Fig. 8, the improvement in threshold (comparing the sets of tones with a single 800-c.p.s. tone) is plotted. The vertical bars cover the 98 percent confidence interval of the mean improvement for each set of tones.

The set of eight tones, F, shows a mean improvement of 0.8 db over a single tone and E, a group of four tones contained in F, shows a mean improvement of 1.2 db. Referring to Fig. 6, the mean improvement for 414 c.p.s., the difference in frequency between two tones that occur in both F and E is 1.5 db, so there appears to be a trend toward less improvement as the number of tones is increased. The trend is not established by the present data, as the scatter of the data is too great. However, the difference between the means of the improvement for 2 and 8 tones is significant at the 2 percent level. If the effect is real, it may be because the set of tones tended to sound more like noise as more tones were added. This would not necessarily be true of other sets of tones; sets that can be heard as a unified auditory object, such as a clang or a chord, might not follow this trend.

The mean improvement in threshold with 4 tones varies from set to set by an amount that would occur by chance less than once in 1000 times. Even among the four relatively homogeneous sets taken with 1-db *RCS* steps, the variation is almost significant at the 1 percent level.

Two hypotheses that might be advanced to account for the observations on several tones are (1) that sets of tones that combine into a single meaningful auditory object are detected better in the midst of noise than others and (2) that "biting" auditory objects are detected better than "bland" ones. "Biting" and "bland" are used rather than "dissonant" and "consonant" because no function in a musical system is considered necessary for auditory objects to produce the results predicted on the hypothesis. The range of auditory objects is not even restricted to chords. The distinction is not absolute, but is represented in the present data by

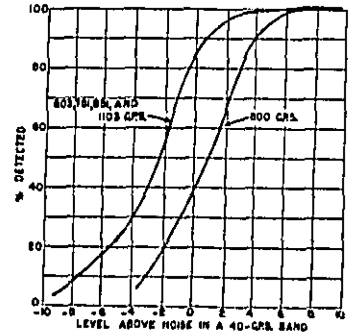

FIG. 5. Typical transition curves.

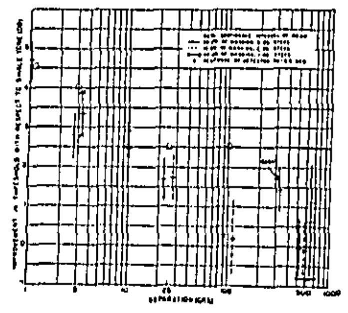
FIG. 6. Improvement of threshold with the addition of a second tone.

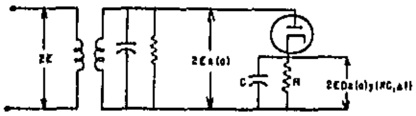

FIG. 7. Circuit analog of the ear in one critical band.

sets C and B. Of course the distinction is not nearly so convincing as if complex tones were used.

The two hypotheses proposed above can only be considered starting points for further research; the data on hand merely suggest them.

The data decisively rule out the possibility that the effect of additional tones in separate critical bands is additive. They also seem to rule out the possibility that the tones entirely fail to reinforce each other. However, the observed improvement in threshold can be accounted for without abandoning the critical band hypothesis.

If several tones not in the same critical band are presented at different levels with respect to their thresholds and are detected independently with probability $p_1, p_2, \cdots, p_n$, the probability P of detecting at least one is the complement of the probability of detecting none, that is,

$$P=1-[(1-p_1)(1-p_2)\cdots(1-p_n)].$$

When all the tones are at threshold, $P=1-(0.5)^n$, which for two tones is 0.75 and for four tones is 0.94. To determine the improvement in threshold, it is only necessary to read the difference in level between the 75 percent and 94 percent points and the 50 percent point on the transition curve for a single tone (Fig. 5, right-hand curve). This is about 1.5 db for two tones and 3 db for four tones. All the improvements in threshold shown in Fig. 6 (for frequency differences of 100, 414, and 500 c.p.s.) and Fig. 8 are within these limits.

Thus the improvement in threshold observed with several tones might be attributable to the increased opportunity for detection, rather than to some mutual reinforcement of the tones in the auditory pathways. This interpretation is not inconsistent with the reported experience of the listeners. Another factor which might explain the observed threshold improvement in part is the possibility that the tones did not all come to threshold precisely at an *RCS* setting. At any rate, the variation in threshold improvement between sets of tones, while an intriguing subject for future research, is not large. It can be said with considerable assurance that the improvement in threshold with four tones is between 0 and 3 db.

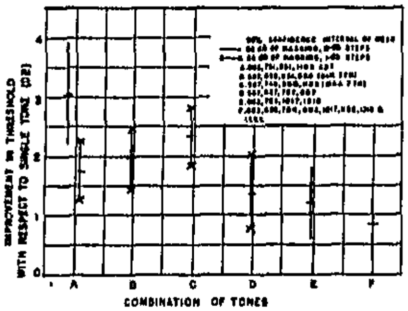

FIG. 8. Improvement of threshold with some combinations of 4 and 8 tones.

TABLE I. The response of a rectifier and low pass *RC* filter to a pair of frequencies.

Δf (c.p.s.)	db above response to single frequency					
	Time constant, *RC* (μsec.)					
	5	20	50	75	100	200
1	6	6	6	5.5	5.5	4.5
3	6	6	5.5	5	5	4
10	6	5	4	3.5	3.5	2.5
25	5.5	3.5	3	3	3	2.5
100	4	3	2.5	2.5	2.5	2.5

CONCLUSIONS

As a result of this work, we conclude that in the frequency region from 600 to 1550 c.p.s.:

(1) As the frequency difference between two tones adjusted in amplitude to subjective equality decreases from well over one critical band to 1 c.p.s., the noise-masked threshold decreases almost 5 db, from a value approximately equal to the single-tone threshold.

(2) This improvement in recognition is to be expected from a system having the ability to analyze the sound spectrum into frequency bands and requiring an appreciable time for the growth of auditory sensation. The data can be approximately fitted by assuming that an analyzer element of the ear acts like a tuned circuit with a Q of 14 followed by a rectifier and a low pass filter with a time constant of 0.2 second.

(3) For 2 tones separated by more than 1 critical band the threshold is probably from 0 to 2 db lower than the single-tone threshold.

(4) For 4 or 8 tones, adjusted in amplitude to subjective equality and all separated by more than 1 critical band, the threshold is probably from 0 to 3 db lower than the single-tone threshold.

(5) Significant but small changes accompany changes in the frequencies used, the observers, and details of testing. The effects of these variables cannot be determined without further extensive testing, but it is suspected that the differences in improvement in threshold observed are related to the ability of the observers to perceive the com-

bination of frequencies as a meaningful configuration, such as a chord or a clang.

(6) There is no steady decrease in the masked threshold as the number of tones in separate critical bands increases. In fact there is probably an increase, at least with some combinations of frequencies.

(7) In predicting the masked threshold from complex signal and background noise spectra, the presence of multiple discrete components in separate critical bands in the signal spectrum does not require modifying the critical band criterion for determining the masked threshold. When multiple components lie within one critical band, their resultant has a higher maximum amplitude than any one component. The critical band criterion cannot be applied directly to the maximum amplitude, however; the effective amplitude is less because of the appreciable time required for the growth of auditory sensation.

ACKNOWLEDGMENT

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The Loudness and Loudness Matching of Short Tones*

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A monaural loudness matching technique was used to study differential sensitivity to intensity as a function of tonal duration. The probable error (p.e.) of the loudness matches was used as the measure of differential sensitivity. With one technique, a standard tone of 500 milliseconds duration was followed by a tone of variable duration (10–500 milliseconds) after a silent interval of 50, 100, or 500 milliseconds. In another technique, both standard and comparison tones were of the same duration (10–500 milliseconds) with the same silent intervals between tones as before. (1) When the standard tone was always 500 milliseconds, the p.e. of the loudness matches increased with a decrease in the duration of the comparison tone from approximately 0.60 to 2.30 db, and the length of the silent interval had no effect on the function. (2) When both the standard and comparison tones had the same duration, the p.e. again increased with a decrease in duration; in this case, however, a silent interval of 500 milliseconds caused an increase from approximately 0.60 to 2.50 db while a silent interval of 50 milliseconds caused an increase to only 1.00 db. These differences are explained in terms of two different processes: a *dissimilarity* effect, and an *interference* effect. (3) When a standard tone of constant duration is used to obtain loudness matches, the mean of the matches becomes a measure of the loudness of tones as a function of duration. These measures showed a clear distinction between the six observers used. For three observers, the change in duration caused practically no change in loudness. For the other three, changes in loudness as great as 8.5 db were recorded. This order of loudness change agrees with that reported by Békésy, but is considerably less than that reported by Munson. Possible explanations for the differences are mentioned.

INTRODUCTION

MANY different techniques have been used to measure the difference threshold for the intensity of tones, and these many techniques have produced different estimates of the size of the difference limen. Montgomery,[1] for example, has shown that the difference limen (measured in decibels) can vary by a factor of four when the experimental procedure is changed. The experimenter who measures differential sensitivity must determine which of the psychophysical methods he shall use, whether the observer has any control over the presentation, whether the two tones shall have an interval of silence between them, and whether the comparison tone should both increase and decrease in intensity relative to the standard. A change in any one of these factors, and others not mentioned here, is apt to change the size of the difference limen he measures.

When differential sensitivity is measured as a function of tonal duration, there are additional complications. A change in the experimental conditions can affect not only the absolute size of the limen, but also the shape and magnitude of the function obtained. Furthermore, another experimental problem arises: should the standard tone have a constant duration and the variable tone change in duration, or should both standard and comparison tone always have the same duration?

PURPOSE OF EXPERIMENTS

The primary purpose of these experiments was to investigate the relation between tonal duration and differential sensitivity to intensity with different

* This research was carried out under Contract N5ori-166, Task Order I, between Special Devices Center, ONR, and The Johns Hopkins University. This is Report No. 166-I-88, Project Designation No. NR-784-001 under that contract.

[1] H. C. Montgomery, "Influence of experimental technique on the measurement of differential intensity sensitivity of the ear," J. Acous. Soc. Am. 7, 39–43 (1935).

methods of tonal presentation. In the investigation we have used a method in which the observer monaurally matches two tones in loudness. With this technique, the probable error of the equality matches becomes the measure of the difference limen, since it indicates the degree of precision with which the equality matches are made. Different silent intervals between the standard and comparison tone have been used, with two basic types of presentation. In one type of presentation, the standard tone always has the same duration, regardless of the duration of the comparison tone; and in the other type, both tones have the same duration at all times.

When a method of loudness matching is used as we have used it, the mean of all the equality judgments is a measure of the relative loudness of the two tones. When the standard tone is kept at a constant duration as the variable tone is changed in duration, the mean equality judgments are then measures of the loudness as a function of duration. This functional relation, then, is a secondary purpose of these experiments—one which is important because of the existing disagreement concerning the magnitude of the effect of duration on loudness.

PROCEDURE

Conditions

The conditions used in these experiments are schematically illustrated in Fig. 1. Under one condition, the first (or standard) tone was always 500 milliseconds in duration, and was followed by the comparison tone which varied in duration from 10 to 500 milliseconds. The interval between the first and second tone was either 50, 100, or 500 milliseconds. The second basic condition was identical except that the first tone always had the same duration as the second tone. In both conditions, the tones were presented once every four seconds. A 1000 c.p.s. tone was used at all times.

Observers

The observers were six male college students, experienced in auditory research. One observer at a time was seated in a sound-deadened room to make observations for an hour at a time, and no longer than two hours in any one day. He was provided with an attenuator calibrated in 1-db steps which controlled the intensity of the first tone. He adjusted his attenuator until both tones sounded equally loud, and then called his attenuation score to the experimenter in an adjacent room by means of an intercom system. The observer's attenuator had a total range of 45 db, but its reading had no meaning to him, since the experimenter added and subtracted attenuation at random. Each observer made a total of ten loudness matches for each condition, but the experiments were so planned that no two observations for the same conditions were made in the same session. This procedure slightly increases variability of judgments, but prevents any systematic influences such as habituation, learning, etc.

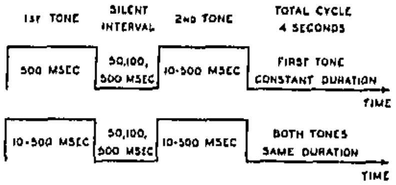

FIG. 1. A schematic illustration of the two basic conditions used in these experiments. See text for further explanation.

Measures Used

The measure of the difference limen (or variability) is the probable error (p.e.) of the judgments. The p.e. was obtained by averaging the variances (standard deviations squared) of each observer's scores for a particular condition, adjusting the variance for degrees of freedom, and then computing the p.e. from this average variance. Thus the measure of the difference limen is taken with reference to the mean of the observer's judgments, not with respect to physical equality. This procedure is necessary, of course, when there are large differences between subjective equality and physical equality, such as when the standard tone is longer in duration than the comparison tone. Under most conditions, the mean scores for the different observers represent random variation. Thus this procedure does not decrease the size of the difference limen because it has already been increased by adjusting it for degrees of freedom.

Measures of loudness of tones are simply the mean score (in db) of all the equality judgments.

APPARATUS

The apparatus used in these experiments is basically the same as that used previously,[2] and a brief description here will suffice. The output of a Hewlitt-Packard oscillator was fed to an electronic timer which provided all the time relations used. Both tones began and ended abruptly, and no later filtering was used to change the transient characteristics. The two tones from the timer were kept separate for purposes of differential attenuation, and then mixed in a resistive network, amplified and fed to a single PDR-8 earphone (mounted in a doughnut cushion) for monaural listening. All

[2] W. R. Garner, "The loudness of repeated short tones," J. Acous. Soc. Am. 20, 513–527 (1948).

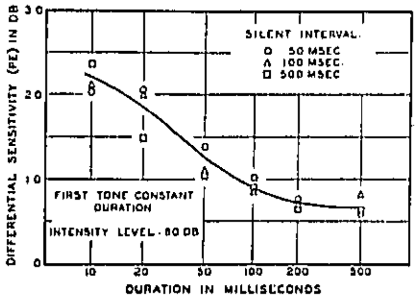

FIG. 2. The effect of tone duration on differential sensitivity for intensity when the first (standard) tone always has a duration of 500 milliseconds. The reference intensity here, and for other intensity levels, is 0.0002 dyne/cm².

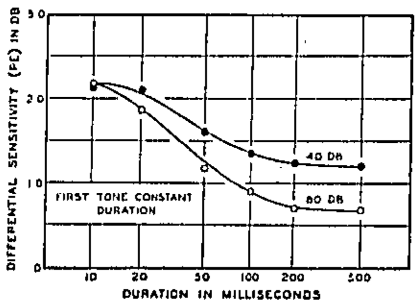

FIG. 3. The effect of tone duration on differential sensitivity for intensity when the first (standard) tone always has a duration of 500 milliseconds, at two different intensity levels. Each plotted point is the average of data obtained with three different silent intervals between tones.

apparatus except the earphone and the observer's attenuator was outside the sound-deadened room.

Intensity levels were determined by measuring voltages across the earphone with a vacuum-tube voltmeter and referring to the calibration curves for the earphone which had been provided by the Permoflux Company. Special calibration circuits, in conjunction with a Standard Electric clock calibrated in 1/100 second, were used to measure the various times. An exception to this procedure was made when very short durations were used, in which case the number of cycles of a 1000 c.p.s. tone were counted on an oscilloscope.

RESULTS AND DISCUSSION

Difference Limens

The rest of the paper is divided into two sections. This section is concerned with measures of differential sensitivity, and a later section will treat the problem of loudness.

First tone constant duration. Figure 2 shows the effect of tone duration on the difference limen when the first tone always has a duration of 500 milliseconds, and the second tone is variable as indicated on the abscissa of the graph. The intensity level for these measurements was 80 db. In general, the difference limen (DL) as measured by the p.e. of the loudness matches increases as the duration of the comparison tone decreases, especially below 200 milliseconds. This relation is not basically different from that obtained previously by Garner and Miller,[3] although the technique used by those authors was quite different from the present technique. The absolute size of the DL, however, is considerably greater in these measures than in the previous ones.

Although some scatter in the plotted points is produced by differences in the interval between the two tones, there is no consistent effect of the interval, and we have drawn a single curve which provides the best visual fit of all the points.

Effect of intensity. The same conditions which were used for the data of Fig. 2 were used at a lower intensity level. Again the size of the interval between the tones had no consistent effect on either the shape of the function or the size of the DL, so that it seemed legitimate to draw one curve for all three intervals. Figure 3 compares the average curves for the two different intensity levels. At the longer durations, the DL is considerably smaller for the higher intensity, although the DL's are essentially the same at the very short durations. The proportional difference in the DL at the longer durations is in good agreement with the data of Riesz,[4] but is in poor agreement with those of Knudsen,[5] which show very little change in differential sensitivity over the range of intensities used in this experiment. It is not clear why this difference should exist.

Both tones same duration. Figure 4 shows how differential sensitivity is affected by tonal duration when both the standard and comparison tones have the same duration. Unlike the previous situation, we now find that the size of the interval between the tones has a marked effect on the shape of the function and on the size of the DL at the shorter durations. When the interval is short (50 milliseconds), the function is approximately the same as it was when the first tone had a constant duration. When the interval is long, however, differential sensitivity changes very slightly as the dura-

[3] W. R. Garner and G. A. Miller, "Differential sensitivity to intensity as a function of the duration of the comparison tone," J. Exper. Psychol. 34, 450–463 (1944).

[4] R. R. Riesz, "Differential intensity sensitivity of the ear for pure tones," Phys. Rev. 31, 867–875 (1928).

[5] V. O. Knudsen, "The sensibility of the ear to small differences in intensity and frequency," Phys. Rev. 21, 84–103 (1923).

tion is decreased. In fact, at 10 milliseconds the DL is only 1.7 times as great as it is at 500 milliseconds as compared to an increase by a factor of 4.2 with an interval of 50 milliseconds.

In a previous paper,[6] the effect of duration on interaural loudness matching was studied when the two tones were presented to the two ears simultaneously. Under those conditions, it was likewise found that duration has relatively little effect on differential sensitivity. But there was another similarity between the functions obtained under those conditions and the functions obtained here when both tones have the same duration. In both cases, the maximum variability in loudness matching occurs with a duration of 20 milliseconds. It would seem, then, that some similar factor is operating in the two conditions. The maximum variability at 20 milliseconds is probably due to the fact that it is the critical duration for tonality. At shorter durations, a definite click is heard, while at longer durations, a clear tone is heard.

Comparison of conditions. We can summarize these results as follows. In monaural listening, if the standard tone is of a long constant duration, differential sensitivity markedly decreases as the duration of the comparison tone is decreased, regardless of the interval between the two tones. Likewise, if both standard and comparison tones have the same duration, with a short interval between tones, differential sensitivity decreases with a decrease in duration. On the other hand, when both standard and comparison tones have the same duration (again with monaural listening), but with a long interval between tones, the effect of duration on differential sensitivity is relatively small. And similarly in interaural comparisons, the effect of duration on differential sensitivity is small when both tones have the same duration and are presented simultaneously.

The best way to explain these results is to assume that there are two different factors operating to produce the decreased differential sensitivity when the duration of the tone is decreased. One of these factors we can call a *dissimilarity* factor, and the other an *interference* factor.

It is generally recognized that minimum difference limens are obtained when the two tones being compared are identical in all respects except that aspect for which differential sensitivity is being measured. If two tones of two different frequencies are used to obtain measures of differential sensitivity to intensity, for example, the sensitivity to differences is less than if both tones have the same frequency. When the standard tone is kept at a constant duration, and the comparison tone is decreased in duration, the two tones become less similar, particularly when the duration of the comparison tone is so short that the tone is heard as a click. Under these conditions, then, we would expect that differential sensitivity would decrease as the comparison tone becomes shorter, and that is exactly what happens. The decreased sensitivity is due to the fact that the tones become less and less similar.

On the other hand, when both tones have the same duration (in either monaural or interaural comparisons) they are the same in all respects except that of intensity, and we would expect a minimum effect of duration on differential sensitivity. We get a minimum effect of duration in interaural comparisons, and in monaural comparisons when the two tones are separated sufficiently in time. When, however, both tones are close together in time with monaural comparisons, we again get a marked effect of duration of the difference limen. This effect we must attribute to an interference of the two tones on each other—an interference which is due to some continued response to the first tone over the period in which the second tone appears. After a period of time the response to the first tone disappears, and we thus get no effect of duration on differential sensitivity.

We would expect the maximum interference to occur with simultaneous presentation of the two tones, which is the condition achieved with the interaural comparisons. Since there was very little effect of duration on the interaural judgment, it seems likely that the interference is relatively peripheral—certainly not occurring as high in the nervous system as the point where the interaural judgment is made. Actually, the critical experiment to determine the locus of the interference would be to use interaural comparisons, but with the two tones separated by a short time interval.

Absolute size of the difference limen. The use of the probable error as the measure of differential

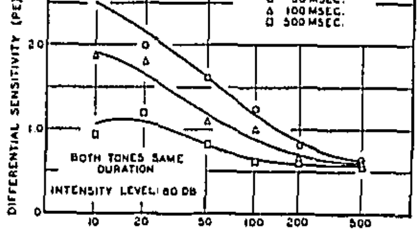

FIG. 4. The effect of tone duration on differential sensitivity for intensity when both the standard and comparison tones have the same duration.

[6] W. R. Garner, "Accuracy of binaural loudness matching with repeated short tones," J. Exper. Psychol. 37, 337–350 (1947).

sensitivity should be equivalent to the limen as measured with other techniques. It is the value which encompasses 50 percent of the equality judgments, and thus is that value which we can assume would produce 50 percent higher or lower judgments and 50 percent equality judgments. In the present experiments, tones of 500 milliseconds have been compared a total of seven times. The average difference limen obtained was 0.60 db, which compares not too unfavorably with the more recent values reported in the literature. Values reported by Knudsen,[5] Riesz,[4] and Churcher, King and Davies[7] for equivalent conditions are on the order of 0.40 db. Postman[8] reports a value close to 0.70 db, which is slightly higher than that obtained by us, while Dimmick and Olson[9] report values considerably higher. It would seem fair to say, then, that the values obtained in these experiments are not out of line with previously obtained values, even though the method used is different from other methods in many respects.

Other effects of interval. Montgomery[1] states that the minimum difference limen is obtained when the interval between the two tones is zero. We obtained data for a zero interval with both tones having a duration of 500 milliseconds, and obtained a difference limen of 0.45 db, which is lower than any of the other limens obtained. Thus Montgomery's statement seems to be justified. Other than this one effect, however, none of the other differences in differential sensitivity due to the size of the interval is statistically significant, in agreement with results obtained by Harris and Myers[10] for noise. Postman,[8] likewise, found little or no effect of interval on the difference limen for the intensity of tones, although his data encompass much greater time intervals than ours.

We obtained no functions with a zero interval between tones because such a function becomes meaningless at the very short durations. In preliminary experiments, attempts were made to obtain data for a zero interval at short durations, and it was evident that judgments under those conditions are impossible. When no time separates the first and second tone with durations of 20 milliseconds, for example, it is impossible to tell which tone came first. Any judgment made becomes one of adjusting to a guessed total loudness, and is completely meaningless in terms of differential sensitivity.

[7] Churcher, King, and Davies, "The minimum perceptible change of intensity of a pure tone," Phil. Mag. 18, 927–939 (1934).

[8] L. Postman, "The time-error in auditory perception," Am. J. Psychol. 59, 193–219 (1946).

[9] F. L. Dimmick and R. M. Olson, "The intensive difference limen in audition," J. Acous. Soc. Am. 12, 517–525 (1941).

[10] J. D. Harris and C. K. Myers, "Intensity discrimination for white noise," Progress Report No. 3, Bureau of Medicine and Surgery Res. Proj. NM-003-020 (1948).

Effect of repetition rate. Additional data were collected to determine if the rate at which the tones were repeated had any serious effect on differential sensitivity. The slowest rate used was once in four seconds (the rate for all data shown here), and the fastest rate was five times a second, which could be used only for some of the shorter durations and intervals. Over this range of rates there was no consistent or reliable effect of the rate on differential sensitivity.

Time errors. Whenever both the standard and the comparison tone are of the same duration, the mean of all the equality matches can be used as a measure of time error. At long durations, the mean differences obtained were always small, and in all cases except one were statistically insignificant (at the one percent level). Even if we assumed that the differences obtained were all significant, there would be no justification for talking about a time error function, since no consistent trends were evident in any of the data.

Loudness as a Function of Duration

In this experiment, the observer adjusted the intensity of the standard (first) tone to give a loudness equal to that of the second tone. Since the frequency was always 1000 c.p.s., the intensity level of the first tone directly becomes the loudness level of the second (variable duration) tone.

Differences between observers. In plotting the data to show the effect of duration on loudness, it was immediately apparent that the observers fell into two distinct groups—one of which showed a consistent change in loudness as a function of duration, and the other of which showed practically no change in loudness as a function of duration. Three observers fell into each group, and were in the same group for all conditions. In other words, the observers were consistent in either showing an effect or not showing an effect.

Figure 5 shows the effect of duration on loudness for these two groups at the two intensity levels used. Although the difference between the two groups is less at the lower intensity, the difference is still clear. We can see no possibility that differences in experimental procedure could have produced the differences shown here between the observers. All observers were given the same instructions, and no observer ever knew the results of his observations. Likewise, since the experimenter added or subtracted attenuation at random, it was impossible for the observer to know how close he was to physical equality in making his judgments. In addition, there were no differences between the two groups in variability of loudness matches. The size of the silent interval had very little effect on any of the loudness equalities, for either group of observers—the maximum difference was about 1 db in favor of less loudness for the longest interval.

Comparison with other results. If we assume that results from those observers who showed a change in loudness with duration represent the true function, there is still a large discrepancy between our results and those of Munson.[11] For example, at an intensity level of 80 db, our results show that a tone of 10 milliseconds duration has a loudness level only 8.5 db less. In contrast, the data of Munson show a loudness level difference of close to 30 db under similar conditions. The maximum difference in loudness level for any one of our observers, under any condition, was 12 db, as compared to the considerably larger average differences of Munson.

In a previous paper,[2] we investigated the loudness of repeated short tones, and found the effect of both duration and repetition rate on loudness. The loudness differences as a function of duration reported in that paper were equivalent to those reported here, but at that time we assumed that the discrepancy between our results and those of Munson was due to the fact that the lowest repetition rate used was five per second. In view of the present results, it must be assumed that there is a fundamental discrepancy between results—a discrepancy which needs some explanation.

In 1929, Békésy[12] reported data on the effect of duration on the loudness of tones. His results (for similar, but not identical, conditions) agree very closely with the results reported here for the three observers who showed the greatest effect. Thus the results of the present paper agree with our previous results and with the results of Békésy, but disagree markedly with the results of Munson.

Possible explanations of differences. Two possible explanations of the discrepancy can be mentioned. The first is that of methodology. Munson used a method of constants, in which the comparison stimuli had fixed and predetermined intensities. With such a method, it is possible that the results are affected by the selection of the intensities which the observer hears. Mr. J. M. Doughty is at present doing experiments in this laboratory which indicate that the equal loudness point can be strongly affected by changing the range and absolute values of the comparison stimuli.

The other explanation—one which needs to be investigated—is in terms of the kind of onset of the tones. In all our experiments the tones had an abrupt onset, which produces many transient frequencies. Békésy likewise used an abrupt onset. In Munson's experiment, however, the tones rose from zero to maximum amplitude in 3 milliseconds, which gives a tone with considerable less click at the beginning and end of the tone. At durations as short as 10 milliseconds, this difference in presentation of tones could certainly have some differential effect, although a difference as large as actually occurs seems unlikely. It seems even more unlikely that this procedural difference could account for the discrepancy in results at durations as long as 100 milliseconds.

It seems appropriate to say at this point that more research needs to be undertaken to rectify these discrepancies.

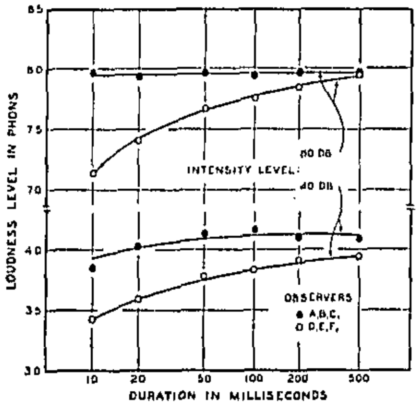

FIG. 5. The effect of tone duration on loudness, at two intensity levels, for two different groups of observers. Each plotted point is the average of data obtained with three different silent intervals between tones.

ACKNOWLEDGMENT

Mr. Harold Schapiro and Mrs. Mary Lamb helped obtain the experimental data, and their contributions are acknowledged with gratitude.

[11] W. A. Munson, "The growth of auditory sensation," J. Acous. Soc. Am. 19, 584–591 (1947).

[12] G. V. Békésy, "Zur Theorie des Hörens," Physik. Zeits. 30, 115 (1929).

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A Study of the Mechanism of the Middle Ear

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I studied principally the mechanics of the middle ear, summarizing the anatomy and the physiology of the ear. I made Diagram I showing the anatomical structure of the ear, and translated it into physical terms of Diagram II, such as mass, spring, and frictional constant, etc. Thereafter I could get equations showing the mechanics of the middle ear through the Lagrangian equation. The complicated treatment for the kinetic, potential, and dissipation energies of the middle ear elements is for the purpose of expressing my opinion about the function of the middle ear which is partly different from the present contradictory medical views. By using the resultant equations of the motion of the ear and Diagram III (showing the electromechanical structure of the middle ear), I studied the mechanism of the middle ear under an assumption that the frequency characteristic curve of the inner ear is flat in a wide range. This assumption may be deduced from Luscher's experiment of the tympanic loading and hearing curves of men with complete defect of the tympanic membrane but with cochlear nerve intact. However, I do not explain it here, and it will be described in detail in further papers. From theoretical results thus obtained I made some experiments and calculation of the natural frequencies of the middle ear elements.

My conclusions are as follows: (a) The air vibration system of the ear which consists of the external auditory canal, the tympanic cavity, and the antrum can be shown electromechanically by Diagram III. (b) The middle ear has four main peaks of resonance on the hearing curves. (c) The middle ear and cochlea appear to be regarded as a displacement receiver and a pressure receiver, respectively. (d) The tympanic membrance has two important roles: (1) that of composing the vibration of the external auditory canals, of the antrum, and of the air cells of the mastoid process; and (2) that of propagating these vibrations to the ossicles. (e) The non-linear vibration of the tympanic membrane, the basilar membrane, and the secondary tympanic membrane, produce combination tones. (f) The air vibration system has an important role in understanding speech sounds. Its magnification of the sound intensity is above 50 db in a range from 700 or 800 cycles to 5000 cycles as in Fig. 5a. (g) This work offers a problem of design of a new audiometer, and is available for diagnosis of otological pathology.

THE so-called Helmholtz theory of hearing, or some modification of it, and others attempted to explain, so far as possible, how the recognized sensations of sound are evoked by stimulation of the ear with sound waves, but they were not successful. On the contrary, modern acoustics and acoustical instruments made a great advance supported by radio technology. It became necessary for even radio technology to investigate the function of the ear as an important problem, because the human ear is the foremost of all receivers of sound. Consequently, the exact function of the ear became of intense interest to physicists as well as otologists. Many important experiments which aim to throw light upon the mechanism of hearing have been performed by physicists. However, the status of our knowledge of the mechanism of hearing may not be satisfactorily applicable in practice in spite of its considerable knowledge.

In my opinion, this means a lack of intimate knowledge between otologists and physicists.

It is hard work for otologists to understand the higher mechanics; however, we otologists must do so in order to make an advance in otology. Therefore, there must be a fundamental mechanics of hearing common to otologists and physicists, and applicable in practice.

I found such a fundamental mechanics of hearing, summarizing the anatomy and the physiology of the ear from the standpoint of physics.

DIAGRAM I

Diagram I is an anatomical diagram of the ear showing the mechanism of hearing. The cochlear labyrinth is surrounded by many air cells of the mastoid process. These air cells have the important role of reflecting on their surface the heart sounds carried by the arteries. The normal reflection coefficient of the air cells is calculated by measuring the impedance of bone and air.

$$r^2=(R_1-R_2)^2/(R_1+R_2)^2,$$

where $R_1=42$ (the impedance of air), $R_2=83\times10^4$ (the impedance of bone), and r^2= the normal reflection coefficient of the air cells. I could not, however, find the value of bone impedance in any table of physics. In 1943, as a result of experiments into the method of which I shall not go at this time, I arrived at 83×10^4 c.g.s. as the value of bone impedance. $r=0.999\cdots$. The percentage of reflex is 99.9 percent; i.e., almost all of the heart sounds which the bone structure propagates from the carotis to the cochlear labyrinth are reflected on the surface of the air cells.

Part of the sounds, however, are admitted through the bone partitions separating the many air cells. Due to this, the actual value of reflex on the surface of the air cells is estimated at 25 db which I have deduced from a study of Harvey Fletcher's experiments in binaural "objective" beats.

The ear has two axes of rotation. One axis, the malleo-incudal rotation axis (11), consists of the process anterior Folii et ligamentum mallei anterius and the short process of the incus. Another axis is behind the posterior pole of the foot plate of the stapes (15).

I consider the combined heads of the malleus and incus to be a sort of counterpoise. This is illustrated by the fact that the weight of two parts, which are obtained by cutting the ossicles in the direction of the ligamentum mallei anterior and the under edge of the short process of the incus is equal. In other words, the moment of inertia of the ossicles is at its lowest value. The equation explaining this fact, from the standpoint of mechanics, is $I_L = I_G + Md^2$, where G is a line which passes through the centroid of a solid whose mass is M, L is a line parallel to G, and the distance between L and G is d. I_L is the moment of inertia with respect to L axis, and I_G is the moment of inertia with respect to G axis. I_L will be at its smallest value when $d=0$. The incudostapedial joint (14) functions as a sort of universal joint. The secondary tympanic membrane (18) is a thin membrane which reduces the impedance of vibration of the cochlear fluid. For instance, in the case of otosclerosis, this membrane becomes rigid; i.e., it means that the impedance of its vibration increases extremely. I consider this to be an explanation of the results of the fenestration operation by Maurice Sourdille.

The difference between the impedance of air and that of liquid is too great, and results in a reflex of sound waves at the boundary of the two media. In order to prevent the reflex of sounds we must insert a transformer between the two media. In other words, the vibration system of the middle ear acts as a sort of transformer of the sound waves between the air and the cochlear fluid. I consider, however, that the difference of impedance between the cochlear fluid and air would be less than that between a large mass of liquid and air, simply because of the extremely small quantity of the cochlear fluid.

The vibration of the stapes is a hinge-like movement which has its axis of rotation behind its posterior pole. The stapedius muscle (20) pulls the anterior pole of the stapes outward, while the tensor tympani muscle (19) pushes the pole inward, through the incudostapedial joint. The result of this is that the two muscles act as antagonists and maintain the ossicles in the neutral position of vibration. The aqueducts (23) and (24) have the role of maintaining the normal pressure of the cochlear fluid. The cellullae hypotympanicae (8) and the other small cells in the tympanic cavity may be considered as suppliers which add air viscosity in their cells to the vibration of the tympanic membrane. Thus, I have explained in outline, by use of Diagram I the functions of the various parts of the ear.

DIAGRAM II

I observed the vibration of the tympanic membrane by introducing the light of a stroboscope into the normal ear. Its vibration has its largest amplitude in the middle zone of the tympanic membrane. The amplitude of the vibration of the malleus is much smaller than that of the middle zone so that I could not exactly determine the ratio between the two amplitudes. However, owing to this fact, I was able to translate Diagram I into Diagram II.

$$T = \tfrac{1}{2}M_G\dot{x}_G^2 + \tfrac{1}{2}M_T\dot{x}_T^2 + \tfrac{1}{2}M_1\dot{x}_1^2 + \tfrac{1}{2}M_2\dot{x}_2^2 + \cdots + \tfrac{1}{2}I_K\dot{\theta}_K^2 + \tfrac{1}{2}I_S\dot{\theta}_S^2 + \tfrac{1}{2}M_C\dot{x}_C^2,$$

where

$\dot{x} = dx/dt$; $\ddot{x} = d^2x/dt$; $\dot{\theta} = d\theta/dt$; $\ddot{\theta} = d^2\theta/dt^2$; t is time.

As a result of my experiments on the normal tympanic membrane, I have been able to observe its non-linear vibrations. These observations, together with the result which Schaeffer published as a result of his clinical experience, have led me to believe the vibration of the basilar membrane and that the secondary tympanic membrane are, like that of the tympanic membrane, non-linear. V = the potential energy of the vibration system of the ear.

$$\begin{aligned} V = {} & \tfrac{1}{2}S_G(x_G - x_T)^2 + \alpha\int(x_G - x_T)^2 d(x_G - x_T) \\ & + \tfrac{1}{2}S_T(x_T - l_1\theta_K)^2 + \beta\int(x_T - l_1\theta_K)^2 d(x_T - l_1\theta_K) \\ & + \tfrac{1}{2}S_l(x_T - x_1)^2 + \gamma\int(x_T - x_1)^2 d(x_T - x_1) \\ & + \tfrac{1}{2}S_1(x_1 - x_2)^2 + \tfrac{1}{2}S_2(x_2 - x_3)\cdots \\ & + \frac{1}{2}\frac{T_L}{l_L}(l_3\theta_K)^2 + \frac{S_L}{2l_L^3}\int(l_3\theta_K)d(l_3\theta_K) \\ & + \tfrac{1}{2}S_M(l_4\theta_K)^2 + \tfrac{1}{2}S_S(r_1\sin\varphi\cdot\theta_S)^2 \\ & + \frac{1}{2}\frac{T_V}{l_V}x_S^2 + \frac{S_V}{2l_V^3}\int x_S^3 dx_S \\ & + \tfrac{1}{2}S_n x_C^2 + \delta\int x_C^2 dx_C + \tfrac{1}{2}S_R x_C^2 + \epsilon\int x_C^2 dx_C, \end{aligned}$$

where α, β, γ, δ, and ϵ are arbitrary constants of very small numbers. F is the dissipation function of the vibration system of the ear.

$$F = \tfrac{1}{2}R_G\dot{x}_G^2 + \tfrac{1}{2}R_l\dot{x}_T^2 + \tfrac{1}{2}R_1\dot{x}_1^2 + \tfrac{1}{2}R_2\dot{x}_2^2 + \cdots + \tfrac{1}{2}R_K\dot{\theta}_K^2 + \tfrac{1}{2}R_C\dot{x}_C^2,$$

where R = the constant of viscous friction of the different elements. From the information in Diagram II, we can deduce the following relation:

$$r_2\theta_S = x_S \qquad \theta_S = (1/r_2)x_S \qquad r_3\theta_S = l_2\theta_K$$

$$(r_3/r_2)x_S = l_2\theta_K \qquad \theta_K = (r_3/r_2 l_2)x_S \qquad \theta_K = K_1 x_S \tag{1}$$

$$\theta_S = K_2 x_S \tag{2}$$

$$x_C = \tfrac{1}{2}x_S, \tag{3}$$

where $r_3/r_2 l_2 = K_1$ and $1/r_2 = K_2$. Substituting Eqs. (1)–(3) for Eqs. T, V, and F, we obtain the following equations:

$$\begin{aligned} T = {} & \tfrac{1}{2}M_G\dot{x}_G^2 + \tfrac{1}{2}M_T\dot{x}_T^2 + \tfrac{1}{2}M_1\dot{x}_1^2 + \tfrac{1}{2}M_2\dot{x}_2^2 + \cdots \\ & + \tfrac{1}{2}I_K(K_1\dot{x}_S)^2 + \tfrac{1}{2}I_S(K_2\dot{x}_S)^2 + \tfrac{1}{2}M_C(\tfrac{1}{2}\dot{x}_S)^2 \\ = {} & \tfrac{1}{2}M_G\dot{x}_G^2 + \tfrac{1}{2}M_T\dot{x}_T^2 + \tfrac{1}{2}M_1\dot{x}_1^2 + \tfrac{1}{2}M_2\dot{x}_2^2 + \cdots \\ & + \tfrac{1}{2}(I_K K_1^2 + I_S K_2^2 + \tfrac{1}{4}M_C)\dot{x}_S^2. \end{aligned} \tag{a}$$

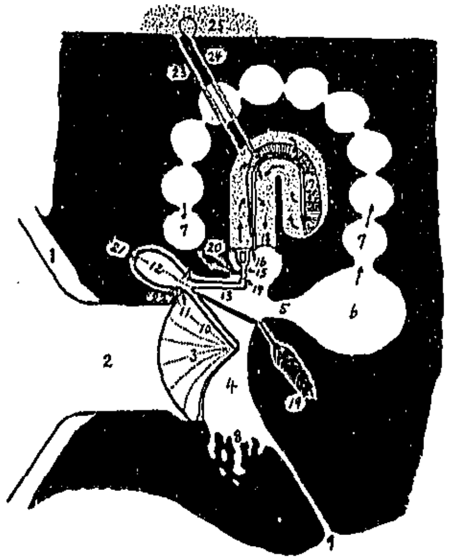

DIAGRAM I. Showing the anatomical structure of the ear.

1. auricle
2. external auditory canal
3. tympanic membrane
4. tympanic cavity
5. aditus ad antrum
6. antrum
7. air cells of mastoid process
8. cellulae hypotympanicae
9. eustachian tube
10. handle of malleus
11. malleo-incudal rotation axis (consists of processus anterior Folii et ligamentum mallei anterius, and short process of incus)
12. combined heads of malleus and incus
13. long process of incus
14. incudostapedial joint
15. stapes
16. annular ligament
17. basilar membrane
18. secondary tympanic membrane
19. tensor tympani muscle
20. stapedius muscle
21. ligamentum mallei superius et ligamentum incudis superius
22. ligamentum mallei laterale
23. aqueductus cochleae
24. aqueductus vestibuli
25. endocranial fluid.

$$
\begin{aligned}
V=&\tfrac{1}{2}S_G(x_G-x_T)^2\\
&+\alpha\int(x_G-x_T)^2d(x_G-x_T)+\tfrac{1}{2}S_T(x_T-l_1K_1x_S)^2\\
&+\beta\int(x_T-l_1K_1x_S)^2d(x_T-l_1K_1x_S)+\tfrac{1}{2}S_P(x_T-x_1)^2\\
&+\gamma\int(x_T-x_1)^2d(x_T-x_1)+\tfrac{1}{2}S_1(x_1-x_2)^2+\cdots\\
&+\frac{1}{2}\frac{T_L}{l_L}(l_3K_1x_S)^2+\frac{S_L}{2l_L^3}\int(l_3K_1x_S)^3d(l_3K_1x_S)\\
&+\tfrac{1}{2}S_M(l_4K_1x_S)^2+\tfrac{1}{2}S_S(r_1\sin\theta\cdot K_2x_S)^2\\
&+\frac{1}{2}\frac{T_V}{l_V}x_S^2+\frac{S_V}{2l_V^3}\int x_S^3dx_S+\tfrac{1}{2}S_H(\tfrac{1}{2}x_S)^2\\
&+\delta\int(\tfrac{1}{2}x_S)^2d(\tfrac{1}{2}x_S)+\tfrac{1}{2}S_H(\tfrac{1}{2}x_S)^2\\
&+\epsilon\int(\tfrac{1}{2}x_S)^2d(\tfrac{1}{2}x_S).
\end{aligned}
\tag{b}
$$

$$
\begin{aligned}
F=&\tfrac{1}{2}R_G\dot x_G^2+\tfrac{1}{2}R_P\dot x_T^2+\tfrac{1}{2}R_1\dot x_1^2+\tfrac{1}{2}R_2\dot x_2^2+\cdots\\
&\qquad+\tfrac{1}{2}R_K(K_1\dot x_S)^2+\tfrac{1}{2}R_C(\tfrac{1}{2}\dot x_S)^2\\
=&\tfrac{1}{2}R_G\dot x_G^2+\tfrac{1}{2}R_P\dot x_T^2+\tfrac{1}{2}R_1\dot x_1^2+\tfrac{1}{2}R_2\dot x_2^2+\cdots\\
&\qquad+\tfrac{1}{2}(R_KK_1^2+\tfrac{1}{4}R_C)\dot x_S^2.
\end{aligned}
\tag{c}
$$

Substitute Eqs. (a), (b), and (c) for T, V, and F in the Lagrangian equation.

$$\frac{d}{dt}\frac{\partial T}{\partial\dot q_i}-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}+\frac{\partial F}{\partial\dot q_i}=Q_i.$$

Then we obtain the following differential equations:

$$
\left[\begin{aligned}
&M_G\ddot x_G+R_G\dot x_G+S_G(x_G-x_T)+\alpha(x_G-x_T)^2=Q_G(\text{sound})\\
&M_T\ddot x_T+R_P\dot x_T-S_G(x_G-x_T)-\alpha(x_G-x_T)^2+S_T(x_T-l_1K_1x_S)\\
&\qquad\qquad+\beta(x_T-l_1K_1x_S)^2+S_P(x_T-x_1)+\gamma(x_T-x_1)^2=0\\
&M_1\ddot x_1+R_1\dot x_1-S_P(x_T-x_1)-\gamma(x_T-x_1)^2+S_1x_1=0\\
&(I_KK_1^2+I_SK_2^2+\tfrac{1}{4}M_C)\ddot x_S+(R_KK_1^2+\tfrac{1}{4}R_C)\dot x_S-l_1K_1S_T(x_T-l_1K_1x_S)\\
&\qquad-l_1K_1\beta(x_T-l_1K_1x_S)^2+\frac{T_Ll_3^2K_1^2}{l_L}x_S+\frac{S_Ll_3^4K_1^4}{2l_L^3}x_S+S_Ml_4^2K_1^2x_S\\
&\qquad+S_Sr_1^2\sin^2\theta\cdot K_2^2x_S+\frac{T_V}{l_V}x_S+\frac{S_V}{2l_V^3}x_S^3+\tfrac{1}{4}S_Hx_S+\tfrac{1}{8}\delta x_S^2+\tfrac{1}{4}S_Hx_S+\tfrac{1}{8}\epsilon x_S^2=0
\end{aligned}\right].
$$

Modifying the above equations we obtain the resultant equations showing the mechanics of vibration of the ear.

$$\left\{\begin{array}{l} M_G\ddot{x}_G+R_G\dot{x}_G+S_Gx_G+\alpha(x_G-x_T)^2=Q_G+S_Gx_T \\ M_T\ddot{x}_T+R_P\dot{x}_T+(S_G+S_T+S_P)x_T-\alpha(x_G-x_T)^2+\beta(x_T-l_1K_1x_S)^2+\gamma(x_T-x_1)^2=S_Gx_G+S_Px_1+l_1K_1S_Tx_S \\ M_1\ddot{x}_1+R_1\dot{x}_1+(S_P+S_1)x_1-\gamma(x_T-x_1)^2=S_Px_T \\ (I_KK_1^2+I_SK_2^2+\frac{1}{4}M_C)\ddot{x}_S+(R_KK_1^2+\frac{1}{4}R_C)\dot{x}_S \\ \qquad +\left(l_1^2K_1^2S_T+\dfrac{T_Ll_3^2K_1^2}{l_L}+l_4^2K_1^2S_M+S_Sr_1^2K_2^2\sin^2\theta+\dfrac{T_V}{l_V}+\dfrac{S_B+S_R}{4}\right)x_S \\ \qquad\qquad -l_1K_1\beta(x_T-l_1K_1x_S)^2+\dfrac{\delta+\epsilon}{8}x_S^2+\dfrac{S_V}{2l_V^3}x_S^2=l_1K_1S_Tx_T \end{array}\right\}. \quad \text{(I)}$$

From the fact that we cannot hear combination tones if the intensities of two tones are less than about 50 db, we can deduce the fact that non-linear vibration does not take place in the vibration system of the ear when the intensities of tones are less than about 50 db. In such a case we can mathematically neglect the terms whose order of x are higher than x'.

Thus, simplifying Eqs. (I), we obtain

$$\left\{\begin{array}{l} M_G\ddot{x}_G+R_G\dot{x}_G+S_Gx_G=Q_G\text{ (sound)}+S_Gx_T \\ M_T\ddot{x}_T+R_P\dot{x}_T+(S_G+S_T+S_P)x_T \\ \qquad =S_Gx_G+S_Px_1+Kx_S \\ M_1\ddot{x}_1+R_1\dot{x}_1+(S_P+S_1)x_1=S_Px_T \\ M_S\ddot{x}_S+R_S\dot{x}_S+\Sigma Sx_S=Kx_T \end{array}\right\}, \quad \text{(II)}$$

where

$$l_1K_1S_T=K$$

$$I_KK_1^2+I_SK_2^2+\tfrac{1}{4}M_C=M_S$$

$$R_KK_1^2+\tfrac{1}{4}R_C=R_S$$

$$l_1^2K_1^2S_T+\frac{T_Ll_3^2K_1^2}{l_L}+l_4^2K_1^2S_M+S_Sr_1^2K_2^2\sin^2\theta+\frac{T_V}{l_V}+\frac{S_B+S_R}{4}=\Sigma S.$$

From my observation of the vibrating tympanic membrane, I found that x_T is too large in comparison to x_S; so we can neglect x_S in Eq. (II). Thus we obtain Eq. (III), showing the mechanism of the air vibration system of the ear; that is to say, in my opinion, the vibration system from the air in the external auditory canal to the air cells of the mastoid process.

$$\left\{\begin{array}{l} M_G\ddot{x}_G+R_G\dot{x}_G+S_Gx_G=Q_G\text{ (sound)}+S_Gx_T \\ M_T\ddot{x}_T+R_P\dot{x}_T+(S_G+S_T+S_P)x_T \\ \qquad =S_Gx_G+S_Px_1 \\ M_1\ddot{x}_1+R_1\dot{x}_1+(S_P+S_1)x_1=S_Px_T \end{array}\right\}. \quad \text{(III)}$$

The mathematically solved answer of x_T is so complicated that we cannot apply it practically.

It is easier for us to study the composition of Eqs. (I)-(III), and deduce the conclusions from them.

From Eq. (I) we can conclude:

(a) The ear is one type of a complicated system consisting of many connected springs.

(b) The vibration of the cochlear fluid which determines the form of the hearing curves is conducted by the tympanic membrane.

(c) The vibration of the ear belongs to a non-linear type.

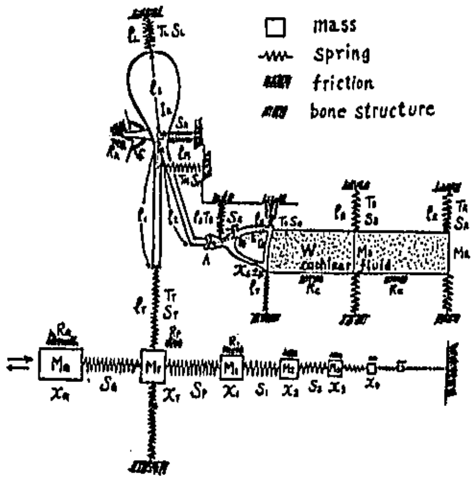

DIAGRAM II. Showing the mechanical structure of the ear. The symbols M, S, X, and T designate mass, spring constant, displacement, and tension. Subscripts designate different elements as follows:

G—external auditory canal
T—tympanic membrane
P—tympanic cavity
1—aditus ad antrum
2—first mastoid cell
3—second mastoid cell, etc.
L—ligamentum mallei et incudis superior
M—tensor tympani muscle
S—stapedius muscle
V—ligamentum annulare
B—basilar membrane
R—secondary tympanic membrane
C—cochlear fluid.

Other symbols:

l—length of each part of the malleus and the incus
r_1—the length between the posterior pole of the stapes and the joining point of the stapedius muscle
r_2—the length between the posterior pole and the anterior pole of the stapes
r_3—the length between the posterior pole and the center of the incudo-stapedial joint
φ—the angle made by the stapedius muscle and r_1
I_K—the moment of inertia of the ossicles malleus and incus concerning its axis
I_S—the movement of inertia of the stapes concerning its axis
θ_K—the angle displacement of the common axis of the malleus and the incus
θ_S—the angle displacement of the axis of the stapes
τ—the kinetic energy of the vibration system of the ear.

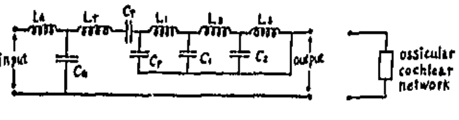

DIAGRAM III. Equivalent electrical circuit for author's Diagram II. Courtesy of Charles T. Molloy of the Bell Telephone Laboratories, who has studied Diagram II.

From the structure of Eq. (II) we can conclude:

(d) The tympanic membrane has an important function of composing the vibration of the external auditory canal, that of the antrum, and that of the air cells of the mastoid process.

(e) The tympanic membrane has its free vibration whose frequency is considered as the lowest in the air vibratory system. In other words, the value $(S_G+S_T+S_P)/M_T$ which determines the frequency of free vibration of the tympanic membrane is smaller than S_G/M_G or $(S_P+S_1)/M_1$, because the value of M_T is extremely large compared to the mass M_G or M_1.

(f) The frequency of the free vibration of the air in the external auditory canal is less than that of the antrum because the air volume of the former is much larger than that of the latter. Consequently, we get the following conception: $f_T<f_G<f_A$, where f_T = the frequency of free vibration of the tympanic membrane, f_G = the frequency of free vibration of the external auditory canal, and f_A = the frequency of free vibration of the antrum.

DIAGRAM III

Through the resultant Eqs. (III), we can translate Diagram II into Diagram III showing the electromechanics of the vibration system of the middle ear. This electrical circuit may be useful in the experimental study of the function of the middle ear elements.

THE ACOUSTICAL EXPERIMENTS

I measured the frequency of free vibration of the tympanic membrane. I pasted a very small and light mirror on the malleus of a cadaver, and registered the light reflected from the mirror on a film. I repeated such experiments, and the results obtained by the experiments show that the frequency of free vibration of the tympanic membrane is approximately 800 cycles as shown in Fig. 1. I consider this value as an exact one because the mirror used in my experiment was extremely small and light, and its weight would hardly affect the frequency of free vibration of the tympanic membrane.

As the frequency of free vibration of the tympanic membrane with the ossicles, Frank reported 1110, 1092, and 1304 cycles; Kobrak, 800 cycles; and Bekesy, 1000 cycles.

Troeger reported that the normal tympanic membrane has its smallest value of mechanical impedance at 800 cycles. These experimental results are very interesting when compared with mine as mentioned above, and they coincide with the frequency of the first peak on the frequency characteristic curve of the normal ear obtained by my new hearing test as will be described later.

I calculated the frequency of free vibration of the external auditory canal and that of the antrum from their anatomical dimensions. The former is $f_G=34000/4(L+0.6R)=34000/4(3.5+0.6\times0.4)\fallingdotseq 2270$ cycles. The latter is $f_A=K(S^{1/2}/V^{1/2})\fallingdotseq 3650$ cycles, because I regard the free vibration of the antrum as that of Helmholtz's resonator. Here, S is the sectional area of the aditus ad antrum, and V is the volume of the antrum. $K=5617$, for example, $S=0.16$ cm², $V=1.0$ cm³ as in Fig. 2.

The dimensions S and V are variable individually but they are in proportional relation. Therefore, the frequency of free vibration of the antrum remains constant in general.

I have not been able to calculate the frequency of free vibration of the ossicles or find its value experimentally.

The ossicles of the ear have many springs such as the tensor tympani muscle, ligamentum mallei et incudis superior, the stapedius muscle, and ligamentum annu-

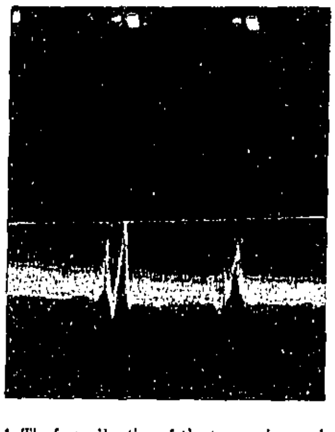

FIG. 1. The free vibration of the tympanic membrane. Time interval 1/100 sec.

FIG. 2. Aditus ad antrum and antrum.

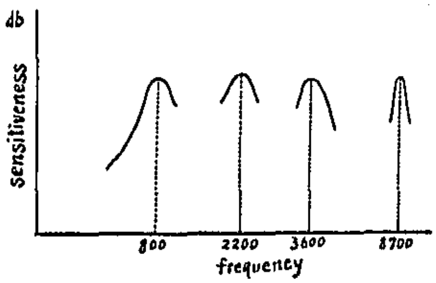

FIG. 3. Imaginary hearing curve.

lare. Due to this fact, we consider that the characteristic curve of the ossicles is probably linear.

However, we may consider that a sort of a tuning fork made by the malleus and the long process of the incus may perhaps have its free vibration period. Following this idea and using Young's modulus in regard to bone, I obtained in 1943, 8700 cycles as a value of its frequency.

$$f=K(A/L)(E/\rho)^{\frac{1}{2}}=0.161\times(0.083/0.5)(E/\rho)^{\frac{1}{2}}$$
$$=8724 \text{ cycles},$$

where A and L are thickness and length of tuning fork, respectively. We can consider practically, that the malleus and the long process of the incus are almost of same length, i.e., $l_1=l_2$ in Diagram II. E=Young's modulus of bone, ρ is its density. $(E/\rho)^{\frac{1}{2}}=328{,}000$ cm.

As mentioned, the ear may have four, main, resonance vibrations. From the fact that Diagram III shows electromechanically the air vibration system of the ear as a type of low pass filters, we are able to consider that after the peak of 3600 cycles the curve would drop sharply. Thus we obtain the imaginary outline of the hearing curve as in Fig. 3. However, the valleys between the peaks of this curve are considered as not very deep because the damping constants of the ear are large, as may be seen in Fig. 1.

THE HEARING CURVES

From the standpoint of my above-mentioned theory, it is necessary to get the measuring method of the frequency-characteristic curve of the ear. In 1944 I used a beat-frequency oscillator, a telephone receiver, and an attenuator of db scale as its measuring instruments. These acoustic instruments are made by RCA and their characteristics are known exactly.

Their circuit is shown in Fig. 4. In this circuit, a

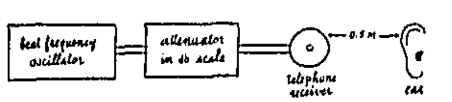

FIG. 4. Circuits of acoustic instruments used for measuring the frequency-characteristic curve of the ear.

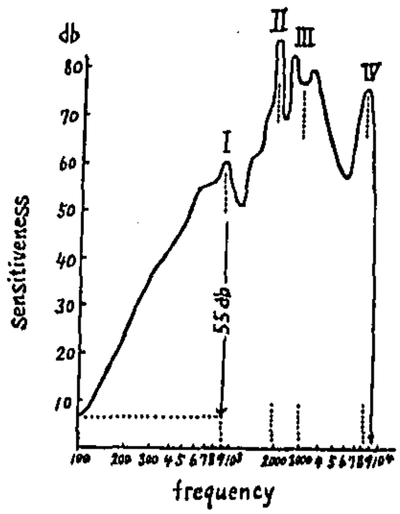

FIG. 5a. Frequency-characteristic curve of normal ear; receiver detached from the ear.

calibrated telephone receiver is placed 0.5 meter before the ear. The output of the oscillator is set at an arbitrary constant value, and the degree of the attenuator is adjusted until the subject cannot hear the tone of the telephone receiver in a soundproof room. When the attenuator readings in db scale (corrected for the receiver calibration) are plotted on the ordinate and the frequency plotted on the abscissa, we obtain a curve.

Thus, we obtain the characteristic curve of the normal ear (Fig. 5). The actual characteristic curve of the normal ear thus obtained is similar to the imaginary curve of Fig. 3. However, in detail, the small differences of the frequency between both curves are recognized

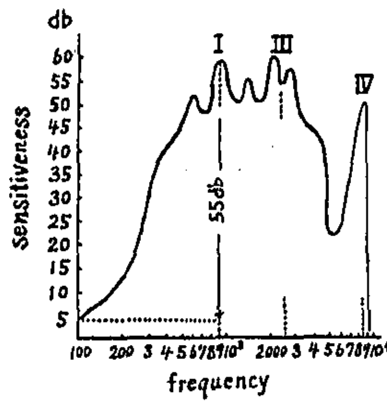

FIG. 5b. Receiver on the same ear.

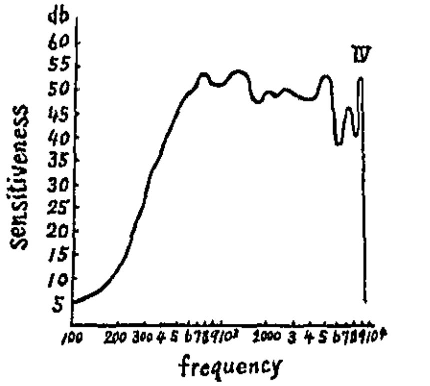

FIG. 6. An example of the normal hearing curve. Only peak IV is prominent on the normal frequency position, i.e., 8500 cycles. The other peaks are undecided with this curve because the receiver is on the ear.

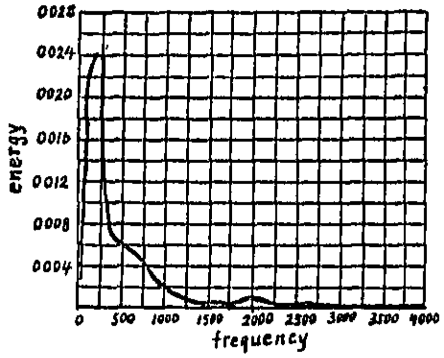

FIG. 7. The frequency-energy distribution for speech by Crandall and MacKenzie. See Phys. Rev. 17, 221–32 (1922).

especially in the peak of the external auditory canal and the antrum. The former difference is due to the approximate calculation which regards the end of the external auditory canal as rigidly closed and neglects the flexibility of the tympanic membrane. The latter difference may be due either to the individual variation of the antrum dimension or to the tympanic membrane pulling the free vibration of the antrum to itself, as is seen in Eq. (III); i.e., $M_1\ddot{x}_1+R_1\dot{x}_1+(S_1+S_P)x_1=0$. Therefore, its natural frequency $1/2\pi[(S_1+S_P)/M_1-R_1^2/4M_1^2]^{\frac{1}{2}}$, where S_1=spring constant of the antrum, S_P= that of the tympanic cavity, R_1= viscous friction, and M_1= the mass of the air in the aditus ad antrum. From Diagram II, $S_1+S_P<S_1$; consequently, the measured value in the hearing curve $1/2\pi[(S_1+S_P)/M_1-R_1^2/4M_1^2]^{\frac{1}{2}}$ is less than the calculated one $1/2\pi(S_1/M_1)^{\frac{1}{2}}$. Therefore, according to the obtained hearing curve, it is reasonable to recognize that the peak of 900 cycles corresponds to the resonance vibration of the tympanic membrane, the peak of 2000 cycles to that of the external auditory canal, the peak of 3000 cycles to that of the antrum, and the peak of 8500 cycles to that of the ossicles.

This idea can be illustrated by the following fact, that when the telephone receiver is attached to the ear of the subject, the peak of the external auditory canal (2000 cycles) disappears in the characteristic curve of the ear as shown in Fig. 5 and Fig. 6.

THE MEANING OF THE HEARING CURVE

(a) The difference of sensitiveness between 100 cycles and the first peak in Figs. 5a, 5b, and 6 is always about 50 db in both tests, with the receiver on the ear and with the receiver detached from the ear. This difference of 50 db on the normal hearing curve appears to me to correspond to the actual amplification of the middle ear. This fact has another important meaning in diagnosis in otology. For instance, this constant difference will decrease in the case of otitis media when the stiffness of the tympanic cavity increases to that of liquid.

(b) The figure of the hearing curve which has three main peaks (*I–II–III*) is significant in the intelligibility of speech of which the energy curve begins to drop at 250 cycles, as shown in Fig. 7. When compared with the energy of average speech power at 200 cycles (0.024), that (0.002) at 1000 cycles is −10 db, that of 1500 cycles (0.0005) is −17 db and so on. From this fact and the above-mentioned hearing curve, we can understand that the important acoustical role of the middle ear is amplification of the speech power in the frequency range of 500–5000 cycles or more. When this amplification was decreased due to the dysfunction of the middle ear, the intelligibility of speech and syllable articulation was decreased in my experiment. This result corresponds with that of Harvey Fletcher's experiment.[1]

(c) The abnormal displacement and damping of the first peak and the fourth peak in the test in which the receiver is detached from the subject's ear may be available in the diagnosis of pathological changes of the tympanic membrane and tympanic cavity.

[1] H. Fletcher, *Speech and Hearing*, Chapter V, pp. 279–89.

The Audibility of Thunder

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(Received December 30, 1948)

The relatively short range of audibility of thunder is explained by use of the observed vertical distributions of temperature and wind velocity in the neighborhood of thunderstorms.

INTRODUCTION

It has been pointed out by Humphreys[1] and frequently verified that thunder is seldom heard at distances greater than about 25 km from the lightning flash. Lightning which is not accompanied by audible thunder is sometimes referred to as "heat" or "sheet" lightning, although the physical characteristics of "heat" lightning and lightning accompanied by thunder seem to be identical. It appears that the inaudibility of thunder is subject to a simple explanation, which, to the writer's knowledge, has not hitherto been given. The explanation requires calculations of the refraction of sound rays resulting from temperature gradient and wind shear, subjects which have been discussed by many investigators.[2] The effects on audibility of attenuation and diffraction are not discussed here.

THEORY

The magnitude of the effects of temperature gradient and wind shear may be determined easily by modifying the theory given by Rayleigh.[3] From Snell's Law we have for the wave train which is propagated horizontally at the earth's surface

$$\sin i = c/c_0, \tag{1}$$

where i represents the angle between the incident ray and the vertical; c, the velocity of sound at the point corresponding to i; and c_0, the velocity of sound at the ground. But since the velocity of sound in air is proportional to the square root of the absolute temperature, (1) may be written

$$\sin i = (T/T_0)^{\frac{1}{2}}, \tag{2}$$

$$\tan i = (T/(T_0 - T))^{\frac{1}{2}}. \tag{3}$$

Since the ratio of the specific heats and the gas constant depends somewhat on moisture content, T and T_0 should be interpreted as *equivalent* temperatures, defined as the temperature at which the velocity of sound in dry air equals the velocity *in situ*. This refinement is unnecessary for the present purpose.

From (3) it follows that

$$dx/dz = (T/(T_0 - T))^{\frac{1}{2}}, \tag{4}$$

where x and z represent, respectively, the horizontal and vertical coordinates. If we assume a linear lapse rate given by $T = T_0 - \alpha z$, equation (4) becomes

$$dx = [(T_0/\alpha z) - 1]^{\frac{1}{2}} dz. \tag{5}$$

Standard tables give as the integral of (5)

$$x = (1/\alpha)(\alpha z(T_0 - \alpha z))^{\frac{1}{2}} - (T_0/\alpha)\tan^{-1}(\alpha z/(T_0 - \alpha z))^{\frac{1}{2}}. \tag{6}$$

However, it is convenient for the present problem to integrate (5) after first expanding in a Taylor series. In this way we find that the path of the critical ray is given by

$$x = 2(T_0/\alpha)^{\frac{1}{2}} z^{\frac{1}{2}} - (1/3)(\alpha/T_0)^{\frac{1}{2}} z^{\frac{3}{2}} - (1/20)(\alpha/T_0)^{\frac{3}{2}} z^{5/2} - \cdots. \tag{7}$$

The ratio test shows that the above series converges rapidly within the troposphere ($z < 10$ km) for the observed range of α and T_0. Since only the first term makes an important contribution, the paths of the critical rays are very nearly parabolas at low elevations.

Integration of (4) in the case of non-linear lapse rates may be accomplished by expressing the temperature by the first few terms of a power series and proceeding as above. For the problem discussed here, only linear lapse rates will be considered since greater accuracy in describing the vertical temperature distribution was not considered necessary.

An estimate of the refraction to be expected from shear may be made by considering the path of a ray propagated horizontally in the positive x direction at the earth's surface in an isothermal atmosphere with negative shear. This ray is refracted away from the earth, and is analogous to the critical ray discussed above. If we assume

$$v_0 = c + V_0, \quad v = c + V \sin i, \quad \text{and} \quad V = V_0 - \beta z \tag{8}$$

we may write in place of (1)

$$\sin i = (c + (V_0 - \beta z)\sin i)/(c + V_0). \tag{9}$$

Here, V_0 represents the horizontal component of

[1] W. J. Humphreys, *Physics of the Air* (McGraw-Hill Company, Inc., New York, 1940), p. 441.
[2] J. W. S. Rayleigh, *The Theory of Sound* (MacMillan Company, Ltd., London, 1896), pp. 129–138; F. J. W. Whipple, Q. J., Roy. Meteor. Soc. 61, 285–308 (1935); P. Rothwell, J. Acous. Soc. Am. 19, 205–221 (1947).
[3] J. W. S. Rayleigh, *The Theory of Sound* (MacMillan Company, London, Ltd., 1896), pp. 129–138.

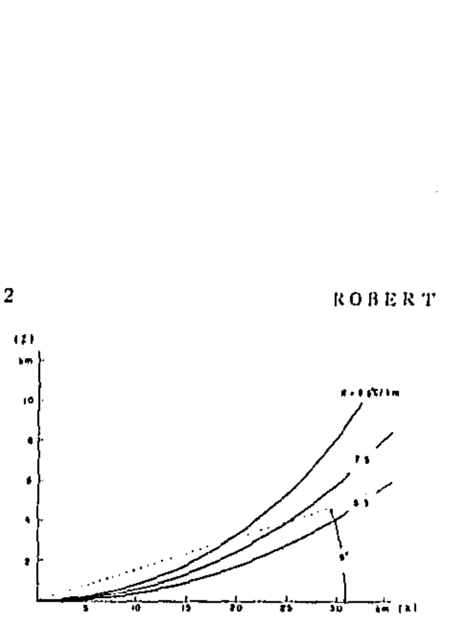

FIG. 1. Path of rays tangent to earth's surface at position of observer for $T=300°K$ and $\alpha=9.8$, 7.5, 5.5°C/km. The coordinates are: horizontal distance (x) and vertical distance (z).

wind velocity parallel to the sound ray at $z=0$, and β represents the horizontal component of vertical shear lying in the plane of the ray. Usually, β depends on z, but an estimate of the maximum curvature due to shear may be made by assuming β constant. Then we have, from (9)

$$\sin i = c/(c+\beta z), \qquad (10)$$

$$\tan i = c/[2c\beta z+\beta^2 z^2]^{\frac{1}{2}}. \qquad (11)$$

The path is given by

$$x=\frac{c}{\beta}\int_0^z \frac{dz}{[(2cz/\beta)+z^2]^{\frac{1}{2}}}. \qquad (12)$$

For values of $z \ll 2c/\beta$, the second term under the radical may be neglected, giving

$$x=[2cz/\beta]^{\frac{1}{2}}. \qquad (13)$$

The paths given by (13) are parabolas. They may be compared approximately with those computed from (7) by comparing $4T_0/\alpha$ and $2c/\beta$. If $T_0=300°K$ and $c=3.3\times10^4$ cm/sec., it follows that the paths of the critical rays resulting from temperature gradient and from shear are nearly identical when $\beta/\alpha=55$ cm²/sec./K. The following values of vertical wind shear (β) and of vertical temperature gradient (α) result in nearly identical paths of the critical rays.

α(°C/km)	β(meters/sec./km)
9.8	6
7.5	4
5.5	3

The values of shear listed here are not larger than those frequently observed in the atmosphere so it must be concluded that shear may modify considerably the curvature due to temperature gradient.

In Fig. 1 the paths of critical rays are shown as computed from (7) or (13) for $\alpha=9.8$, 7.5, 5.5°C/km or $\beta=6$, 4, 3 meters/sec./km and $T_0=300°K$. For each lapse rate or value of shear, sounds which originate to the right of the corresponding curve are inaudible at the origin, whereas sounds which originate to the left are audible. Symmetry with respect to the origin exists for the rays resulting from temperature gradient (but not for rays resulting from wind shear) so that similar regions of audibility and inaudibility exist within any vertical plane passing through the observer. It is, of course, equally true that sounds which originate at the origin are inaudible within the region to the right of the appropriate curve.

Since the curves shown in Fig. 1 are very nearly parabolas, in the simplest cases of negligible shear the regions of audibility approximate paraboloids of revolution with the observer at the vertices. The region of inaudibility is the portion of the atmosphere outside the paraboloid of audibility.

The ranges indicated in Fig. 1 are often modified by factors not considered here. For example, the range of audibility is extended by diffraction of the wave front in the region to the left of the origin and by layers of stable lapse rate. On the other hand, the range is reduced by attenuation within the atmosphere and by features of the terrain which hinder the horizontal propagation of the critical ray in its final several kilometers and also by super-adiabatic lapse rates frequently present during the day just above the ground.

CONCLUSION

Within thunderstorms the vertical distribution of temperature approximates the pseudoadiabatic rate, about 5 to 6°C/km. In the unsaturated air surrounding the thunderstorm the lapse rate is appreciably greater, approximately 7 to 8°C/km.[4]

From Fig. 1 it is apparent that if an average lapse rate of 7.5°C/km is assumed, thunder which originates at a height of 4 km has a maximum range of audibility of 25 km if shear is ineffective.

The vertical wind shear near the base of mature and dissipating thunderstorms studied by Byers and Braham is directed toward the storm and appears to be about 1.5 to 3 meters/sec./km;[5] consequently, sound which originates within the storm must be refracted away from the earth. In this case the shear lies in the plane of the critical ray throughout a large portion of its path so that the maximum range of audibility as computed from (6) or (7) probably is reduced appreciably and uniformly in all directions. The observed small range of audibility for thunder, therefore, follows directly from the normal temperature and wind distributions within and in the neighborhood of thunderstorms.

[4] S. Petterssen, *Weather Analysis and Forecasting* (McGraw-Hill Company, Inc., New York, 1940), pp. 50–85.

[5] H. R. Byers and R. R. Braham, J. Meteor. 5, 71–86 (1948).

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The Lined Tube as an Element of Acoustic Circuits*

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(Received December 4, 1948)

This paper presents a method for calculating the performance of acoustic circuits containing lined ducts. The diameters of the ducts must be less than one-half the wave-length of sound in free air. The cases of long and short ducts are treated. For the latter, equivalent electrical circuits are given, and some discussion is given concerning the equivalent electrical circuits for long ducts. A brief list of formulas applicable to filters employing lined tubes is given.

INTRODUCTION

IT is the purpose of this paper to consider the lined tube as an acoustical element and to show how this element may be combined with other acoustic elements (see Figs. 1 and 2). The assumption will be made here that the wave-length is long compared to the largest transverse duct dimension. Distributed parameter formulas will be given for the lined duct which enable one to follow a systematic procedure for calculating the performance of various combinations of lined ducts and other acoustic elements. The use of these formulas will be illustrated by a few examples. One of the important applications of lined tubes is in their use as acoustic filter elements, and it is shown here how the theory of acoustic filters developed by W. P. Mason[1] for rigid tubes can be adapted for use with lined tubes.

The assumption described above puts no restriction on the length of the duct. However, in some applications the length of the duct is small compared to the wave-length of the sound propagated in it (duct length $\leq \frac{1}{8}$-wave-length). In these cases it is possible to make further simplifications and to derive equivalent electric circuits using lumped parameters to represent a lined tube. Several examples are given showing the applications of these circuits. It is to be pointed out that lumped parameter representation can be applied to tubes of arbitrary length but that the procedure requires caution. A brief discussion of this is given for rigid walled tubes, and it is believed that the results obtained are applicable to most cases met in practice.

DISTRIBUTED IMPEDANCE METHOD

When a lined tube is part of an acoustic circuit there is a straightforward method for calculating pressures, velocities, and impedances. It is simply this: Start at the end of the system farthest from the sound generator. Then, working towards the generator, compute the acoustic impedance at the various junctions in the circuit until the impedance at the generator is obtained. Then, working forward from the generator, calculate the various velocities and pressures. In this process the composite impedance which exists at the end of a lined tube is used as z_x in Eq. (3) and the impedance at the end of the tube nearest the generator is taken as z_0. Other impedances are calculated by standard acoustic theory. When computing pressures, velocities, and impedances at the ends of lined tubes, the following formulas can be used. They may be derived by making suitable approximations in the general solution to the duct problem. This reduction is not given here because it is not necessary for the purposes of this paper.**

$$v_l = v_0(A_h/A_d) \cdot [z_c/(z_c \cosh i\Gamma_{x00}l + z_x \sinh i\Gamma_{x00}l)], \quad (1)$$

$$p_l = p_0[z_x/(z_x \cosh i\Gamma_{x00}l + z_c \sinh i\Gamma_{x00}l)], \quad (2)$$

$$z_0 = z_c[(z_x + z_c \tanh i\Gamma_{x00}l)/(z_c + z_x \tanh i\Gamma_{x00}l)], \quad (3)$$

$$z_c = k/\Gamma_{x00}, \quad (4)$$

where:

A_h = area of opening into tube on side nearest the sound generator (cm²).
A_d = area of duct cross section (cm²).
v_0 = linear velocity of air at entrance to tube on side of hole A_h nearest the generator (cm/sec.). If the hole A_h were closed by a piston, v_0 would be the linear velocity of the piston.
v_l = linear velocity of air inside the tube at end farthest from generator (cm/sec.).
p_0 = acoustic pressure inside tube at end of tube nearest the generator (dynes/cm²).
p_l = acoustic pressure inside tube at end of tube farthest from generator (dynes/cm²).
l = tube length (cm).
Γ_{x00} = longitudinal propagation constant.
$k = \omega/c$.
z_c = characteristic impedance of tube in (ρc) units; pressure/($\rho c \times$linear velocity).
z_x = impedance of tube termination in (ρc) units; pressure/($\rho c \times$linear velocity).
z_0 = input impedance of tube in (ρc) units; pressure/(ρc $\times$linear velocity). Pressure and velocity are taken inside the tube and not inside hole A_h.

* Part of a dissertation presented for the degree of Doctor of Philosophy in New York University.
[1] W. P. Mason, Bell Sys. Tech. J. 6, 258 (1927).

** Complete details may be found in C. T. Molloy, *The Propagation of Sound in Tubes Lined with Sound Absorbing Material*, Doctoral Dissertation, New York University, 1948.

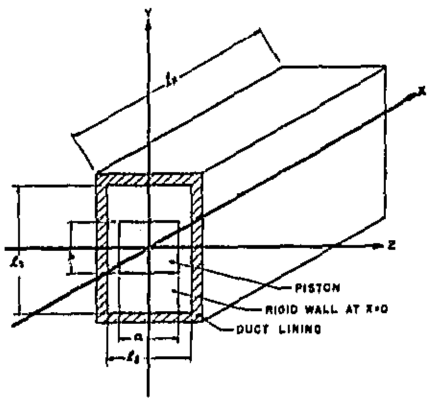

FIG. 1. Rectangular duct.

The writer[1] has given an approximate formula for Γ_{x00} applicable to ducts of arbitrary cross section where the lining is formed of strips of absorbing material of constant width whose long edge is parallel to the duct axis. In the present notation the formula is:

$$\Gamma_{x00}=[k^2-ik(L/A_d)\cdot(1/z_s)]^{\frac{1}{2}}, \quad (5)$$

$$1/z_s=\sum_{i=1}^{i=n}(1/(L/l_i)z_i), \quad (6)$$

where:

L = duct perimeter (cm).
l_i = width of (ith) strip.
z_i = acoustic impedance of (ith) strip in (ρc) units.
n = number of strips.

The preceding Eqs. (1)–(6), together with standard acoustic theory, permit the analysis of any acoustic circuit in which lined tubes are present. The use of these formulas will be illustrated by a few examples.

LINED DUCT OPEN AT ONE END

Consider a lined duct having a circular cross section of length (l), cross-sectional area (A_d), and radius (a) driven by a piston of area A_k. At the end of the duct remote from the piston a large flange is assumed to exist. The acoustic impedance at this end is therefore the well-known impedance of a piston mounted in a baffle and radiating out into space.[2]

$$z_x=[1-(J_1(2ka)/ka)]+[S_1(2ka)/ka]i \quad (7)$$

[1] C. T. Molloy, J. Acous. Soc. Am. 16, 31 (1944).
[2] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1936), p. 259.

where J_1 is Bessel's function of the first kind and order one and S_1 is a Struve Function. Equation (7) inserted into formulas (1)–(3) will yield any of the pertinent acoustical quantities directly.

LINED DUCT COUPLED TO A VOLUME AT ONE END

Consider the same system as above except that the end of the duct remote from the piston is closed with a thin rigid plate having a hole of area (S) cm². The duct communicates through the hole with a rigid walled closed container of volume V cm³.

If (v) is the linear velocity of air in the hole (S), and (v_l) is the linear velocity of air in the duct a very short distance in front of the hole (S), then by continuity of volume flow we have:

$$vS=v_lA_d, \quad (8)$$

and since the pressure is constant we have:

$$S(v/p)=A_d(v_l/p). \quad (9)$$

Or

$$S/z_v=A_d/z_x \quad \text{and} \quad z_x=(A_d/S)z_v, \quad (10)$$

where z_v is the acoustic impedance of the volume V and is given by:

$$z_v=-(S/Vk)i \quad (11)$$

and

$$z_x=-(A_d/Vk)i. \quad (12)$$

This (z_x) can be inserted in the formulas (1), (2), and (3) and the acoustical quantities of interest computed.***

FILTER THEORY

The theory of acoustic filters in which wave propagation occurs in the filter elements was first given by W. P. Mason[2] in 1927. In his lucid article Mason treats filters composed of a main tube having rigid walls connected to regularly spaced side branches. His treatment includes the effect of viscosity. In his paper Mason shows that the equivalent electrical circuit for a rigid wall tube is an electrical transmission line in which there is no leakance. However, in developing the theory he

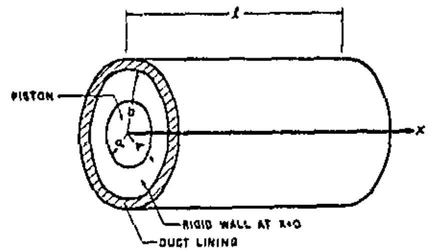

FIG. 2. Circular duct.

*** The mass and resistance of the hole have been neglected.

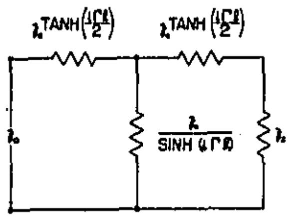

FIG. 3. Exactly equivalent "T" network.

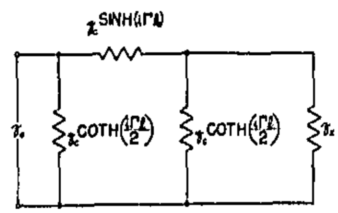

FIG. 4. Exactly equivalent "Pi" network.

makes use of the fact that the leakance is zero only in the calculation of the propagation constant of the main tube of the filter and in the calculation of the characteristic impedance of the filter main tube. The form of his pressure and velocity functions remain unchanged even if leakance is present except that new definitions are required for his propagation constant and characteristic impedance. Since Sivian[3] has shown that the lined tube is equivalent to an electric transmission line in which leakance is present, it follows that Mason's theory of filters can be taken over *in toto* when the changes noted above are made. Since the derivation of filter formulas using lined tubes as elements would precisely parallel that given by Mason for rigid tube elements, only a list of formulas giving the main results of lined tube filter theory is presented here.

SUMMARY OF LINED TUBE FILTER THEORY FORMULAS

$$p_n = p_0\left[\cosh(in\sigma) - \left(\frac{z_f}{z_0}\right)\sinh(in\sigma)\right], \tag{13}$$

$$v_n = v_0\left[\cosh(in\sigma) - \left(\frac{z_0}{z_f}\right)\sinh(in\sigma)\right], \tag{14}$$

$$z_0 = z_f\left[\frac{z_n + z_f\tanh(in\sigma)}{z_f + z_n\tanh(in\sigma)}\right], \tag{15}$$

$$z_f = z_c\left[\frac{1+\left(\dfrac{z_cA_B}{2z_BA_d}\right)\tanh(i\Gamma l)}{1+\left(\dfrac{z_cA_B}{2z_BA_d}\right)\coth(i\Gamma l)}\right]. \tag{16}$$

$\Gamma = \Gamma_{x00}$ defined by equation (5).

$$z_c = \frac{k}{\Gamma}, \tag{17}$$

$$\cosh i\sigma = \cosh(2i\Gamma l) + \left(\frac{z_cA_B}{2z_BA_d}\right)\sinh(2i\Gamma l), \tag{18}$$

$$\delta = 10\log_{10}\left[\frac{|z_0+z_s|^2}{|z_n+z_s|^2}\cdot\frac{1}{\left|\cosh(in\sigma)+\left(\dfrac{z_0}{z_f}\right)\sinh(in\sigma)\right|^2}\right]. \tag{19}$$

The type of filter to which the above formulas apply is the same as that shown in Mason's[1] Fig. 2. It comprises (n) identical filter sections. Each section has a main tube and a side branch. The side branch is placed at the middle of the main tube. Thus, in one section the main tube has a length ($2l$) while the branch is placed at a distance (l) from each end of the section.

p_n = pressure at end of nth section.
v_n = linear velocity at end of nth section.
p_0 = pressure at input of 1st filter section.
v_0 = linear velocity at input of 1st filter section.
z_0 = impedance at input of 1st filter section.
z_n = terminating impedance of nth filter section.
z_f = characteristic impedance of filter.
z_c = characteristic impedance of main tube.
z_B = impedance of side branch of filter.
z_s = impedance at input to 1st filter section looking toward the source.
σ = propagation constant for filter.
Γ = propagation constant for main tube.
n = number of filter sections.
l = half the length of one filter section.
A_d = area of main filter tube.
A_B = area of side branch opening.
L = main tube perimeter.
δ = insertion loss due to filter (db)—(assumes filter main tube has the same cross-sectional area as the tube of the system in which it is inserted).

Note—All impedances are in (ρc) units, i.e., pressure/($\rho c\times$linear velocity).

EQUIVALENT ELECTRICAL CIRCUITS

In the previous paragraphs restrictions were placed only on the diameter of the duct (Sivian[4] finds on the basis of limited experimental data that the duct diameter must be less than ½ the wave-length of the sound propagated in free space). If to this restriction we add that the length of the duct be less than ⅛-wave-length, it is possible to

[4] L. J. Sivian, J. Acous. Soc. Am. 9, 135 (1937).

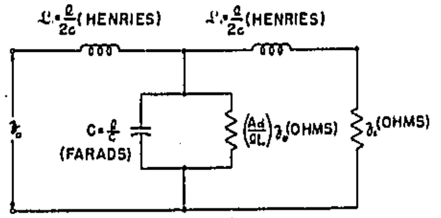

FIG. 5. Lumped parameter equivalent circuit for lined tube.

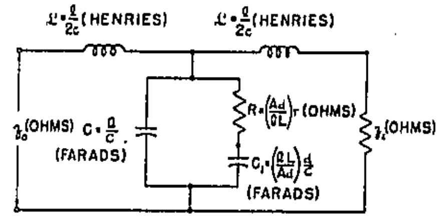

FIG. 6. Lumped parameter equivalent circuit for lined tube showing lining impedance as resistance and capacitance in series.

write an equivalent electrical "Tee" for the duct using lumped parameters. As a matter of fact this equivalence is not "one to one" since a given duct system can be represented by several equivalent electrical circuits all of which are valid. For example a lined tube may be represented as a "Tee" network, a "Pi" network, a Lattice, etc. The discussion here will be limited to a "Tee" network. The other types can readily be worked out from this by use of well-known transformations.

References[5] on electrical transmission line theory give the derivation of the exactly equivalent "Tee" and "Pi" networks for the long electrical line. In the nomenclature of this paper these networks are shown in Figs. 3 and 4.

If the impedance (z_0) is calculated from either of the above two networks it will be exactly that given by Eq. 3. If now we impose the condition that $l<\lambda/8$ then the series arms are given correct to about 5 percent each by using only the first term in the expansion of $\tanh(i\Gamma l/2)$ and the shunt arm is given correct to about 10 percent by using only the first term in the $\sinh(i\Gamma l)$ expansion. Under these conditions we have:

$$z_c \tanh(i\Gamma l/2) \approx (k/\Gamma)(i\Gamma l/2) \approx ikl/2, \qquad (20)$$

$$\frac{z_c}{\sinh(i\Gamma l)} \approx (k/\Gamma)(1/i\Gamma l) = (k/il)1/\Gamma^2. \qquad (21)$$

Combining (5) and (21) there results:

$$\frac{1}{\text{Shunt arm impedance}} = \frac{\sinh(i\Gamma l)}{z_c}$$

$$= \frac{1}{[1/(l/c)\omega i]} + \frac{1}{(A_d/lL)z_s}. \qquad (22)$$

The lumped parameter representation of the "Tee" becomes as shown in Fig. 5. For low frequencies and the case of uniform lining on all duct walls the following approximation is valid:

$$z_s \approx r - i\cot kd \approx r + 1/ikd, \qquad (23)$$

[5] W. L. Everitt, *Communication Engineering* (McGraw-Hill Book Company, Inc., New York, 1937), p. 171.

where r = resistive component of lining impedance and (d) is the thickness of lining. (This approximation is valid only for essentially homogeneous linings, such as are mostly used in practice. It does not hold if the lining is a composite structure.) For some commercial materials the (r) has been determined[6] and can be read directly from curves. Equation (23) shows (z_s) to be a resistance and capacity in series. When this is inserted in the previous circuit there results the network shown in Fig. 6.

Note—In all the equivalent circuits used here the impedances are in "ρc" units. If it is desired to utilize these circuits so that pressure is analogous to impressed voltage and velocity analogous to current in a branch then all *impedance* values must be multiplied by (ρc = 41.5). This means that all inductances and resistances must be multiplied by (41.5) and all capacitances divided by (41.5).

NUMERICAL EXAMPLE

The following calculation of circuit constants is for a 6-inch diameter circular duct lined with Johns-Manville "Airacoustic" whose length is 15 inches and which is terminated in a large flanged open end. Reference frequency = 100 c.p.s.

Length (l) = 15 inches = 38.1 cm.
Diameter = 6 inches.
r = 5.0 cm, d = 3.29 (reference 6, Fig. 2, curve c).
$(A_d/lL) = 1/38.1 \cdot 7.62/2 = 0.1$,

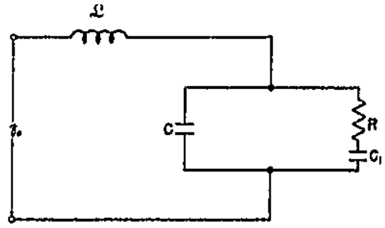

FIG. 7. Lumped parameter equivalent circuit for lined duct with rigid termination.

[6] L. Beranek, J. Acous. Soc. Am. 12, 14 (1940).

$\mathcal{L}=38.1/(2\times 3.44\times 10^4)=5.54\times 10^{-4}$ henries,
$C=38.1/(3.44\times 10^4)=1.11\times 10^{-3}$ farads,
$R=0.1\times 5=0.5$ ohm,
$C_1=(10\times 3.29)/(3.44\times 10^4)=9.56\times 10^{-4}$ farads,
$z_t=0.01+0.10i$ ohms (Eq. 7).

If the length of a duct is greater than ⅛-wave-length, at the highest frequency for which lumped parameter representation is desired, it may be divided into sections and each section represented as an equivalent "Tee" terminated by the succeeding section. It is evident that some consideration must be given to the choice of the length of each of these sections. Since the performance of each "Tee" deviates from that of the tube section which it represents it is clear that the network, representing the whole tube, being itself a composite of several "Tees" in tandem, will deviate in its performance from that of the tube which it represents. Further it is to be expected that the deviation of the network will reflect the cumulation of errors of the individual "Tees" composing it. It is this consideration which makes the proper sub-division of a long tube so important. The case of a rigid walled tube has been studied and estimates of the cumulative error obtained. It is believed that these results can be used for most practical cases even when the tube walls are not rigid. If a rigid walled duct of length (l) is subdivided into (n) equal sections each of length (l/n), then the equivalent circuit is composed of (n) "Tees" in tandem, each "Tee" of the form shown in Fig. 5. In this case $z_t=\infty$. The relations between the input and output, voltage and current have the same form for this network as do the input and output pressure and velocity in the tube. The differences between the two cases lie in their propagation constants and characteristic impedances. For the tube we have:

$$\Gamma l=\omega l/c;\quad \text{Characteristic: impedance}=1.$$

For the network we have:

$$\Gamma l\approx(\omega l/c)+(n/24)(\omega l/nc)^3;\quad (\omega l/nc)\ll 1;$$
$$\text{Characteristic impedance}=(1-(\omega l/2nc)^2)^{\frac{1}{2}}.$$

The network (Γl) and characteristic impedance can be inserted in the tube distributed parameter formulas and the deviations for any particular case computed. The error in the argument of the hyperbolic functions for the network as well as the characteristic impedance can be expressed in terms of the ratio of the length of one section to the wave-length and the number of sections.

$$\text{Error in }\Gamma l=(n/24)(\omega l/nc)^3=10.34nm^3\text{ (radians)},$$
$$\text{Characteristic impedance}=(1-(\omega l/2nc)^2)^{\frac{1}{2}}=(1-9.870m^2)^{\frac{1}{2}},$$

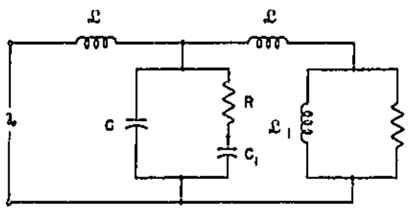

Fig. 8. Lumped parameter equivalent circuit for lined duct with open end.

n = number of sections into which tube is subdivided, and
$m=l/n\lambda$.

This concludes the discussion of the theory of lumped parameter representation for a lined duct. For purposes of illustration there are given below the equivalent electrical circuits for some acoustical systems which include lined ducts.

SYSTEM #1

Lined duct with rigid termination.
The network for this case is shown in Fig. 7.

$z_0=(\rho c$ units),
$\mathcal{L}=l/2c$ (henries),
$C=l/c$ (farads),
$R=(A_d/lL)r$ (ohms), and
$C_1=(lL/A_d)d/c$ (farads).

SYSTEM #2

Lined duct with an open end and a large flange on open end.

For this case the terminating impedance is given by Eq. (7). However, it has been shown[7] that this impedance can be represented approximately by a resistance and an inductance in parallel. The circuit for the flanged open ended tube then becomes the one shown in Fig. 8.

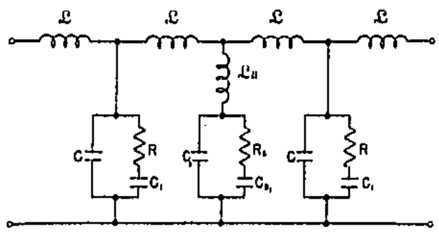

Fig. 9. One section of filter having main tube lined and side branches which are lined tubes rigidly terminated.

[7] B. B. Bauer, J. Acous. Soc. Am. 15, 223 (1944).

z_0, $\mathfrak{L}$, C, R and C_1 exactly as defined in System # 1,
$R_1 = 1.44$ ohms, and
$\mathfrak{L}_1 = 2.47 \times 10^{-6} \times$ (duct radius in centimeters).

SYSTEM #3

Acoustic filter having main tube lined and side branches which are lined tubes rigidly terminated (see Fig. 9).

$\mathfrak{L} = l/2c$ (henries),
$C = l/c$ (farads),
$R = (A_d/lL)r$ (ohms),
$C_1 = (lL/A_d)d/c$ (farads),
l = Length from beginning of section to side branch (i.e., half the section length) (cm),
A_d = Cross-sectional area of main filter tube (cm²),
L = Perimeter of main filter tube (cm),
r = Acoustic resistance of main tube lining (ρc units),
d = Thickness of main tube lining (cm),
$\mathfrak{L}_B = (A_d/A_B) \cdot (l_B/2c)$ (henries),
$C_B = (A_B/A_d)(l_B/c)$ (farads),
$R_B = (A_d/A_B) \cdot (A_B/l_B L_B) \cdot r_B = (A_d/l_B L_B) r_B$ (ohms),
$C_{B1} = (A_B/A_d)(l_B L_B/A_B) \cdot d_B/c = (l_B L_B/A_d)(d_B/c)$ (farads),
l_B = Length of side branch tube (cm),
A_B = Cross-sectional area of side branch tube (cm²),
L_B = Perimeter of side branch tube (cm),
r_B = Acoustic resistance of side branch tube (ρc units), and
d_B = Thickness of side branch lining (cm).

RESULTS

1. A method for calculating the performance of acoustic circuits containing lined ducts is given. The diameter of the ducts must be less than one-half the wave-length of the sound in free air but there is no restriction on the duct length.

2. The relationship between Mason's rigid tube acoustic filter theory and the theory of acoustic filters composed of lined tubes is pointed out and a brief list of filter formulas is given.

3. The lumped parameter equivalent circuit for a lined tube of length less than one-eighth wave-length is given and examples are given of its use. Some discussion of the equivalent circuits of long tubes is also given.

ACKNOWLEDGMENT

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Sound Transmission through Multiple Structures Containing Flexible Blankets*

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A general theory of sound transmission for normal wave incidence is developed for a structure containing two impervious layers, an air space, and two acoustical blankets. Equations for more simple structures are derived from the general case by setting some of the parameters equal to zero. A number of design charts are presented giving attenuation in decibels *vs.* frequency for different structures with specific acoustic resistivity of the blanket material as a parameter. Experimental results are found to be in good agreement with the theoretical predictions. It is found that panels deviate from mass law behavior as their thickness is increased. Mass law behavior is obtained for panels of any thickness when a laminated construction is used to damp out flexural waves.

I. INTRODUCTION

As part of a World War II program for quieting aircraft, Nichols *et al.* studied experimentally[1,2] a series of six attenuating structures. Each of these structures is a special case of a more general multiple structure consisting of two impervious layers (panels), an air space, and two flexible acoustical blankets. In the present paper an exact theory is developed for predicting the attenuation characteristics of these structures as a function of frequency for the special case of normal wave incidence. The resulting equation gives the ratio of the pressure incident on the structure to that radiated into a perfectly absorbing termination. The air space and blankets are treated as elements with distributed parameters and acoustic impedance boundary conditions at the interfaces.

Use is made of the results obtained in an earlier paper[3] relating the propagation constant and characteristic impedance of homogeneous, porous blankets to the fundamental parameters of specific acoustic resistivity, density, and, to a lesser extent, structure factor, volume coefficient of elasticity and porosity. From the charts presented in reference (3) the propagation constant and characteristic impedance were found and then inserted into the formulas derived here to give design charts of attenuation as a function of frequency.

Experimental attenuation-frequency data obtained on 18×18-inch panels using a modified form of an apparatus described in reference (2) is compared with the theoretical predictions. Some data obtained on single panels are also presented to indicate the increased attenuation made possible through the use of a "sandwich" type construction, such as bonded layers of duraluminum and mica. The apparatus appears to be a suitable method for measuring attenuation at normal incidence. Extension to larger panels and other angles of wave incidence appears to be feasible. A cooperative effort to develop a similar type of apparatus for measuring panels as large as 8×8 feet is now underway at the Acoustics Laboratory at the Massachusetts Institute of Technology.

II. THEORY

The structures to be considered are shown in Fig. 1. First, let us derive a general equation for the basic system, Structure V. Results for the more simple structures (O–IV) will then be obtained by setting some of the parameters equal to zero. In Fig. 1, Structure V is composed of: (1) an impervious layer (e.g., dural sheet) of surface density σ_1, (2) an air space of thickness l, (3) a flexible blanket of thickness d_1, (4) an impervious layer

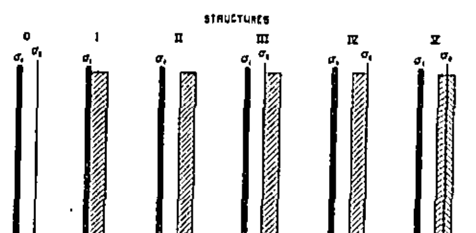

FIG. 1. Sketches of the six principal types of structures discussed in this paper.

* This paper was presented at the Thirty-Fourth Meeting of the Acoustical Society of America, December 12–13, 1947.
** During the course of this research this author held a John Simon Guggenheim Fellowship and worked jointly at M.I.T. and Harvard University.
*** Now at the Los Alamos Scientific Laboratory, Los Alamos, New Mexico.
[1] Nichols, Sleeper, Wallace, and Ericson, J. Acous. Soc. Am. 19, 428 (1947).
[2] Wallace, Dienel, and Beranek, J. Acous. Soc. Am. 18, 246 (1946).
[3] L. L. Beranek, J. Acous. Soc. Am. 19, 556 (1947).

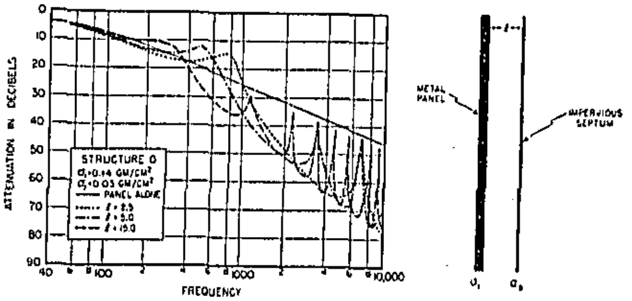

FIG. 2. Ratio of pressures on the primary and secondary sides of Structure O expressed in decibels. The secondary side is assumed to be terminated in the characteristic impedance of air.

(e.g., septum) of surface density σ_s, and (5) a second flexible blanket of thickness d_2.

Sound is to be propagated through the structure as a plane wave, starting with a magnitude p_0 at the surface of the left hand impervious layer. It will emerge from the right hand side of the d_2 blanket with a magnitude p_6 and is assumed to continue on into an "infinite" (ρc) medium. In order to calculate the ratio of p_0 to p_6, and hence, the attenuation or transmission loss, we will need the ratio of pressures at successive interfaces. The pressure in the blankets or air space will be given by the well-known solution of the one-dimensional wave equation

$$p = A \cosh(bx + \psi_b), \tag{1}$$

where x is the distance from a terminal impedance Z_T, b is the propagation constant for the medium, and

$$\psi_b = \coth^{-1}(Z_T/Z_0). \tag{2}$$

Z_0 is the characteristic impedance of the medium, i.e., of an infinite medium. Equation (2) follows directly from the impedance relation

$$Z = Z_0 \coth(bx + \psi_b) \tag{3}$$

by setting $Z = Z_T$ at $x = 0$.

In the air space $b = j\omega/c$ and $Z_0 = \rho c$. In the blanket b and Z_0 will be complex functions of the fundamental parameters of the blanket: specific acoustic resistivity R_1 (i.e., specific acoustic resistance of a unit cube of the material), density ρ_m and, to a lesser extent, structure factor k, volume coefficients of elasticity K and Q and porosity Y. In a previous paper charts (Figs. 1 and 2 of reference (3)) were presented, giving the magnitude and phase angle of the propagation constant, b, for the case of flexible (soft) blankets. A flexible blanket is defined as one for which the ratio K/Q is greater than twenty, where K and Q are the volume coefficients of elasticity of the air in the blanket, and of the acoustical material, respectively. In this case we are neglecting the highly attenuated wave

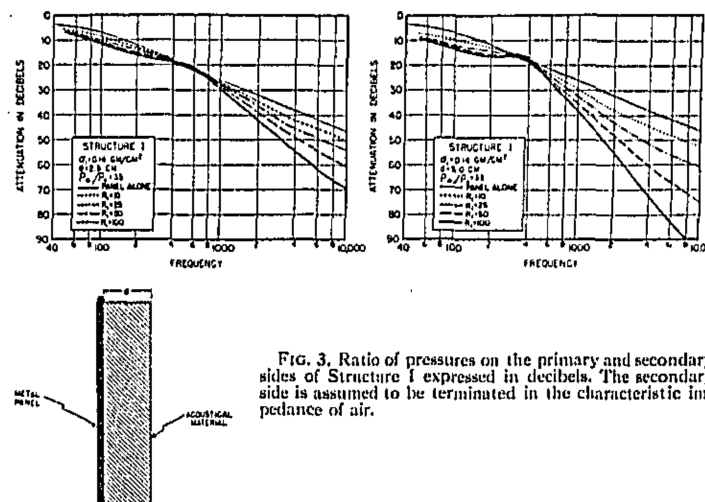

FIG. 3. Ratio of pressures on the primary and secondary sides of Structure I expressed in decibels. The secondary side is assumed to be terminated in the characteristic impedance of air.

associated with the skeleton, so that it is possible to express the pressure, p, as a hyperbolic function involving a *single* propagation constant (see Eq. 1). If we make the further assumption that the porosity Y is greater than 0.95, then the characteristic impedance of an infinite sample is found from Eq. (27) of reference 3,

$$Z_0 = (jK/\omega Y)\cdot b. \qquad (4)$$

The impervious layers will be assumed to have no stiffness so that their impedances can be represented by $j\omega\sigma$, where σ is the surface density. With the help of Eq. (3) we can determine the impedances at the interfaces of the layers of the structure. In the following we designate the impedance looking from left to right by Z_i, where the subscript i refers to the interface. Proceeding from left to right (see V of Fig. 1), Z_1 is the impedance as seen from the right side of the first (σ_1) impervious layer, Z_2 the

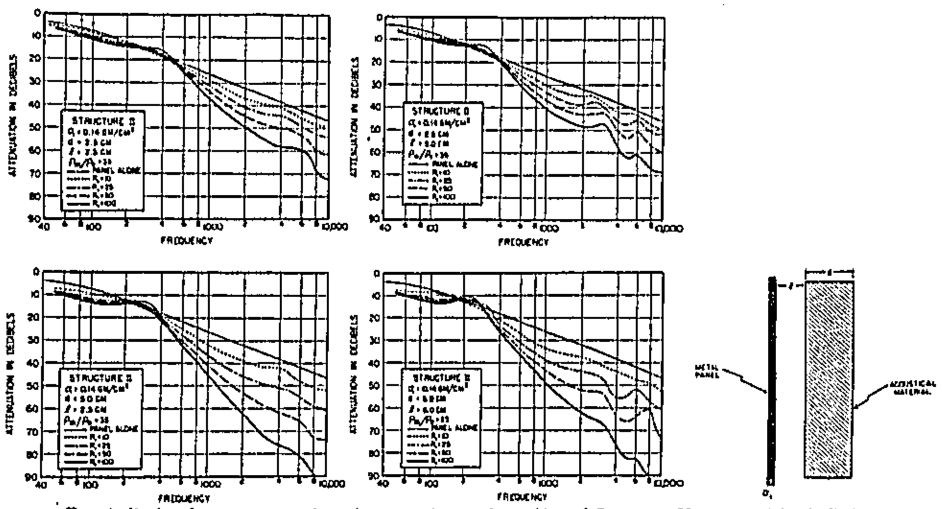

FIG. 4. Ratio of pressures on the primary and secondary sides of Structure II expressed in decibels. The secondary side is assumed to be terminated in the characteristic impedance of air.

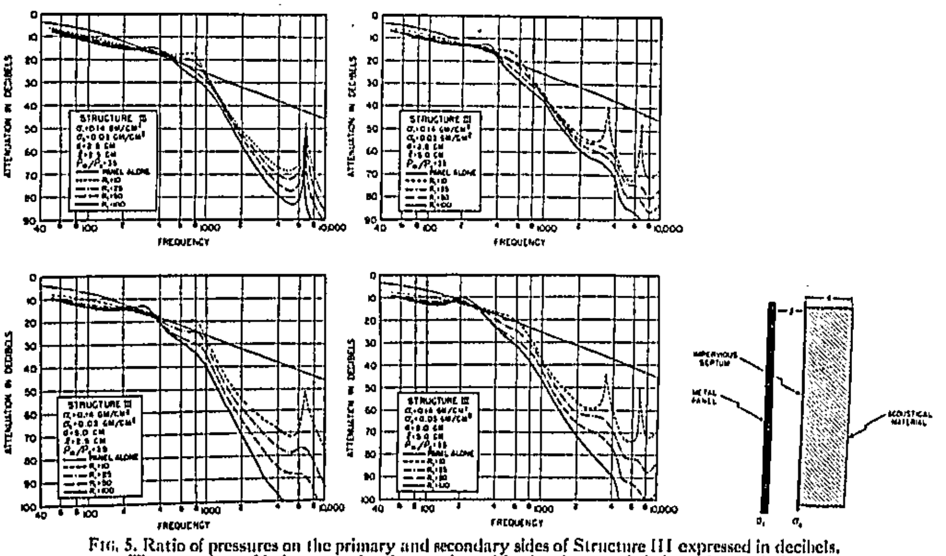

FIG. 5. Ratio of pressures on the primary and secondary sides of Structure III expressed in decibels. The secondary side is assumed to be terminated in the characteristic impedance of air.

impedance seen from the air space-blanket interface, etc. We find

$$Z_5 = \rho c, \tag{5}$$

$$Z_4 = Z_0 \coth(bd_2 + \psi_4), \tag{6}$$

$$Z_3 = Z_4 + j\omega\sigma_4, \tag{7}$$

$$Z_2 = Z_0 \coth(bd_1 + \psi_2), \tag{8}$$

$$Z_1 = \rho c \coth(j\omega l/c + \psi_1), \tag{9}$$

where the ψ's are, from Eq. (2),

$$\psi_4 = \coth^{-1}(\rho c/Z_0), \tag{10}$$

$$\psi_2 = \coth^{-1}(Z_3/Z_0), \tag{11}$$

$$\psi_1 = \coth^{-1}(Z_2/\rho c). \tag{12}$$

The ratio of pressures on opposite sides of each impervious layer is given by the impedance ratio because the velocity is continuous. The ratio of pressures on opposite sides of the air layer or blankets can be found from Eq. (1). As was done for the impedance, let us designate the pressure at a particular interface by an appropriate subscript, proceeding in order from left to right (p_0 and p_6 have already been defined). The pressure ratios are seen to be:

$$p_0/p_1 = 1 + j\omega\sigma_1/Z_1, \tag{13}$$

$$p_1/p_2 = [\cosh(j\omega l/c + \psi_1)]/\cosh\psi_1, \tag{14}$$

$$p_2/p_3 = [\cosh(bd_1 + \psi_2)]/\cosh\psi_2, \tag{15}$$

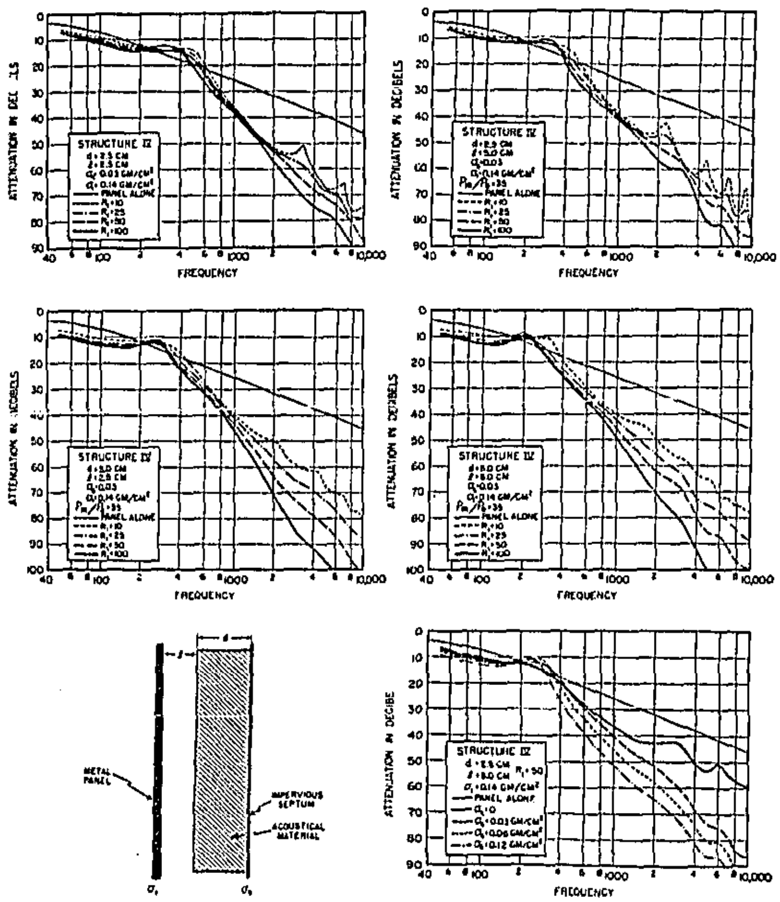

FIG. 6. Ratio of pressures on the primary and secondary sides of Structure IV expressed in decibels. The secondary side is assumed to be terminated in the characteristic impedance of air.

$$p_3/p_4 = Z_3/Z_4 = 1 + j\omega\sigma_s/Z_4, \quad (16)$$

$$p_4/p_5 = [\cosh(bd_2+\psi_4)]/\cosh\psi_4. \quad (17)$$

Multiplying together Eqs. (13)–(17) gives, of course, the desired ratio p_0/p_5. Utilizing Eqs. (5)–(12) to reduce the involved product, one obtains, after several expansions of the hyperbolic functions, the following general result for Structure V.

Structure V

$$p_0/p_5 = [x_1 \cosh bd_1 + x_2 \sinh bd_1] \cosh bd_2 + [x_3 \cosh bd_1 + x_4 \sinh bd_1] \sinh bd_2 \quad (18)$$

where

$$x_1 = A + j(B + A\omega\sigma_s/\rho c) \quad (19)$$

$$x_2 = AZ_0/\rho c - B\omega\sigma_s/Z_0 + jB\rho c/Z_0 \quad (20)$$

$$x_3 = AZ_0/\rho c + j(B\rho c/Z_0 + A\omega\sigma_s/Z_0) \quad (21)$$

$$x_4 = A - \omega\sigma_s\rho c/Z_0^2 B + jB \quad (22)$$

and,

$$A = \cos(\omega l/c) - (\omega\sigma_1/\rho c)\sin(\omega l/c) \quad (23)$$

$$B = \sin(\omega l/c) + (\omega\sigma_1/\rho c)\cos(\omega l/c). \quad (24)$$

Setting various parameters equal to zero in Eqs. (18)–(24) we obtain the following results for the derived family of structures.

Structure IV; $d_2=0$, $d_1=d$

$$p_0/p_5 = x_1 \cosh bd + x_2 \sinh bd \quad (25)$$

Structure III; $d_1=0$, $d_2=d$

$$p_0/p_5 = x_1 \cosh bd + x_3 \sinh bd \quad (26)$$

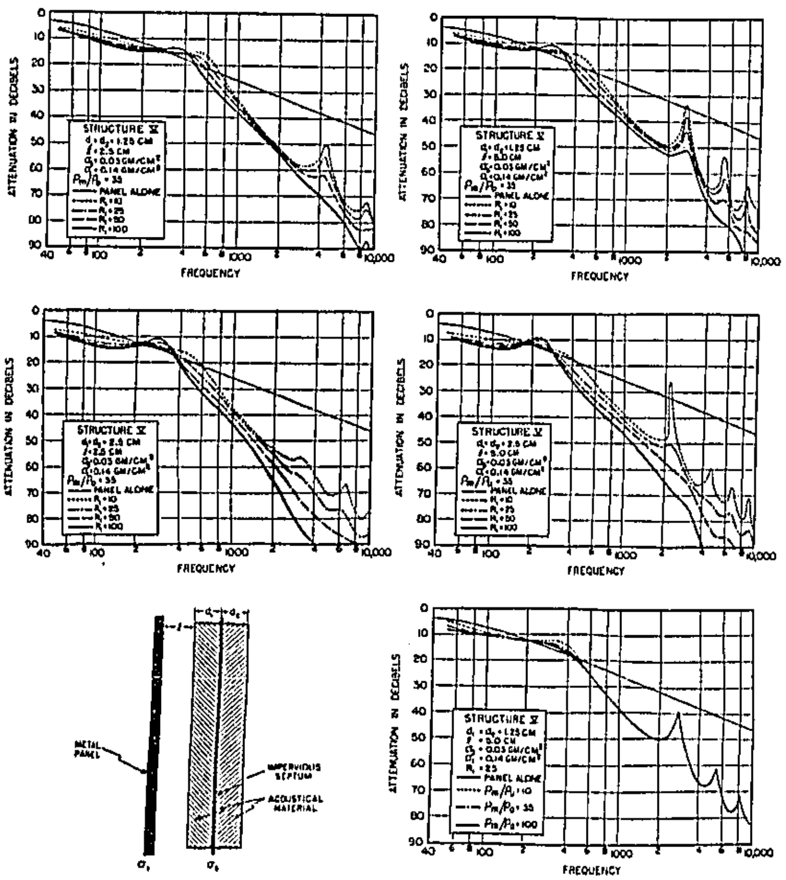

FIG. 7. Ratio of pressures on the primary and secondary sides of Structure V expressed in decibels. The secondary side is assumed to be terminated in the characteristic impedance of air.

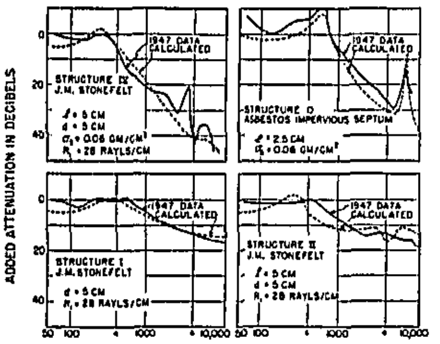

FIG. 8. Difference in decibels between the attenuation provided by the structure shown on the graph and that provided by the damped metal panel of density σ_1. These graphs illustrate the agreement between theory and measurement for four different types of structures.

Structure II; $d_2=0,\ \sigma_s=0,\ d_1=d$

$$p_0/p_5=(A+jB)\cosh bd+(AZ_0/\rho c+jB\rho c/Z_0)\sinh bd \quad (27)$$

Structure I; $d_2=0,\ \sigma_s=0,\ l=0,\ d_1=d$

$$p_0/p_5=(1+j\omega\sigma_1/\rho c)\cosh bd+(Z_0/\rho c+j\omega\sigma_1/Z_0)\sinh bd \quad (28)$$

Structure O; $d_1=d_2=0$

$$p_0/p_5=x_1=A+j(B+A\omega\sigma_s/\rho c). \quad (29)$$

Two additional structures, designated Partitions Type 1 and 2, are special cases of Structures O and IV, respectively,

Partition Type 1; $d_1=d_2=0,\ \sigma_1=\sigma_s=\sigma$

$$p_0/p_5=A+j(B+A\omega\sigma/\rho c) \quad (30)$$

where, $\sigma_1=\sigma$ in Eqs. (23) and (24).

Partition Type 2; $d_2=0,\ l=0,\ \sigma_1=\sigma_s=\sigma$

$$p_0/p_5=(1+2j\omega\sigma/\rho c)\cosh bd+(Z_0/\rho c-\omega^2\sigma^2/Z_0\rho c+j\omega\sigma/Z_0)\sinh bd. \quad (31)$$

Because the expressions above for attenuation are quite complicated, design curves for each type of structure have been prepared. The design charts, presented in Figs. 2–7, give the attenuation of each structure for various values of the depth of the air space l in cm, the thicknesses of the two blankets d_1 and d_2 in cm, and the specific (unit volume) resistance of the blanket R_1 in rayls/cm.**** It is shown in reference 3 that R_1 varies only slightly with frequency, and is approximately equal to R_f, the flow resistance per cm thickness measured by a steady air flow method. It has also been found that variation of the density alone of an acoustic blanket has very little effect on the propagation constant for the materials.

III. EXPERIMENTAL RESULTS

In Figs. 8–10 the theoretical predictions are compared with data obtained using the modified form of the apparatus described in Fig. 3 of reference 2. An array of nine speakers, forming an essentially zero acoustic impedance source, transmits sound through an 18×18-inch sample which is terminated by a ρc impedance. The incident and transmitted energy are measured by microphones on opposite sides of the structure under test. The principle modifications to the apparatus consisted

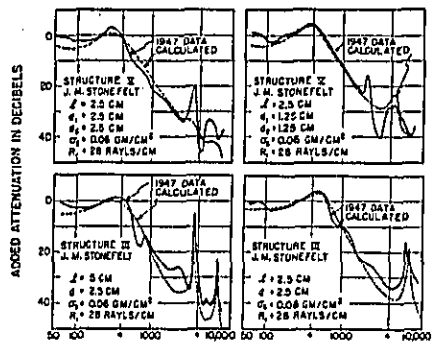

FIG. 9. Difference in decibels between the attenuation provided by the structure shown on the graph and that provided by the damped metal panel of density σ_1. These graphs show the difference between theory and measurement for two types of structures with different blanket thicknesses and air space depths.

in filling the air space between the loudspeakers and the (σ_1) panel with an acoustical blanket in which nine holes equal in diameter to the loudspeaker diameters were cut. Also, absorbing material was placed around the edges of the air space between the (σ_1) panel and the blanket, or σ_s panel. These added absorbing materials prevent lateral standing waves in those spaces. As will be shown later, it is also necessary to use a well-damped panel in the σ_1 position of Fig. 1 in order that weight-law attenuation will be achieved.

Figure 8 shows a comparison of theory and of measurement for the same material (J. M. Stonefelt) in three different structures. In Fig. 9 there is shown for two structures the effect of changing either the thickness of the absorbing blankets or the depth of the air space. The ordinates of these graphs indicate the increased attenuation obtained

**** The word *rayl* was adopted in reference 3 as the unit for the ratio of sound pressure to linear particle velocity. In the c.g.s. system it symbolizes the units dyne-sec./cm³.

over that which would be obtained for the single sheet of dural (σ_1) alone. The agreements are seen to be satisfactory.

In Fig. 10 we show comparisons of calculated and measured data for two structures using three widely different types of materials for the absorbing blankets. In these cases the data were taken by Mr. H. F. Dienel† with the earlier version of the 18×18-inch apparatus.[2] The agreement between measured and calculated data for Structure V is good. For Structure III, however, systematic differences exist at high frequencies. These systematic differences occur because transverse standing waves in the air space between the panel and the loudspeakers were not damped out as was the case in the modified form of the apparatus and as was assumed in the theory.

In airplane applications the principal concern is to reduce the high frequency components in the airplane noise. Such reduction improves speech intelligibility and comfort for passengers and crew. A comparison among the five different structures at 1000 and 5000 c.p.s. are shown in Figs. 11 and 12. It is seen that Structures III, IV, and V differ little from each other. However, Structures III and V present an absorbing face to the interior of the cabin which reduces reverberant sound. At both 1000 c.p.s. and 5000 c.p.s. the improvement of Structure IV over Structure III is of the order of three decibels. If a non-absorbent interior finish is permissible because of the existence of rugs and upholstered seats in the cabin, then Structure IV is the best at all frequencies. The relative importance of transmission loss and sound absorption in airplane quieting may be understood by study of previous papers.[4]

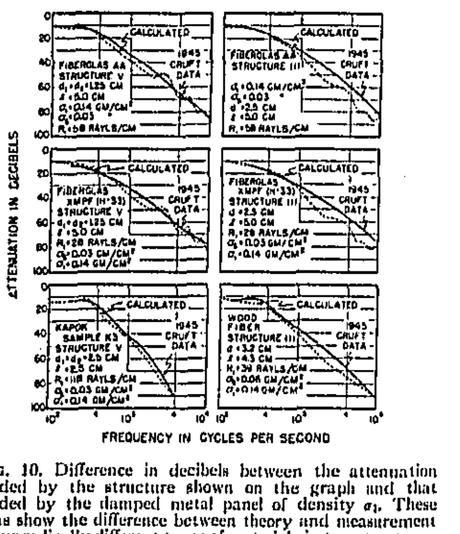

FIG. 10. Difference in decibels between the attenuation provided by the structure shown on the graph and that provided by the damped metal panel of density σ_1. These graphs show the difference between theory and measurement for four radically different types of materials in two structures. The systematic difference between high frequency data and theory for Structure III is explained in the text.

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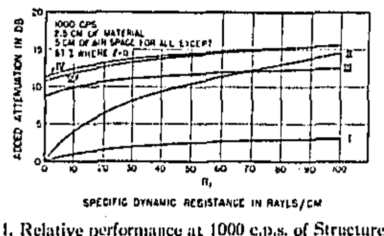

FIG. 11. Relative performance at 1000 c.p.s. of Structures I–V as a function of specific dynamic resistance of the blanket.

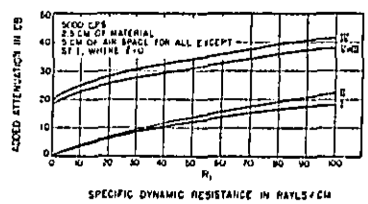

FIG. 12. Relative performance at 5000 c.p.s. of Structures I–V as a function of specific dynamic resistance of the blanket.

IV. WEIGHT LAW ATTENUATIONS FOR PANELS

In the basic theory it is assumed that weight law attenuation is obtained for the impervious septa. This assumption requires checking. The attenuation of a series of dural panels with surface densities varying from 0.25 to 5.5 lb./ft.^{2} was measured in the 18×18-inch apparatus. Out to a frequency of about 1000 c.p.s., each panel appears to follow weight law reasonably well above its major resonant frequency. Above 1000 c.p.s., however, it was soon discovered that for certain surface densities the attenuation is actually less than that for lower surface densities. A curve illustrating this fact for undamped single panels is shown in Fig. 13. These data are the measured attenuations in db at 3000 c.p.s. as a function of surface density. The zero on the ordinate refers to the attenuation obtained for the 0.25 lb./ft.^{2} panel. The attenuation which should be obtained from weight law considerations is given by Eq. (32) and is shown by the straight line of Fig. 13.

Theoretical weight law attenuation

$$= 10 \log_{10}[1+(\omega\sigma/\rho c)^2] \text{ db.} \quad (32)$$

[4] L. L. Beranek and H. W. Rudmose, Trans. A.S.M.E. 69, 89–97 (1947). L. L. Beranek and H. W. Rudmose, J. Acous. Soc. Am. 19, 357 (1947). H. W. Rudmose and L. L. Beranek, J. Aero. Sci. 14, 79 (1947).

TABLE I. Transmission loss in db.

Normal incidence Eq. (32)	Normal incidence Eq. (33)	Random incidence Eq. (32)	Random incidence Eq. (33)	Difference between random and normal incidence
15.7	10	11.6	5.9	4.1
26	20	19.3	13.3	6.7
36	30	27.6	21.6	8.4
46	40	36.4	30.4	9.6
56	50	45.4	39.4	10.6
66	60	54.6	48.6	11.4

At high frequencies, the difference between theory and measurement is of the order of 17 db.

It should be noted that Eq. (32) differs from the classical formula for weight-law attenuation expressed by Eq. (33).

Classical weight law attenuation

$$=10\log_{10}[1+(\omega\sigma/2\rho c)^2]\text{ db.}\quad(33)$$

The explanation for this difference, is that Eq. (32) gives the ratio (in db) of the pressures on the two sides of the panel, while Eq. (33) gives the ratio (in db) of the pressure at a point before and after the panel is inserted between the point and the source. Because, for $(\omega\sigma)^2 \gg (2\rho c)^2$, pressure doubling occurs when the panel is present, the attenuation is six decibels greater for Eq. (32) than for Eq. (33).

The deviation from theoretical weight law mentioned in the paragraph preceding the last is attributable to the existence of flexural sound waves traveling in transverse directions in the panel. This conclusion was checked by making the panels highly damped, such that the transverse waves were made ineffective. For the damping, sheets of mica weighing about 0.06 lb./ft.^{2}, were cemented to thin aluminum panels and these panels were then combined to form sandwiches covering a range of densities from 0.25 to 2.5 lb./ft.^{2}. Attenuations measured for these panels were in good agreement with weight law, except at very high frequencies (above 6000 c.p.s.) where the attenuation became somewhat higher than weight law. The results of these tests at 3000 c.p.s. are shown plotted in Fig. 13 as open circles.

V. RANDOM WAVE INCIDENCE

Even when panels are sufficiently damped so that they follow weight law for normal wave incidence, London[5] has shown that the average attenuation for random wave incidence is considerably lower than that given by Eqs. (32) and (33). He derives a comparison between the normal and random transmission loss values for panels which follow weight law for normal incidence. The results taken directly from his paper are shown in columns 2, 4 and 5 of Table I. Columns 1 and 3 have been added to give attenuation in terms of the ratios of the pressures on the two sides of the panel. That is to say, columns 1 and 2 are direct comparisons of Eqs. (32) and (33) for the same surface density.

VI. OFFICE PARTITIONS

It is interesting to compare the calculations of this paper with data published by Gorton[6] on an office partition composed of two sheets of metal, separated by about three inches, in which an absorbing blanket was introduced. Calculated and measured data for this structure are shown in Fig. 14. A comparative curve correcting for the difference between random and normal sound incidence is shown by the heavy dashed curve in Fig. 14. The corrections of Table I were applied to each panel separately. The difference between the calculated and measured data of that figure is due, perhaps, to the fact that the edges of the Gorton panels were not vibration isolated from each other. Hence, there was a flanking path through which sound could be transmitted. Further experimentation would seem to be profitable to determine if a significant improvement for that type of demountable panel could be affected, using a different type of construction incorporating vibration isolation between the two faces.

VII. CONCLUSIONS

A study of the graphs and the equations leads to the conclusions:

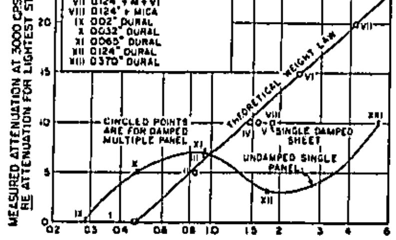

FIG. 13. Chart illustrating the increase in attenuation at 3000 c.p.s. as a function of surface density for damped and undamped metal panels.

[5] Albert London, "Transmission of reverberant sound through single and double walls," submitted to the International Symposium on Noise, London, July 1948. To be published by the Physical Society of London together with other papers of the Symposium.

[6] W. S. Gorton, J. Acous. Soc. Am. 17, 236 (1946).

(1) The theory and measurement of sound transmission through the structures of Fig. 1 are in good agreement for normal wave incidence.

(2) The charts given in Figs. 2–7 show that for best results at low frequencies the over-all depth of the structure should be as great as possible.

(3) At high frequencies the absorbing blanket between the two impervious layers, σ_1 and σ_s, removes the resonant peaks and appreciably increases the transmission loss.

(4) When there are no acoustical blankets (Structure O) the attenuation at low frequencies is that of a simple filter comprised of a series mass σ_1, a shunt condenser ($l/\rho c^2$), and a series mass σ_s working into a ρc termination. These elements will resonate at some frequency (see Fig. 2) where the attenuation will be substantially less than that of the panel alone. Above that frequency, the attenuation increases rapidly until the wave-length approaches twice the separation l. When $l=n\lambda/2$, where n is an integer and λ is the wave-length of the driving frequency, resonance occurs. At the resonance peaks the attenuation is a minimum and is given approximately by the formula:

$$\text{Min. atten.}=10\log_{10}\left[1+\frac{\omega^2(\sigma_1+\sigma_s)^2}{\rho^2c^2}\right]\text{db.}\quad(34)$$

This equation says that at the resonant peaks, the two panels act as though they are one panel with a surface density $(\sigma_1+\sigma_s)$. Between these resonant peaks the maximum attenuation is given approximately by

$$\text{Max. atten.}=10\log_{10}\left[1+\frac{\omega^2\sigma_1^2}{\rho^2c^2}\right]+10\log\left[1+\frac{\omega^2\sigma_s^2}{\rho^2c^2}\right]\text{db.}\quad(35)$$

(Note that because of the definition of attenuation used the values (for $\omega^2\sigma^2\gg\rho^2c^2$) of each of the logarithmic terms is six decibels higher than that usually given in the literature.) This equation says that at the points of maximum attenuation, the two panels act as though each was terminated by ρc and that their attenuations were linearly additive.

(5) With the acoustical blanket in the airspace between the two septa the attenuation at the low frequencies is nearly the same as that without it, except that the value of the shunt capacitance is increased by about 40 percent because of the change from adiabatic to isothermal compression in the gas (see reference 3). At the high frequencies, the

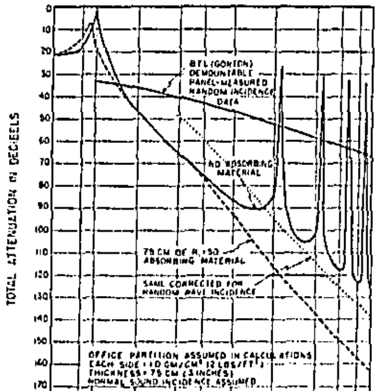

FIG. 14. Comparison of calculated and measured data on a typical metal office partition. The difference between measurement and theory at high frequencies may be due to flanking transmission.

maximum attenuations for Structures IV and V are given by the equation:

$$\text{Max. atten.}=\text{Eq. (35)}+[8.686\cdot Re(b)\cdot d]-6\text{ db,}\quad(36)$$

where $Re(b)$ is the real part of the propagation constant of the acoustical blanket, and d is the sum of the thicknesses of the blanket in the structure in cm. The six db term in Eq. (36) is included because of pressure doubling at the interface between the absorbing blanket and the σ_s interface layer. This equation says that the effect of the blanket in the airspace is to provide a ρc termination for the σ_1 panel. Also, it decouples the two panels from each other so that their attenuations are separately additive except for the 6 db pressure doubling term just mentioned. In addition, there is an attenuation loss in the blanket itself given by the second term in Eq. (36). At these frequencies, the airspace l in Structures IV and V has no effect on the attenuation because the wave traverses the space suffering only a phase shift.

(6) For random wave incidence at high frequencies the values in the fifth column of Table I should be subtracted from the weight law attenuation of Eq. (32) for *each* impervious septum used in the structure.

(7) In the design of office partitions, the panels on the opposite sides of the partitions should probably be vibration isolated to obtain full value from the elements of the structure.

(8) At normal incidence, weight law attenuation

is achieved for thin panels only if they are sufficiently thin or sufficiently damped that flexural waves in the panel are highly attenuated.

(9) These studies suggest that a series of panels of different surface densities made of alternate layers of metal and bonded mica would serve as calculable standards of transmission loss for use in test laboratories.

(10) The agreement between measurement and theory is consistently satisfactory enough that the extension of the 18×18-inch apparatus to the measurement of the transmission loss of large panels is indicated.

VIII. ACKNOWLEDGMENT

The authors are deeply indebted to Mida N. Karakashian and Margaret Z. Freeman of the Computation Laboratory at the Massachusetts Institute of Technology for their efforts, extending over the greater part of a year, in converting the equations given in this paper into charts contained in Figs. 2–7. Without their help the principal conclusions of this paper would not have been possible. The authors also wish to express their appreciation to Mr. Ralph Beatty, who assisted in the preparation of the manuscript and in checking the equations for accuracy.

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Building to the Acoustical Optimum New Mutual-Don Lee Broadcasting Studios

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(Received February 2, 1949)

The acoustical design and construction of the new Mutual-Don Lee Broadcasting Studios in Hollywood were done under novel and very favorable arrangements. First, a careful check was made of the optimum acoustical characteristics to which it was decided to design the studios. Some of the best auditoriums for broadcasting were measured, and from these data as well as comments regarding the excellence and shortcomings of these auditoriums, the optimum characteristics for the new studios were determined.

During the course of construction, acoustical measurements were made several times for the purpose of "tailor-making" the acoustical characteristics of the studios. On the basis of the measurements, it was found that only a few minor modifications in acoustic treatment were necessary, such as changing the mountings of the Acousti-Celotex materials in order to change the low frequency absorption characteristics as desired, and the areas used of these materials.

The results are, close agreement between optimum and finally measured acoustical characteristics, and very satisfactory broadcast programs.

IN contemplating the building of the new Mutual-Don Lee Broadcasting Studios in Hollywood, California, a design philosophy was developed for producing optimum acoustical characteristics. The design procedure is described herein for the four principal studios, used primarily for large musical broadcasts, which is the same general plan followed in the design and construction of the smaller studios.

I. OPTIMUM STUDIO CHARACTERISTICS

Important considerations under this heading are the answers to the three questions discussed in the following paragraphs.

A. What is the Size Requirement?

The size of a studio should be determined by the type of programs to be broadcast from it. In order to fulfill the desire of the management for highest quality musical broadcasts, it was necessary to provide larger studios than had previously been in general use. Dr. Stokowski once said "Art is a habit of mind." Since we have usually heard orchestras in halls bearing certain relationships in size to the orchestras playing in them, there is an emotional satisfaction when this effect is reproduced. The quality of the reverberation of a large room is successfully reproduced at the present time only in rooms of commensurate size.

A person listening to a broadcast program requires for realistic enjoyment that the direct sound reach his ear augmented by reverberation characterized by the natural surroundings. Lack of this effect lessens his ability to visualize the orchestra. As the number of orchestral instruments is increased in a studio of insufficient volume, the impression of added orchestral size is not increased proportionally; however, where the number of instruments is small compared to the volume normally associated with a studio having optimum reverberation, it is possible with proper pick-up to create an effect of numbers in excess of those actually employed. In order to satisfy the acoustical requirements of a symphony orchestra, a volume of 170,000 cubic feet was chosen, in accordance with data regarding optimum size, for each of the four major studios.

B. What is the Optimum Reverberation Characteristic for the Studio?

In 1936, Morris and Nixon[1] described curves representing optimum reverberation times for broadcasting studios of various sizes. Maxfield, Colledge, and Friebus[2] described in 1938 a family of optimum reverberation time curves for motion picture scoring stages as shown in Fig. 1. These times are slightly higher than those recommended by Morris and Nixon,[1] particularly for small rooms. Potwin's[3] data, published in 1939, was in agreement with that of Maxfield, Colledge, and Friebus.[2] In 1947, Gurin and Nixon[4] reaffirmed the optimum reverberation times of Morris and Nixon.[1]

In order to correlate objective data with the subjective experiences of management, producers, artists and engineers, some first-hand information was required. In the growing art of broadcasting, the optimum reverberation characteristic for a given room size has been somewhat controversial like most things dealing with the aesthetic. However, on the basis of our most satisfactory listening experiences, programs from certain music halls and studios were generally considered best. The reverberation time frequency characteristics of some of these music halls were measured. There was an unmistakable trend in the results towards higher reverberation times than previously had been considered optimum for broadcasting. An average of these measurements, modified by comments about each relative to desirable changes, became the reverberation characteristic taken as a basis for the design of the new studios.

It is to be noted that the reverberation time frequency characteristic determined from these measurements to be optimum and shown as the dotted line on Figs. 2 and 4 (plotted to a larger scale on 4), is substantially higher than the broadcast optimum that has been used in years past. For instance, the curves of Fig. 1 indicate that a

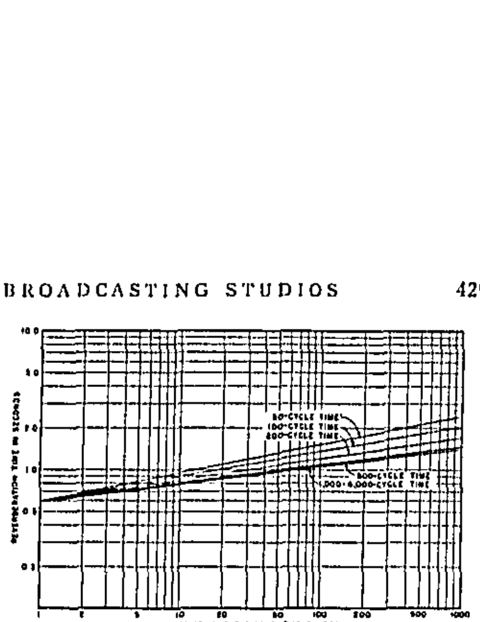


Fig. 1. Optimum reverberation times for broadcasting.

studio having a capacity of 170,000 cubic feet should have a reverberation time of slightly more than 1.1 seconds from 1000 to 4000 c.p.s., and 1.7 seconds at 50 c.p.s. The optimum curve of Fig. 4 indicates that the reverberation time from 1000 to 4000 c.p.s. should be nearly 1.4 seconds, and at 50 c.p.s. approximately 2.4 seconds.

It has been customary in describing optimum reverberation time characteristics to show them to be flat above 1000 c.p.s. Measurements that have been made of various auditoriums indicate that quite generally the measured reverberation time characteristics slope downward at frequencies above 4000 c.p.s. (to 8000 c.p.s.). This is attributable not only to the absorption of the acoustical materials on the walls, ceiling, and floor of the auditoriums, but also to the absorption of the air which is quite appreciable at high frequencies. Because of the fact that a study of the exact shape of the optimum characteristics at the high frequencies has not been made, within the knowledge of the authors, the optimum curve is not shown above 4000 c.p.s. It is recognized, however, that the most satisfactory auditoriums from which orchestras have been broadcast have characteristics which slope downward at the high frequencies. It is the firm conviction of the authors that an auditorium of the approximate size of the four largest Don Lee studios would have an unnaturally sharp and undesirably brilliant characteristic if this downward tendency at the high frequencies did not exist.

C. What Factors in Construction are Important for Sound?

It is important in the construction of a studio to eliminate external noise and vibration. To this end, each studio was designed basically as an isolated outer 8-inch thick concrete enclosure within which was the inner studio of wood construction with dimensions conforming to the optimum ratio 2:3:5. In order to minimize undesirable reflections between walls, the inner walls were angled to avoid parallel

[1] R. M. Morris and G. M. Nixon, "NBC studio design," J. Acous. Soc. Am. 8, 81 (1936).
[2] Maxfield, Colledge, and Friebus, "Pickup for sound motion pictures," J. Soc. Mot. Pict. Eng. 30, 666 (1938).
[3] C. C. Potwin, "Architectural acoustics," Archit. Forum, September 1939.
[4] H. M. Gurin and G. M. Nixon, "A review of criteria for broadcast studio design," J. Acous. Soc. Am. 19,[404 (1947).

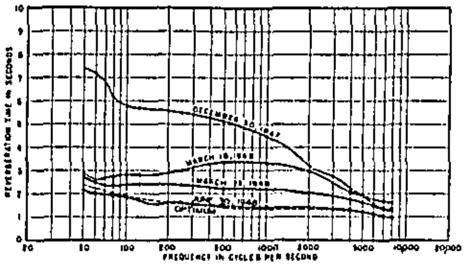


FIG. 2. Studio 1, reverberation time characteristics.

surfaces. The ceiling and floor surfaces were also made non-parallel.

In order to provide adequate, but avoid excessive, sound diffusion and give room character, the walls and ceiling were alternately treated with different areas of convex and flat surfaces. Wood was used on the stage floor, on the curved surfaces of the walls and ceiling and on some of the flat areas in order to provide the rich tonal effects which experience has indicated are obtained by the use of wood.

The studios were designed to have not only optimum acoustical characteristics but architectural beauty as well.

II. BUILDING TO THE ACOUSTICAL OPTIMUM CHARACTERISTICS

The next problem was to design the studios to conform to the optimum acoustical characteristics. In earlier building history, such an undertaking would have been difficult. With the increased information now available regarding the acoustical characteristics of materials, it is possible to calculate the sound treatment for a given studio more accurately than formerly. Because of the lack of information regarding some of the materials, however, the polycylindrical surfaces being one of the materials in particular about which not enough information was available, it was decided to make acoustical measurements during the course of construction and make slight adjustments in the acoustical treatment which the measurements might indicate to be desirable. Arrangements were made in connection with the construction program, to install one type of material at a time as far as practicable, and make acoustic measurements before and after each installation. The results are a unique family of curves, and tables of acoustic absorption characteristics described below. This "tailor-making" of the acoustical treatment proved to be very valuable in adjusting the acoustical characteristics accurately to the optimum.

In calculating the reverberation time frequency characteristics, it was essential to realize that the polycylindrical surfaces, provided primarily for the diffusion of the sound, had a maximum absorption at a low frequency. These polycylindrical diffusers were constructed of sheet plywood bent over convex forms. The convex forms were made of ribs which were spaced at irregular intervals in order to avoid plywood diaphragms of similar sizes, which otherwise would vibrate at frequencies within a narrow band, and thereby produce undesirable resonance vibration effects. The panels were damped by means of a Celotex lining. In addition, the seats, carpet, drapes, and acoustical treatment, all were considered carefully in the calculation of the acoustical characteristics of the studios.

Acousti-Celotex was chosen as the acoustic treatment because of the wide variety of acoustical characteristics available with different thicknesses and mountings, and because of the fact that these materials can be painted with oil paints or otherwise redecorated without impairing the acoustical characteristics. Arrangements were made in connection with the design of the studios to make changes in the Acousti-Celotex during the course of construction, if the acoustic measurements indicated minor modifications to be desirable. Changes in the type of Acousti-Celotex and its mounting (directly against hard surfaces or on furring strips either 12″ or 24″ apart) were contemplated. Under these conditions, the absorption frequency characteristic, at the low frequencies in particular, could be varied over wide limits.

III. ACOUSTIC MEASUREMENTS AND CONSTRUCTION

The following is an essentially chronological account of the measurements made in Studio 1.

December 30.—The studio consisted of a concrete shell with a rough wooden roof (see Fig. 3). The reverberation time varied from 7.4 seconds at 50 c.p.s. to 1.6 seconds at 8000 c.p.s. (see Fig. 2).

March 18.—The installation of the polycylindrical surfaces on the ceiling and walls was complete, and the entire installation of the ceiling was finished,

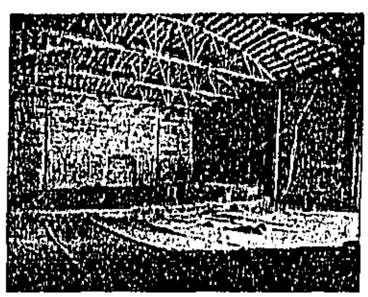


FIG. 3. Picture of Studio 1, at the time of the first measurements, December 30, 1947.

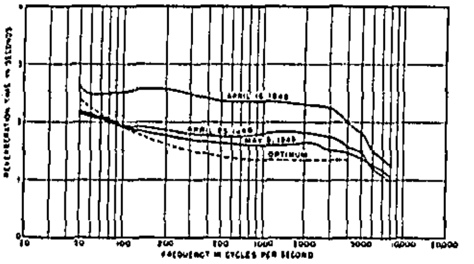


FIG. 4. Studio 1, reverberation time characteristics.

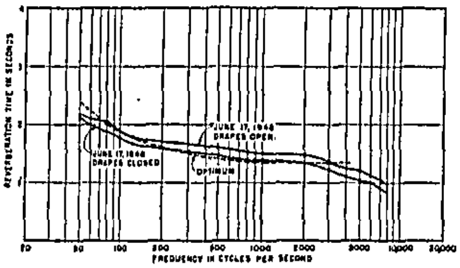


FIG. 5. Studio 1, reverberation time characteristics with stage drapes open and closed.

with Acousti-Celotex or other materials installed in strips between the polycylindrical diffusers. The reverberation time at 50 c.p.s. was reduced from 7.4 to 2.8 seconds (see Fig. 2). Above 2000 c.p.s. the absorption was increased only a small amount. The absorption at the low frequencies was attributable largely to panel vibration of the polycylindrical and flat surfaces of the inner walls of the stage and audience section.

It was decided at this time, as a result of a study of the measurements, to install the remainder of the Acousti-Celotex directly in contact with the hard wall surfaces instead of on furring strips. The purpose of this change was to reduce the absorption of the Acousti-Celotex to a minimum at the low frequencies.

March 25.—The Acousti-Celotex installation on the walls was complete. The mid-range frequency absorption was increased in particular. The low frequency absorption was little changed, as was desirable (see Fig. 2).

April 16.—The construction of the wood floor of the stage was complete. There were no undesirable resonance vibrations of the floor to change the smoothness of the reverberation time frequency characteristic. This had been accomplished in particular by putting extra stringers between some of the wood floor supports, which caused the natural vibration frequencies of the floor, therefore, to be varied as is desirable.

The reverberation time under these conditions was slightly greater than during the previous measurements, largely because of the removal of the building paper that had covered the floor of the audience section and the removal of the Acousti-Celotex that had previously been stored in cartons on the stage. The doors had been hung and were closed.

April 23.—The seats had been installed in the audience section. The absorption of the seats reduced the reverberation times substantially at the low as well as the mid-range and high frequencies (see Fig. 4). Reverberation chamber measurements of the seats during the planning period had indicated good absorption over the entire frequency range.

May 5.—Ozite lined carpet had been installed in the audience section. In order to avoid absorption at the low frequencies as completely as practicable, as previous measurements had indicated to be desirable, a single rather than double thickness of carpet lining was used. This produced the desired results (see Fig. 4).

It is to be noted that above 5000 c.p.s. the reverberation time was higher than during the previous measurements. These data were rechecked and the effect was found to be a real one, undoubtedly attributable to higher humidity conditions during the latter measurements, which would result in less high frequency air attenuation. Such conditions will not recur in these studios due to the operation of the humidity control equipment, the installation of which had not been completed at the time of these measurements.

June 17.—The windows in the monitor and client's booths, and four heavy drapes as well as a strip of carpet on the stage, had been installed prior to these measurements. The measurements were made with the two rear drapes open, and also closed so as to shut off the rear portion of the stage. The upper curve of Fig. 5 indicates that with the drapes open, the reverberation time characteristic was above optimum over the frequency range from

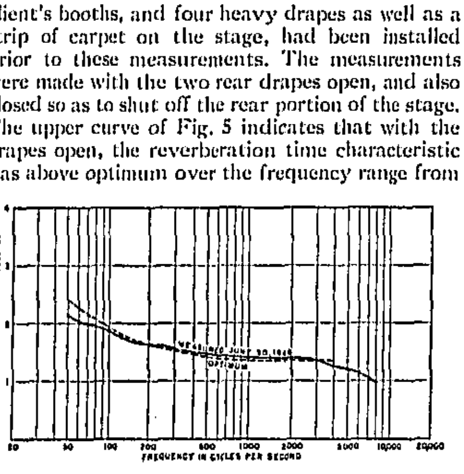


FIG. 6. Studio 1, reverberation time characteristics after completion, June 30, 1948.

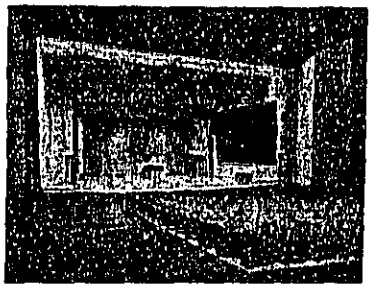


FIG. 7. Picture of Studio 1, after completion, June 30, 1948, taken from the rear of the audience section.

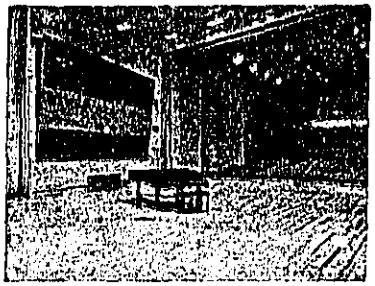


FIG. 8. Picture of Studio 1, after completion, June 30, 1948 taken from the stage.

200 to 3000 c.p.s. With the drapes closed, the characteristic coincided almost exactly with the optimum over the range from 200 to 2000 c.p.s.

June 30.—Acousti-Celotex had been installed on two additional panels of the stage to complete Studio 1. The final measurements, made with the drapes open, are in close agreement with the optimum characteristic (see Fig. 6). Two pictures of the completed studio are shown as Figs. 7 and 8.

The measurements, represented by the characteristic of Fig. 6, were made in three groups. First, the loudspeaker and the microphone were each placed in five different positions on the stage. The average of these measurements is shown as the heavy line of Fig. 9. Next, the loudspeaker and microphone were each placed in five different positions in the audience section. The average of these measurements is shown on the same figure as the dotted line. Next, the loudspeaker was placed in five different positions on the stage and the microphone in five different positions in the audience section. The average of these measurements is shown as the light solid line on the same figure. In making practically all the reverberation time measurements referred to in this article, the loudspeaker and microphone were each placed in ten different locations as far apart as practicable. The characteristic curves are the average of ten sets of data in practically every case.

Measurements were made June 30 of Studio 2, which had been designed to be practically the same as Studio 1. This characteristic is shown as the solid curved line of Fig. 10. Measurements had been made in Studio 2 previously, which were in good agreement with the corresponding measurements that had been made in Studio 1 at essentially the same stage of completion. Studio 3 was measured August 17, the results of which are shown as the dotted curved line of the same figure. The characteristic of Studio 4, August 4, 1948, is shown as the dot dashed curved line. The characteristics of these three studios are essentially alike and are in good agreement with Studio 1 and the optimum.

IV. ACOUSTIC ABSORPTION CHARACTERISTICS

At the time that the Don Lee Studios were designed, less was known about the acoustic absorption frequency characteristics of polycylindrical diffusers than most of the other materials used in the studio construction. This was one of the most important materials about which information was required because of the relatively large areas in the studios. Two other important parts of Studio 1 equipment were the 350 deeply cushioned theater seats, and the carpet. From the acoustic measurements made in the studio before and after the installation of the polycylindrical diffusers, before and after the installation of the Acousti-Celotex on the walls, before and after the installation of the seats, and before and after the installation of the carpet, the absorption frequency characteristics have been calculated. These materials were measured in this instance under the conditions of use rather than under the unnatural conditions of the reverberation chamber. These measurements, therefore, are of the absorption contributed by these

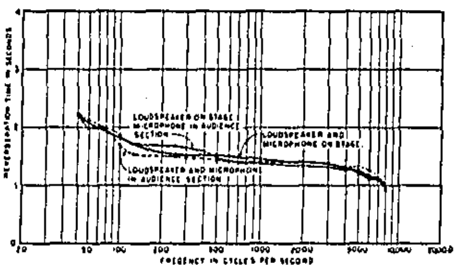


FIG. 9. Studio 1, reverberation time characteristics after completion, June 30, 1948, measured with the loudspeaker and microphone in various locations on the stage and in the audience section.

TABLE I. Polycylindrical diffusers Studio 1 calculated absorption coefficients.

Frequency in cycles per second	Absorption coefficients in percent
50	27
60	31
80	24
100	23
125	23
250	17
500	10
1000	8
2000	2
3000	6
4000	7

materials to the acoustical characteristics of the studio.

A. Polycylindrical Diffusers

The first measurements of Studio 1 were made December 30, 1947, when it was virtually a concrete shell. The characteristic is shown on Fig. 2, and the Studio is pictured as Fig. 3. The next measurements were made March 18, 1948, after all of the polycylindrical diffusers had been installed on the ceiling and walls, and the Acousti-Celotex acoustic treatment had been installed on the stage ceiling. From these data, together with the information obtained regarding the absorption frequency characteristic of the Acousti-Celotex, the characteristic of the polycylindrical diffusers was calculated. This is shown in Table I. It will be noted that the peak absorption of 31 percent was obtained at a frequency of 60 c.p.s., and that the absorption averaged about 8 percent above 125 c.p.s.

B. Acousti-Celotex

The next major change made was the addition of the Acousti-Celotex acoustic treatment to the walls. Almost exactly equal areas of ½" thick and 1½" thick Acousti-Celotex were installed between the measurements of March 18 and March 25, 1948. Both of these measured characteristics are shown on Fig. 2. From these data, the average absorption coefficients at the various frequencies for the two types of Acousti-Celotex were calculated. The

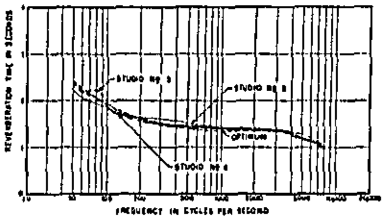


FIG. 10. Reverberation time characteristics of Studios 2, 3, and 4, following completion.

TABLE II. Acousti-Celotex ½" and 1½" (equal areas) screwed to Acousti-Lock board. Studio 1. Calculated average absorption coefficients.

Frequency in cycles per second	AMA absorption coefficients in percent: ½" Acousti-Celotex screwed to A-L board	1½" Acousti-Celotex screwed to A-L board	Average	Average absorption coefficients of ½" and 1½" Acousti-Celotex calculated from reverberation measurements
125	9	14	12	26
250	15	42	29	37
500	61	99	80	71
1000	77	74	76	75
2000	70	60	65	72
3000				58
4000	64	50	57	51

results are given in Table II. It will be noted that the principal differences between the Average AMA, coefficients and those calculated from the measurements, are that the latter are higher at 125 and 250 c.p.s. and lower in value at 500 c.p.s.

C. Seats

The seats were installed between the acoustic measurements made April 16 and April 23 in Studio 1 (Fig. 4). From these data, the absorption frequency characteristic of the 350 seats was determined. These coefficients are compared in Table III to those determined in a reverberation chamber prior to the construction of the studios. It is to be noted that the low frequency absorption of the seats, measured after installation in the studio, is higher than the absorption as measured in the reverberation chamber; and that the absorption at the high frequencies is lower as measured in the studio.

D. Carpet

The carpet was installed in Studio 1 between the acoustic measurements of April 23 and May 5 (Fig. 4). It had been determined prior to the installation of the carpet that a minimum low frequency absorption was desired from the carpet, in order to make the studio characteristic conform closely to the optimum. The carpet lining was, therefore, limited to one thickness of material. The calculated absorption frequency characteristic of the lined carpet is shown in Table IV. It is to be noted that the carpet absorption is less than 30 percent below 1000 c.p.s. The absorption reaches its maximum value of 50 percent at 3000 c.p.s.

V. CONCLUSIONS

In the design and construction of the new Mutual-Don Lee Broadcasting Studios, the desired studio characteristics were first determined by means of measurements of some of the auditoriums which broadcasting experience indicated to be best. Next, the new studio acoustical characteristics were

TABLE III. Seats Studio 1 calculated absorption.

Frequency in cycles per second	Absorption in units measured in a reverberation chamber	Absorption in units measured in Studio 1
125	2.3	3.0
250	2.4	3.7
500	2.7	3.4
1000	3.3	3.1
2000	3.2	2.7
3000	3.2	3.0
4000	4.5	3.1

"tailor-made" to the desired characteristics by means of acoustic measurements made at intervals during construction, and by minor adjustments in the acoustic treatment indicated to be desirable by these measurements. The results were close agreement of the final characteristics of the studios to the optimum. Furthermore, the design optimum was found to be a more reverberant characteristic than had been in general use.

TABLE IV. Carpet Studio 1 calculated absorption coefficients.

Frequency in cycles per second	Absorption coefficients in percent
125	11
250	15
500	26
1000	36
2000	36
3000	50
4000	31

The results that have been obtained, including comments of management, artists and radio listeners, particularly regarding orchestra broadcasts, have been very excellent. It is realized, however, that it may take some time for studios of this type to gain general acceptance. It is expected as experience is gained in the use of the Don Lee Studios, that additional information will be obtained which will lead to the preparation of a supplementary article.

Erratum: On Diffraction through a Circular Aperture

JOHN W. MILES
Department of Engineering, University of California, Los Angeles 24, California
[J. Acous. Soc. Am. 21, 140 (1949)]

WITH reference to our recent Letter to the Editor, Dr. Harold Levine of Harvard University has pointed out to us that the variational principle, when applied to the static field in the aperture, is exact through terms of order $(ka)^2$ and not "virtually exact," as we stated. Our arithmetic was in error, and the coefficient of the term $(ka)^2$ in the transmission coefficient should have been

$$(4/9-4/\pi^2)=0.03915971$$

which is equivalent to Bouwkamp's figure of 0.039160 to the same number of significant figures. In addition, Eq. (8) in reference 1, should have read

$$T/T_0=R=G(G^2+B^2)^{-1}.$$

Erratum: Adaptation of the Ear to Sound Stimuli

E. LÜSCHER AND J. ZWISLOCKI
Basle, Switzerland
[J. Acous. Soc. Am. 21, 135 (1949)]

MILLISECONDS, *not* microseconds. Throughout this paper, the unit of time is the millisecond, which through error was abbreviated as μsec.

Letter to the Editor

Noise

RALPH MARTIN MCGRATH
Hawthorne Works, Western Electric Company, Chicago, Illinois
April 2, 1949

PERMIT me to join with Mr. Frank Massa[1] in urging that more time in the meetings and more space in the Journal be devoted to the "applied" phases of the field. The coming meeting in New York is scheduling papers under the heading "Acoustics in Safety and Comfort" and I am sure, after reading over the abstracts of the papers that will be presented, that only the surface will be scratched. I am interested in the effects of noise on human beings and I would like to hear papers on what the limits of safety are for industrial exposures, what the effect of noise is on labor turnover, quality of product, and absenteeism. I am interested in what practical steps can be taken to correct adverse environmental conditions. I am interested in actual rulings by industrial commissions on claims made for hearing losses; in the type of legislation that should be enacted, and in the viewpoint of labor and management toward the "noise" problem. I am interested in noise level measurement and the hearing acuity measurement of the individual. I am interested in standards of comfort as well as annoyance and what can be done to increase the comfort of workers and reduce the "threat" to his safety.

In my opinion fully half of the program at our meetings should be devoted to the "applied" phases of the field. In this way we can learn how others are attacking the problems with which we in industry live day after day. As I see it, the industrial noise problem is one of the most unexplored fields in acoustics.

[1] Frank Massa, "Theory *versus* Practice," J. Acous. Soc. Am. 21, 141 (1949).

Acoustical Society News

Dates of Future Meetings of the Acoustical Society

The following dates have been set for meetings of the Acoustical Society and Chairmen of the Program committees have been appointed:

November 17–19, 1948, St. Louis, Missouri. *Chairman*: PROFESSOR KERON C. MORRICAL, Washington University, St. Louis, Missouri.

June 22–24, 1950, State College, Pennsylvania. *Chairman*: PROFESSOR HAROLD K. SCHILLING, The Pennsylvania State College, State College, Pennsylvania.

November 9–11, 1950, M.I.T., Cambridge, Massachusetts. *Chairman*: PROFESSOR RICHARD H. BOLT, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Spring 1951, Atlantic City, N. J. *Chairman*: DR. HARRY F. OLSON, RCA Laboratories, Princeton, New Jersey.

Fall 1951, Chicago, Illinois. *Chairman*: DR. HALE J. SABINE, The Celotex Corporation, Chicago 3, Illinois. This will be a joint meeting with the other societies of the American Institute of Physics.

Cumulative Index to Volumes 11–20

In 1939 the Acoustical Society published a Cumulative Index to Volumes 1–10 covering all papers which had appeared in our Journal from 1929 to 1939 as well as the contemporary papers in other journals from 1937 to 1939. These were classified according to subject and also indexed according to author. This index was mailed at that time to all members, but copies are still available at the American Institute of Physics at $4.50.

It had been the plan to extrapolate that experience and to publish a similar cumulative index covering the next ten volumes. This turned out to be a very serious financial problem. The field of acoustics had become very active during the period, the number of titles of acoustical papers compiled from other journals had vastly multiplied, the reviewing of acoustical patents had been started, and the *Journal* itself was publishing an increasing number of papers per year. It was reluctantly decided that the Cumulative Index would have to be abandoned for financial reasons.

Meantime, however, the ONR had made plans to support the preparation of a bibliography of acoustics. By good fortune the plans of each group became known to the other. Inasmuch as the *Journal of the Acoustical Society of America* was the only journal devoted to acoustics which had been published throughout the entire period from 1939 to 1948 and, since this journal had already prepared references to acoustical literature appearing elsewhere and to acoustical patents, it was concluded that the cumulative index of this material would meet the needs of the ONR. Compilation of this material into integrated lists has therefore been supported by ONR contract.

The new Cumulative Index to Volumes 11-20, 1939-1948, is a volume of approximately 500 pages covering papers in our own Journal, contemporary literature from other journals, and patents. These are classified separately according to subject and again by author or inventor. This very useful volume has been made possible by the many hours of hard work during the past ten-year period by those members of the Society who have compiled the material, namely, Arthur Taber Jones, Floyd A. Firestone, Herbert A. Erf, and Robert W. Young and his staff of eleven patent reviewers whose names are listed in the Index itself.

This new Cumulative Index is being sent free of charge to all members of the Acoustical Society. Others may obtain copies by sending $5.00 to the American Institute of Physics, 57 East 55th Street, New York 22, New York.

Results of Questionnaire on Journal Policy

The advice of the Society membership was sought last year in a questionnaire designed to assist the officers and the Executive Council in determining a suitable publication policy in the face of rising publication costs and continuing shortage of paper. Over 480 replies were received representing more than one-third of the total Society membership. The check lists on "recommended action" and "degree of interest" were diligently executed, and a large number of cogent suggestions were received on the questionnaire and in forwarding letters. A brief statistical analysis of the check lists, with due respect for correlation coefficients and power spectra, suggested that more value would accrue from a detailed study of the appended comments and suggestions. The statistics, however, provided a validating confirmation of the conclusions drawn from the verbal discussions.

The replies were gratifying and informative. The constructive criticisms and suggestions are proving useful in giving guidance to the members of the Council, and a number of the suggestions have already been incorporated in the Journal.

A stricter editorial policy is now being followed and continual scrutiny given to matters of layout and size of cuts. The new "Suggestions to Authors" on the inside front cover encourages practices that aid editorial economy. Unnecessary duplications and verbosity are to be discouraged. Although the Editorial Board looks for a high standard of material, it will not reject papers of real interest to the Society in the interests of economy.

Space will soon be saved by placing the Table of Contents on the back cover, where it is really easier to use. Arrangements have been made to reduce the number of patents reviewed and the number of figures reproduced. The questionnaire replies emphasized that too many of the patents listed are simply gadgets and not of scientific interest; that information appears too late to be of much use to the patent minded; and that persons really interested in such matters obtain the patent bulletins directly. Other frequently expressed opinions, which are under consideration, include reduction of papers which describe only facilities and not research as such; reduction of material on musical instruments (with a few notable objectors); and reduction of material from contract-sponsored work that is given fully elsewhere.

Several pointed out the need for more Sustaining Memberships, an arrangement that is mutually beneficial to the Society and to the Sustaining Member.

A very wide range of interests and opinions was brought out by the questionnaire; this is a natural outcome of the broad scope of the Society's activities and the wide range of subject matter encompassed by the field of acoustics.

In summary, it was clear that most readers like the Journal and would be willing to pay more if necessary to maintain a high standard of well presented articles. At the same time, many useful suggestions for economy were given and are being put into practice.

The assistance of Avis M. Clarke in analyzing and reporting the results of the questionnaire is gratefully acknowledged.

RICHARD H. BOLT

Certificate of Appreciation to Dr. Hallowell Davis

A Certificate of Appreciation was recently awarded to Dr. Hallowell Davis "for outstanding contribution to the work of the Office of Scientific Research and Development during World War II." Dr. Davis is Director of Research at Central Institute for the Deaf, and Professor of Physiology and Research Professor of Otolaryngology at Washington University in St. Louis.

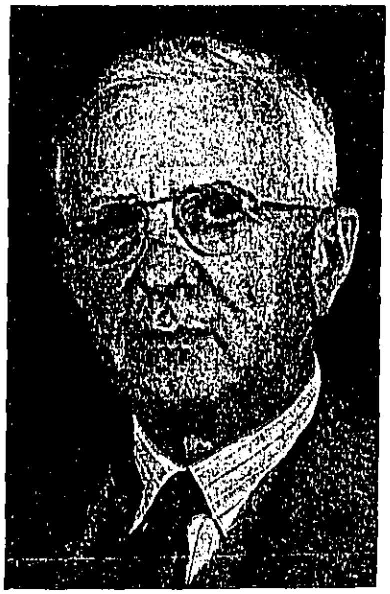
Dr. L. L. Beranek to Lecture on Acoustics at the University of Buenos Aires

Dr. Leo L. Beranek has accepted a teaching position as lecturer in acoustics at the Instituto Radiotecnico of the University of Buenos Aires for a three-month term this summer. The Instituto Radiotecnico has been in existence as a division of the University of Buenos Aires for five years and they are now attempting to expand their curriculum to include graduate work. Dr. Beranek is being asked to organize a course in acoustics that will be carried on by the school after he leaves. He also will be expected to render assistance in the setting up of a general post-graduate course in radio communications.

Death of Dr. Edward B. Stephenson

Dr. Edward B. Stephenson, Superintendent of the Mechanics Division of the Naval Research Laboratory, died Friday, May 6, at the age of 67, in San Francisco. Dr. and Mrs. Stephenson were visiting their son, in that city, following Dr. Stephenson's participation at the All Navy Laboratories Conference at the Navy Electronics Laboratory in San Diego, where he was chairman and discussion leader of the program entitled "Educational Programs in Navy Laboratories."

Dr. Stephenson came to NRL in 1924. He served as Associate Superintendent of the Sound Division until 1948 when he became the first head of the newly established Mechanics Division. While in the Sound Division, Dr. Stephenson was responsible for the research undertaken on Subaqueous System of Artillery Fire Control and the Virtual Target for Underwater Sound Echo Ranging; he held the patent rights for both. In 1931 he received the Navy Department's Bene-



ficial Award of $2000 (one of the highest monetary awards given by the Navy) for research in "Quartz Crystals." In addition to his research, Dr. Stephenson took a very active interest in administrative activities. In 1945, he received the Meritorious Civilian Service Award for outstanding service to the Navy. His citation read as follows:

> For outstanding administrative and organizational ability for meeting the war need of rapidly expanding the technical work of the Sound Division with the simultaneous need for the technical development and orientation of new personnel to its work. This effective coordination made possible efficient service to the Fleet.

For many years, he served as Chairman of the NRL Efficiency Rating Committee and was Problem Secretary of the NRL Scientific Program Board. He helped organize the Civil Service Board of Examiners for Scientific and Technical Personnel in the Potomac River Naval Command. He served on the MIT Committee on Thesis Accrediting and was a member of the Navy Advisory Committee for Scientific Personnel.

Before coming to the Laboratory, Dr. Stephenson was a Physicist in the Office of the Chief of Engineers in the War Department; during World War I he served as a Major in the Engineer Corps of the U. S. Army. A former college professor, Dr. Stephenson taught at Knox College (where he received his B.S. and M.S. in Physics) for several years, at the University of Illinois (where he received his doctorate), and at the University of North Dakota. In 1943 his alma mater, Knox College, honored him with the honorary degree of Doctor of Science.

Dr. Stephenson presented many scientific papers to various scientific and technical societies and was frequently invited as speaker at many national and international meetings; he was a member of the American Physical Society, the American Association for the Advancement of Science, the Acoustical Society of America, the Geophysical Union, the Philosophical Society of Washington, the Cosmos Club, and Phi Beta Kappa, Sigma Xi, Phi Delta Theta, Gamma Alpha fraternities. The most recent occasions when he represented the Laboratory at international meetings were in August 1948 when he attended the International Union of Geodosy and Geophysics in Oslo and in September 1948 when he attended the International Congress of Applied Mechanics in London.

Born in Sparta, Illinois, on January 19, 1882, Dr. Stephenson spent most of his early career in the midwest that he loved so dearly, and was very proud that he was a "farm boy." He will be long remembered not only for his scientific contributions and achievements, but also for his wholehearted human relationships. Of him, it can truly be said:

> "He was a leader, not a driver;
> He was forceful, not arrogant;
> He was helpful, not patronizing;
> He was friendly, not familiar;
> He was sympathetic, not sentimental;
> He was confident, not dogmatic."

Chicago Acoustical and Audio Group

This is written to describe briefly the history of the Chicago Acoustical and Audio Group. About two years ago, a group of acoustical and audio men in this area agreed that the A.S.A. and the I.R.E. were not in a position to satisfy their desire for local activity in their fields of endeavor. A technical society was discussed and initial organizational meetings were held. One of the persons who devoted much effort to the formation of the society was Dr. Vincent Salmon. He was the original president pro tem but in January, 1949, left the Chicago area for the West Coast.

Formal meetings with speakers commenced in October, 1948, and continued monthly. A constitution was accepted at the February, 1949, meeting. An important part of the constitution was the provision of means to become affiliated with a society such as those mentioned above when the membership so desired. With the acceptance of a constitution, four officers and three members of the executive council were elected, the seven then constituting the executive council. These are: *President:* H. C. Hardy, *Vice President:* J. S. Boyers, *Treasurer:* S. J. Klapman, *Secretary:* G. L. Bonvallet.

Remaining Executive Council Members: H. J. Sabine, for 3 years; R. E. Samuelson, for 2 years; M. A. Smith, for 1 year.

An annual meeting was held in May, 1949.

Under the original president pro tem, the above officers, and a large number of interested and hard-working enthusiasts who became members, the program for its first year was found to have been very satisfying in content and also to have set a high precedent. The program for the approaching year is now being formulated and appears interesting and beneficial. We hope to achieve the purpose of the society which is to foster within the Chicago area the diffusion and increase of the scientific and engineering knowledge of acoustics and audio engineering, and to encourage the interchange of ideas and the promotion of high professional standards among its members.

New Associates

Wayne A. Beaverson, Electro-Voice, Inc., Buchanan, Michigan
Robert W. Benson, 10 Faculty Lane, St. Louis 5, Missouri
Robert W. Carr, 2929 Burling Street, Chicago 14, Illinois
(Shure Brothers, Inc. 225 W. Huron, Chicago, Illinois)

Cassius M. Clay, Jr., Room 509, MIT Graduate House, Cambridge 39, Massachusetts
Kenneth S. Cook, Department of Physics, University of Connecticut, Storrs, Connecticut
Edwin A. Doane, 1156 Constance Street, Pasadena 5, California (Western Trundeau Streptomycin Laboratory, Olive View, California)
Percy W. Gatz, 73 Miles Avenue, Great Kills, Staten Island 8, New York (Teachers College, Columbia University, New York, New York)
Robert M. Hoover, Ordnance Research Laboratory, P. O. Box 30, State College, Pennsylvania
Kenneth R. Larson, U. S. Gypsum Company, 1253 West Diversey Parkway, Chicago 14, Illinois
Edred T. Marsh, 281 W. Bagley Road, Berea, Ohio (NACA, Cleveland Airport, Cleveland, Ohio)
David Mintzer, Acoustics Laboratory, Massachusetts Institute of Technology, Cambridge 39, Massachusetts
Lionel O. Schott, Bell Telephone Laboratories, Murray Hill, New Jersey
William M. Seeley, Jr., 733 Craig Avenue, La Canada, California
Martin I. Sperber, 9507 Euclid Avenue, Cleveland 6, Ohio
Glenn E. Tisdale, 915 Westover Road, Wilmington 79, Delaware
Victor Twersky, 1244 Lincoln Place, Brooklyn 13, New York
George P. Wakefield, The F. W. Wakefield Brass Company, Vermilion, Ohio
William T. Watkins, The Hampshire Corporation, P. O. Box 82, Roanoke, Virginia
Louis Zernow, Ballistic Research Laboratories, Aberdeen Proving Ground, Aberdeen, Maryland
Reinstated Members: John W. Brocks, Guy S. Cook, Marcus R. Tarrant
Resigned: Reuel S. Alford, R. D. Mugg, Douglas F. Winnek, Members
Deceased: Edwin H. Colpitts, Fellow, Walter F. Smith, Jr., Member

Fellows	207
Members	1036
Associates	185
Total Membership	1428

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 21, NUMBER 4 JULY, 1949

Current Publications on Acoustics

F. A. Firestone
3318 Fessenden Street, NW, Washington 8, D. C.

Book Reviews

Carillon. Arthur Lynds Bigelow, pp. 91+xiv. Princeton University Press, Princeton, New Jersey, 1948. Price $2.00.

The author of this book is the Bell-Master at Princeton University, and the book was written because enthusiastic visitors to the Princeton carillon often ask about the carillon and about bells in general. The book serves admirably its purpose of providing a straightforward and interesting account of bells and the possibility of bell music, and it will be read with pleasure by many persons beside those who hear the Princeton carillon. There are three chapters. The first tells the history of the Princeton carillon. The second deals with the origin and development of bells and carillons. The third is on the carillon in America.

Mr. Bigelow defines a carillon as "An instrument comprising at least two octaves of fixed cup-shaped bells arranged in chromatic series and so tuned as to produce, when many such bells are sounded together, concordant harmony. It is normally played from a keyboard which controls expression through variation of touch." This is the definition adopted at the Sixth Congress of the Carillon Guild, held at Princeton in 1946. Thus a carillon requires bells in which the partial tones of one do not jangle with those of another that is heard at the same time, and the carillonneur must develop the art of making use of these bells in such a way as to take into account strong partial tones, and must combine the bells in such a way as to obtain effects that are musical. No mere reproduction of harmonies that were written for other instruments is adequate.

Mr. Bigelow sketches the history of the varying shapes of bells, from the time of Nineveh and Babylon, through Roman tintinnabula and early Irish bells, to the thirteenth century, and then to the fifteenth century and more modern bells.

In the carillon at Princeton there are forty-nine bells, and the clavier is in an unusually satisfactory location—close to all the bells, above the larger ones and below the smaller. For the larger bells the action is of the Dutch "broek" or "breeches" type, and for the smaller it is of the Flemish "tuimelaar" or "tumbler" type. These two types of action are explained clearly with the aid of excellent diagrams. During the war the Princeton carillon was enlarged to its present size by the addition of twelve small bells. With foundries at home and abroad converted for war, it became necessary for Mr. Bigelow himself not only to find metal for the additional bells, but also to cast and tune them. He already had a wide understanding of such matters, but he can hardly have carried through this work without learning much while doing it. In fact, he states that there are probably not more than six or seven men living who are capable of tuning satisfactorily the bells of a carillon.

As to the pleasure and satisfaction to be derived from a carillon Mr. Bigelow points out that the number of bells "is in itself no criterion of its musical qualities or of the amount of pleasure to be derived from it. Some smaller sets of two or three octaves may charm the listener as much as carillons twice their size. There is, however, a limit to the type of music which may be played on a smaller instrument." On a carillon with a larger number of bells it is, however, possible to "achieve effects through scales and extensive arpeggios which a smaller instrument cannot produce."

The book is enriched by twelve pages of photographs, and by some information about fifty-eight carillons in this country and eight in Canada. It is a book that every one who takes any interest in bells will want to read.

Arthur Taber Jones
Smith College,
Northampton, Massachusetts

Proposed American Standard Acoustical Terminology. February 1949. American Standards Association, 70 East 45th Street, New York 17, N.Y. Price $1.00.

The new trial edition of the proposed revision of the 1942 Standard Acoustical Terminology was prepared under the sponsorship of the Acoustical Society of America with special cooperation of the Institute of Radio Engineers, Inc. As indicated in the foreword, this edition is issued for trial and study for a period of six months, after which it will be proposed for adoption as an American Standard with whatever corrections the trial has indicated.

A comparison of the new proposed standard with the 1942 standard shows that it has over five hundred definitions compared to about one hundred and fifty. The increase is largely in six new sections reflecting recent scientific developments. These are: Ultrasonics, Recording and Reproducing, Underwater Sound, General Acoustical Instruments, Shock and Vibration. New material is also included in the six sections—General, Architectural Acoustics, Hearing, Sound Transmission, Transmission Systems, and Music—which appeared in the earlier version.

Many of the definitions in the earlier edition have been revised to conform with the results of researches made during the intervening busy seven years. As an example, major changes appear in the concept and method of specification of Articulation and Intelligibility. These are now recognized as being less definite and more intimately tied in with the method of measurement. The new definitions indicate more exactly what factors must be specified.

The basic principles followed in the new version are essentially the same as for the older version. Multiple meanings for a term have generally been avoided, and each definition is as complete as possible, using only words found in a standard dictionary except where specific reference is made to another term within the glossary. To make the standard more useful to nonspecialists many of the terms have, in addition to an accurate statement of the meaning, explanatory paragraphs in the form of notes containing background material and discussion. Two additional tables also appear in this issue: one a table on Standard Water Conditions listing the velocity of sound, the density and the acoustical impedance of fresh and sea water as a function of salinity and temperature; and the second a table of conversion factors for the present acoustical units and the mks units.

This standard should have widespread value to all persons interested in acoustics and allied fields. It gives the specification writer, the manufacturer, the research worker, the field worker and the customer a common language which is at once accurate and yet simple compared to that in many other technical fields. Few foreign words and few "coined" words appear which would mar the feeling of understanding in discussions between the specialist and the less technical person.

The proposed standard should be carefully studied by all persons interested in acoustics in order to locate all errors so that these can be corrected in the final edition.

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References to Contemporary Papers on Acoustics

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IN most of the following references the name of a journal is followed by the volume number, in black face, then the page reference, and lastly, in parentheses, the date. Where reference is made to abstract journals which number their abstracts the abstract number is given instead of a page reference. Abstracts in Annales des Télécommunications are in French, and in Physikalische Berichte are in German. Most of the other abstracts to which reference is given are in English. The abbreviations for the names of journals follow those used in the World List of Scientific Periodicals. The numbers to the left of the references are those given in the Classification of Subjects on pages 894–896 in the issue of this Journal for November, 1948. The number at the right, at the end of each reference, designates that particular reference.

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Review of Acoustical Patents

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Patents reviewed below have been issued by the United States Patent Office on the dates indicated. Any opinions expressed are those of the individual reviewers and do not necessarily reflect official views of organizations with which the reviewers are associated. Statements of fact are ordinarily based on the patents alone without independent verification.

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LYNN C. HOLMES, *Stromberg-Carlson Company, Rochester 3, New York*

GEORGE KIS, *Triad Transformer Manufacturing Company, Los Angeles 4, California*

FRED W. KRANZ, *Sonotone Corporation, Elmsford, New York*

C. E. NELSON, *Nelson Muffler Corporation, Stoughton, Wisconsin*

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D. B. SHOTWELL, *Caterpillar Tractor Company, Peoria 8, Illinois*

FRANK H. SLAYMAKER, *Stromberg-Carlson Company, Rochester 3, New York*

WEIANT WATHEN-DUNN, *Naval Research Laboratory, Washington 25, D. C.*

2,450,911
2.5 ACOUSTICAL STRUCTURE

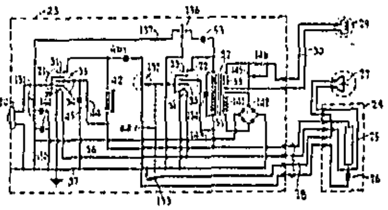
Arthur D. Park and Norman A. Johnson, assignors to Armstrong Cork Company.
October 12, 1948, 6 Claims (Cl. 20-4).

The acoustical treatment described consists of blocks or tiles of an incombustible felted material, such as glass fiber board, cemented to a wall or ceiling surface, and a decorative surfacing of larger sheets of glass fibre bonded mat applied by adhesive to the sound absorbent material. The advantages of a monolithic, incombustible, decorative treatment are cited.—HJS

2,420,686
4.5 HEARING AID AMPLIFIER

Harry B. Shapiro, now by judicial change of name Harry B. Shaper, assignor to Sonotone Corporation.
May 20, 1947, 9 Claims (Cl. 179-171).

A circuit for a hearing aid amplifier is described such that the gain of the amplifier will remain constant for sound inputs up to a level of 70 or 80 db, but the gain will decrease for sound inputs above about 80 db so as to reach a maximum output with about 100-db input level. Thus the amplifier will act in the usual manner for inputs below about a 70-db level and so background noise and other relatively undesired sounds will not be overemphasized in the absence of speech input, such as would be the case with an automatic volume control operating to give a constant power output. The circuit includes a rectification of a portion of the output from the power tube, to provide a change of bias voltage on the control grid of the



first tube, but in such a way that the bias is not sufficient to cause rectification and accompanying distortion. The effect is achieved in part by the screen grid circuit.—FWK

2,424,348
4.5 HEARING AID EQUIPMENT FOR CONFESSIONALS

Nicholas V. Casson.
July 22, 1947, 1 Claim (Cl. 179-1).

This describes a circuit for use in a church confessional in which there is a microphone in the priest's cubicle and a receiver in the cubicle of the penitent, so that the priest may converse with a penitent having deficient hearing. When the receiver is taken off its hook, the electrical circuit is closed and the hearing aid system is put into operation. The present modification provides that when the receiver is taken off its hook, there will be a visual signal as a lighted lamp in the priest's cubicle and also another signal such as an electric bell at some distance from the confessional, as at the rectory. The object is to give ample notice when unauthorized persons, such as playful children, might tamper with the hearing aid system. The distant signal can be turned off by the priest by means of a switch in his cubicle.—FWK

2,424,935
4.5 HEARING AID ATTACHMENT FOR SPECTACLES

George P. Kimmel.
July 29, 1947, 9 Claims (Cl. 179-107).

This describes the mounting of a bone conduction receiver of a hearing aid on the bow of a pair of spectacles. The object is to avoid the use of the usual headband because this is conspicuous and because it is sometimes uncomfortable. The spectacle bow should preferably have a substantial cross section, and can be made of plastic or other suitable material which will resist torsion. The rear-end portion of the bow has an aperture extending downwardly and inwardly toward the head, this aperture portion being reinforced by a metal sleeve around the bow. A rod or tongue fits into this aperture and carries the bone conduction receiver which rests against the mastoid bone with a pressure contributed by the torsion of the bow. Adjustment is accomplished by moving the tongue in the aperature.—FWK.

2,437,049

4.5 EAR PROTECTOR

Robert H. Sailsbury and Edward M. Oehser, assignors to Consolidated Vultee Aircratf Corporation.
March 2, 1948, 1 Claim (Cl. 128-152).

This ear protector comprises a pair of cups of soft or flexible rubber adapted to enclose the ears and also to extend around the bony structure back of the ear. These rubber cups are held in place by a headband with flexible connections to the cups so that the rims of the cups may adapt themselves to the contours of the ear and its vicinity. A stiffening disk is held in the flat back of each cup and covered with a sound absorbing material.—FWK

2,447,470

4.5 NOISE INSULATING RING FOR EARPHONES

Joseph E. Valentine, assignor to Oxzyn Company.
August 17, 1948, 5 Claims (Cl. 179-156).

This is a construction to hold a small type earphone to the ear and to protect the ear from external noise. A circular carrier plate with a diameter on the order of the major axis of the ear has mounted in it centrally a holder of a small receiver, which has attached to it an ear insert of soft material with a central hole for sound transmission to the ear canal. The carrier plate has mounted on it a soft annular pad to rest against the side of the user's head to conform itself to the shape of the user's ear, and large enough to cover most of the ear. The carrier plate with its attachments is held in place by suitable means.—FWK

2,459,325

4.5 BONE CONDUCTION UNIT

Hugh S. Knowles, assignor to Zenith Radio Corporation.
January 18, 1949, 2 Claims (Cl. 179-107).

This relates to a bone conduction receiver for use with a hearing aid and designed to transmit mechanical vibrations to the bony structure of the user's head, being usually held rather tightly over the mastoid bone back of the ear. The present arrangement is designed to prevent excessive force of the unit against the head of the user by giving a warning of such excessive force. This warning or notification consists of a reduction in output due to a change in the magnetic air gap when the pressure becomes excessive. A screw and spring arrangement provide an adjustment of the pressure point at which this reduced efficiency becomes effective.—FWK

2,436,384

5.1 SOUND RECORDING DEVICE

Harvey Fletcher, John F. Müller, and Karl D. Swartzel, Jr., assignors to Bell Telephone Laboratories, Incorporated.
February 24, 1948, 12 Claims (Cl. 274-1).

Claim 6. "A device for recording sound waves in a sound field produced by a moving or remote sound source comprising a casing or shell adapted to be projected into said sound field in proximity to said source, a sound responsive member mounted within said casing, a sound record medium in cooperative relation thereto to record the sounds causing responses from said sound responsive member, and means operated by the movement of said casing to and through the sound field to move said recording medium relative to said sound responsive element, said last-mentioned means being located inside said casing or shell."—RWY

2,447,018

5.1 THREE-MAGNITUDE RECORDER

George Keinath.
August 17, 1948, 8 Claims (Cl. 128-2.05).

This is an automatic means for showing the relationship between three variables, two of which are represented in the usual Cartesian coordinates and the third is represented by the thickness or frequency of the recorded marks. In one embodiment, the sound of pulse beats is recorded as a function of time and the tightness of a tourniquet fastened to the arm.—RWY

2,450,933

5.1 HORN CONTROL

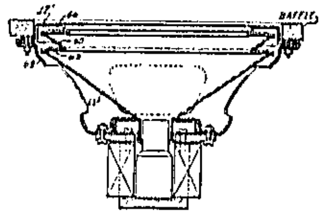
Lawrence D. Bell, Assignor to Bell Aircraft Corporation.
October 12, 1948, 5 Claims (Cl. 177-7).

"The invention contemplates a horn control system for automobiles or the like whereby when travelling at relatively low speeds operator-actuation of the usual horn control pushbutton on the automobile steering column will result in production of a modulated horn signal of the intermittent and/or subdued "courtesy toot" type; whereas under higher travel speed conditions the same actuation of the horn control button will procure usual, full volume horn signals for as long as the horn control button is depressed."—RWY

2,439,666

5.8 LOUDSPEAKER DIAPHRAGM SUPPORT

John F. Marquis, assignor to Radio Corporation of America.
April 13, 1948, 2 Claims (Cl. 181-31).

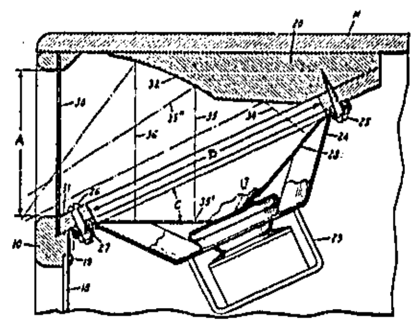


This is the patent on the RCA accordion-edge speaker.—FHS

2,440,078

5.8 RADIO CABINET AND SPEAKER MOUNTING

George F. Devine, assignor to General Electric Company.
April 20, 1948, 3 Claims (Cl. 181-31).



By tilting the loudspeaker, it is possible to install it in a radio cabinet having limited vertical space. A reflector 32 directs the high frequencies out of the cabinet.—FHS

2,442,791
5.8 ACOUSTIC DEVICE

Edward C. Wente, assignor to Bell Telephone Laboratories, Incorporated.
June 8, 1948, 11 Claims (Cl. 181-31).

A domed diaphragm for horn-type loudspeakers is described in this patent. Radial corrugations extend from the domed portion of the diaphragm to the supporting structure. The depth of the corrugations increases in proportion to the distance from the dome.—FHS

2,445,276
5.8 ELECTRODYNAMIC LOUDSPEAKER

Frank Massa.
July 13, 1948, 8 Claims (Cl. 179-115.5).

The inventor has analyzed the performance of the conventional direct-radiator moving-coil loudspeaker by familiar and straightforward methods. From the analysis the inventor concluded that by a suitable choice of voice coil mass and diaphragm size, it is possible to make a direct-radiator loudspeaker having an efficiency of 50 percent.—FHS

2,435,031
5.9 DETONATION PICK-UP

John R. Burns and John M. Whitmore, assignors to General Motors Corporation.
January 27, 1948, 5 Claims (Cl. 171-209).

This is a magnetostrictive pick-up designed for measuring the vibrations caused by detonations within an internal combustion engine.—RWY

2,435,231
5.9 ACCELERATION PICK-UP

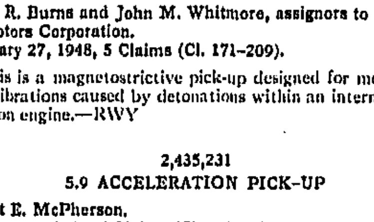
Albert E. McPherson.
February 3, 1948, 5 Claims (Cl. 201-48).

This is an acceleration pick-up of the strain gage type in which "the pick-up component has strain sensitive wires functioning simultaneously as an elastic suspending element and an axial guide."—RWY

2,435,254
5.9 DYNAMIC STRAIN PICK-UP

Walter Ramberg.
February 3, 1948, 3 Claims (Cl. 201-52).

This dynamic strain pick-up, suitable for recording aircraft vibration, is characterized by large voltage sensitivity. The



leaf springs 15 and 19 are so designed that there is nearly a linear relationship between the displacement of element 9 and the current in indicator 23.—RWY

2,441,975
5.9 ELECTROMAGNETIC THROAT MICROPHONE

James Samuel Paterson Roberton, assignor to International Standard Electric Corporation.
May 25, 1948, 6 Claims (Cl. 179-114).

The microphone is an inertia-type magnetic microphone. To reduce external noise pick-up, the case is isolated from the transducer element, and also from the plate which touches the throat, by a flexible coupling.—FHS

2,443,969
5.9 VIBRATION PICK-UP

John M. Tyler and Vincent E. Thornburg, assignors to United Aircraft Corporation.
June 22, 1948, 9 Claims (Cl. 171-209.)

A vibration pick-up is disclosed in which the "movable element," a pivoted arm, remains more or less stationary in space while the frame of the instrument vibrates with the object to which it is secured. A coil on the pivoted arm threads an air gap in a magnetic circuit consisting of a permanent magnet with inner and outer pole-pieces curved in partially toroidal shape. This construction provides maximum generation of voltage in the coil and permits magnetic damping of the pivoted arm.—GK

2,428,168
5.10 SEISMIC WAVE DETECTOR

George B. Loper, assignor to Socony-Vacuum Oil Comapny.
September 30, 1947, 11 Claims (Cl. 177-352).

The invention relates to means for insuring the firm engagement of a seismic wave detector with the wall of a drill hole at any depth. It is operated from the surface in such a way that by the control of tension in a suspension rope or the connecting cable, spikes fulcrummed at suitable points engage and dig into the wall of a hole while the detector is pressed against the opposite wall.—GK

2,449,085
5.10 SUBMERSIBLE SEISMOMETER SYSTEM

Raymond A. Peterson, assignor to United Geophysical Company, Incorporated.
September 14, 1948, 7 Claims (Cl. 177-352).

The invention relates to means for the suspension of seismometers in water, giving efficient transmission or coupling to the seismometer of the seismic waves in the water, while eliminating most of the undesirable extraneous waves. It is carried out by the use of a non-resonant platform suspended at some distance beneath the surface of the water from a float. A number of such floats may be grouped into a system along a course at which it is desired to receive seismic waves.—GK

2,461,344
5.13 SIGNAL TRANSMISSION AND RECEIVING APPARATUS

Harry F. Olson, assignor to Radio Corporation of America.
February 8, 1949, 6 Claims (Cl. 179-1).

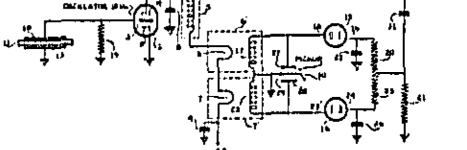
This is concerned with a personalized sound system, so that an individual may receive a sound signal without disturbing others who do not wish to listen, or without the knowledge of those for whom the sound is not intended. An electrical current of the desired audio signal frequency is modulated by an ultrasonic frequency and the resultant is connected to an ultrasonic loudspeaker which will produce modulated ultrasonic compressional waves in the air. These ultrasonic waves are received and demodulated by a small device in the ear of the listener, so that the listener will hear the original audio-frequencies. Such a device might consist of two parts, first a transducer with the ultrasonic sound input and with a corresponding electrical output, and secondly a square law transducer with the modulated ultrasonic electrical input and with a demodulated sound output containing the audio frequencies. These transducers might be of electromagnetic type similar to telephone receivers, except that the square law unit would not have a permanent magnet. Other types of transducers are also possible. It is suggested that the ultrasonic loud speaker be directed toward the listener.—FWK

2,404,026

5.16d METHOD OF AND SYSTEM FOR TRANSLATING SIGNALS

Joseph G. Beard and Robert W. Harralson, assignors to Radio Corporation of America.
July 16, 1946, 12 Claims (Cl. 179-100.4).

This patent is another in that group in which r-f oscillations are used to obtain a voltage or current proportional to a physical displacement. Its particular advantage lies in the fact that the resultant signal is independent of amplitude and/or small frequency changes in the r-f oscillator and of physical shock and microphonics in the oscillatory circuit. In addition, the oscillator may be crystal controlled if such stability is desirable. These objectives are realized by means of a balanced rectifier circuit, composed of diodes or other rectifiers 15 and 16 which have a common load resistor 21 and separate tunable input circuits 17-27 and 23-28. The inputs are tuned to the same frequency, which differs slightly from the frequency of the r-f oscillator, and they are fed by the coupling coils 6 and 7 in such a manner that normally equal and opposite voltages



are generated across the resistor 21. However, the mobile element 29 is a common part of the tuning capacitors of the input circuits, and any motion imparted to it will decrease the resonant frequency of one input and increase that of the other, thus causing a larger voltage in the first and a smaller voltage in the second, or vice versa, depending on whether the oscillator frequency is below or above the frequency to which the inputs are normally tuned. The net result is to generate a differential voltage across 21 which is representative of the displacement of the movable element. The claims cover numerous combinations and include the use of the method for phonograph pick-ups and microphones.—WW-D

2,418,591

5.16d PHONOGRAPH STYLUS MOUNT

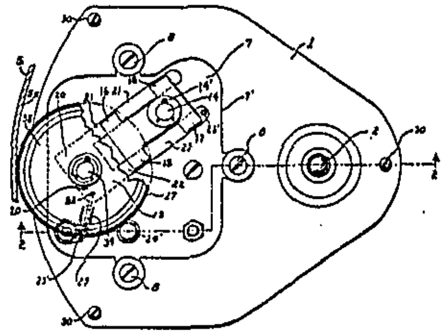
Richard A. MacDonald, assignor to Flexograph, Incorporated.
April 8, 1947, 6 Claims (Cl. 274-38).

In this invention the basic idea is that the stylus point is carried on the end of a short offset arm which is free to rotate about the axis of the main shank. The construction reminds one of a bed caster. It is claimed that such a stylus, when playing a record, will automatically arrange itself tangent to the record groove by an action similar to that in which a caster trails behind the leg of a bed when the latter is moved laterally. Several constructions are shown, but all are characterized by having the arm inseparably associated with the main shank and having the latter of such size as will fit a standard pick-up chuck. In one form the swivel element carries a second chuck in which a standard stylus may be inserted. The inventor claims that the device is equally efficacious for preventing unequal sylus wear and groove damage in both lateral and vertical recordings, though no mention is made of how well such a device will transmit lateral vibrations to the associated transducer.—WW-D

2,421,910

5.16d TURNTABLE DRIVE FOR PHONOGRAPHS

Herbert L. Hartman, assignor to The General Industries Company.
June 10, 1947, 2 Claims (Cl. 74-206).



This invention pertains to an improved mounting for the idler wheel in rim-driven phonograph turntables. Older designs are said to be deficient in that the idler wheel pressed with unequal forces against the turntable rim and the driving roller and/or the idler mounting plate allowed too much vertical play, which gave rise to speed variations and rumble. The present design is specifically concerned with eliminating vertical play in the particular mounting wherein the idler wheel is allowed a lateral motion along a horizontal axis which is free to rotate about an offset vertical axis. The idler wheel 35 is carried on the vertical shaft 24 which is firmly attached to the U-shaped mounting plate 20. The two sides of the U, 21 and 22, slide in horizontal slots accurately cut in the projections 16, 17, 18, and 19, of an I-shaped block pivoted on the vertical shaft 14, which in turn is firmly attached to the motor mounting plate 7. The I-block is provided with a vertical tubular flange which forms a sufficient bearing for the shaft 14 so that any rocking motion of the block is eliminated. A spring 23 pulls the idler mounting plate and hence the wheel evenly against the rim 5 and the motor spindle 24′. A stop 23′ prevents disassembly when the turntable is removed.—WW-D

2,426,061

5.16d ELECTRIC PHONOGRAPH PICK-UP OF THE CAPACITY TYPE

René Snepvangers, assignor to Radio Corporation of America.
August 19, 1947, 15 Claims (Cl. 179-100.41).

This invention covers an improved type of phonograph pick-up, wherein the electrical signal is obtained from the variations in the capacity of a small condenser formed by a stationary plate and a plate fastened to the movable element. The former is mounted vertically on an insulating block carried by the pick-up arm and with such an orientation that its plane is parallel to the axis of the arm. The movable element may be formed from variously shaped pieces of sheet metal bent at right angles to make a vertical plate and an extended essentially-horizontal reed, or it may be shaped from rectangular wire to give a horizontal flat section and a vertical slightly wedge-shaped portion. In every case the vertical plate, or a portion thereof, provides the necessary lateral compliance. The stylus point may be affixed to either end, though generally it is carried by the horizontal part, and the opposite end is firmly anchored to the insulating block. It is claimed that the various forms of the design result in (1) low driving-point mechanical impedance to reduce record wear and the effects of arm resonances, (2) high torsional stiffness to prevent rotational motion, (3) low vertical stiffness to accomodate the motion due to pinch effect, (4) wide-range response, and (5) simplicity of fabrication.—WW-D

2,424,697

5.16m MAGNETIC RECORDER

William P. Lear, assignor to Lear, Incorporated.
July 29, 1947, 13 Claims (Cl. 179-100.2)

The mechanical design for a magazine type of magnetic wire recorder is disclosed in this patent. The magazine encloses two wire reels, a level wind mechanism, and a magnetic head structure. Also incorporated in the magazine are locking brakes which are automatically released when the magazine is attached to the recorder, an elapsed time indicator with limit switches, and a spring loaded idler which should tend to prevent wire breakage during periods of high acceleration. —LCH

2,425,213

5.16m MAGNETIC WIRE TELEGRAPHOPHONE SYSTEM

David E. Sunstein, assignor to Philco Corporation.
August 5, 1947, 16 Claims (Cl. 179-100.2

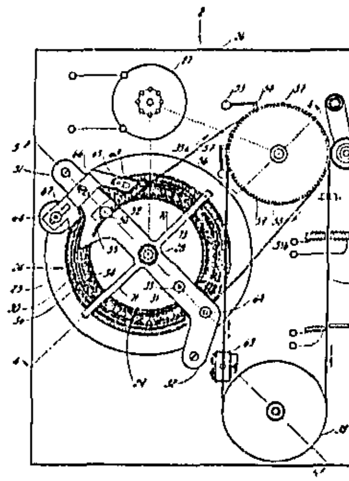
This is an extension of some of the ideas disclosed in Patent 2,458,315. Transverse wire recording is used. In this case, two recording heads are used as well as two reproducing heads. In each case the heads are disposed at right angles to each other. This essentially gives two recording and two reproducing channels. One channel is used for the audiosignal. The other is used for recording a control signal. The secondray signal can be used to control an expander if compression is used during recroding. It can also be used to actuate a motor which rotates the reproducing head structure to insure that the audio reproducing head will always be aligned along the axis of flux corresponding to the audio recording.—LCH

2,426,838

5.16m ENDLESS TAPE MAGNETIC RECORDING-REPRODUCING DEVICE

Harry B. Miller, assignor to The Brush Development Company.
September 2, 1947, 15 Claims (Cl. 179-100.2)

This is a patent for a drive mechanism for an endless tape. The tape is taken from the inside of a spirally wound coil, then threaded around a driven fly-wheel 37, a guide wheel 38,

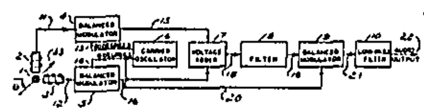


then again around the fly-wheel 37, and finally back to the outside of the spirally wound coil.—LCH

2,458,315

5.16m METHOD AND APPARATUS FOR REPRODUCTION OF ANGULAR MAGNETIC RECORDING

David E. Sunstein, assignor to Philco Corporation.
January 4, 1949, 18 Claims (Cl. 179-100.2).



This patent discloses a reproducing system which is intended to compensate for the twist effect in reproducing a transverse magnetic wire recording. Signals from two reproducing heads 2 and 3 are disposed at right angles to each other. The signals from these two heads are used to modulate a pair of balanced modulators 4 and 5. The outputs from the modulators are added in a voltage adder and one of the side bands is separated from the other in a filter 8. The output from the filter is fed into another balanced modulator 9 where it is demodulated and then filtered in a low pass filter 10 to give finally an audio output represented by the following equation:

$$e_{21} = \frac{AB^2}{4} \cos(Mt + \theta).$$

In this equation, A and B are the amplitudes of the audio signal and the carrier, respectively. M is the angular velocity corresponding to the audiofrequency, and θ is the angular position of the flux axis relative to one of the heads. This equation shows that the amplitude of the final audio signal is independent of the angle θ.—LCH

2,440,439

5.17 PERMANENT MAGNET ELECTRODYNAMIC TRANSDUCER

Walter E. Gilman, assignor to Permoflux Corporation.
April 27, 1948, 2 Claims (Cl. 179-115.5).

This is a very detailed patent describing the construction of moving-coil earphones of the insert type.—FHS

2,445,486

6.4 DRUM PEDAL APPARATUS

Frederick J. La Londe.
July 20, 1948, 14 Claims (Cl. 84-422).

This drum pedal mechanism is supported on a bar nearly as long as the diameter of the drum. The bar is clamped at its ends to the counter hoop.—RWY

2,446,508

6.4 DRUM PEDAL

Milton E. Crowell, assignor to H. & A. Selmer, Incorporated.
August 3, 1948, 9 Claims (Cl. 84-422).

"The primary object of the invention is to provide a device having a beater, an operating pedal, and a tensioning spring wherein the relationship of beater to the operating pedal is adjustable to regulate the stroke of the pedal required to actuate the beater without varying the tension of the spring." —RWY

2,449,032

6.7 PLAYING BAR

Olen H. Yates.
September 7, 1948, 5 Claims (Cl. 84-319).

Intended for the assistance of beginners playing Hawaiian or similar guitars, the playing bar is so shaped that it facilitates the correct holding of the bar.—GK

2,449,124

6.7 MUSICAL INSTRUMENT

Arthur Horden Kimmons.
September 14, 1948, 1 Claim, (Cl. 84-267).

This guitar-like instrument has but three strings. The two on the outside are supposed to be tuned to the same pitch permitting the playing of the instrument by either a right- or a left-handed player.—GK

2,449,890

6.7 PICK FOR STRINGED INSTRUMENTS

Robert C. Garlick.
September 21, 1948, 2 Claims (Cl. 84-322).

The characteristics of several picks of different thickness are combined into one, the thickness of which is tapered from one end to the other.—GK

2,450,210

6.7 STRING DEPRESSOR FOR STRINGED MUSICAL INSTRUMENTS

Howard L. Sprague.
September 28, 1948, 1 Claim (Cl. 84-315).

The invention consists of push-buttons supported in a frame that may be strapped to the neck of a lute type instrument. Connected with the buttons are depressors that may have one or more fingers for depressing one or more strings simultaneously between frets for the production of tones or chords. It is intended for persons having large finger tips, who cannot accurately depress a string without interfering with adjacent ones.—GK

2,452,307

6.9 MUSICAL KEY CONTROL

James A. Koehl, assignor to Central Commercial Comapny.
October 26, 1948, 7 Claims (Cl. 201-55).

The invention relates to electric switches intended for the use in connection with playing keys of electrical musical instruments. A plurality of such switches is arranged in such a way that, as a key is depressed, the switches open in sequence, each inserting an electrical resistance into a shunt across the amplifier input. This will cause a gradual increase in volume of the tone produced, and also enables the player to control the volume by only partly depressing the key.—GK

2,457,886

6.10 TREMOLO DEVICE FOR ACCORDIONS

Walter Gerber.
January 4, 1949, 7 Claims (Cl. 84-376).

An intermediate partition is provided between the bellows and the sounding section of an accordion. This partition has aperatures in it, one of which is covered by a reed. The reed is vibrated by the air flow from the bellows, thus causing an amplitude tremolo. An operating button near the keyboard permits the damping of the reed and exposes another opening in the partition permitting the free passage of air for steady tones without vibrato.—GK

2,458,653

6.10 DOUBLE VALVE FOR ACCORDIONS AND LIKE MUSICAL INSTRUMENTS

René Seybold.
January 11, 1949, 2 Claims (Cl. 84-376).

The invention discloses a double valve for accordions so arranged that the excess pressure between the playing blast and the atmosphere is first relieved before the sound holes are completely open. Closing is similar in reverse order.—GK

2,450,212

7.7 MUFFLER

Joseph J. Thomas.
September 28, 1948, 4 Claims (Cl. 181-43).

In this muffler construction cold air is drawn into the muffler by a sort of venturi action or "injection cones." It is claimed that this action will eliminate smoke by burning unused gases and will also cool the muffler—CEN

2,452,723

7.7 SPARK ARRESTER SILENCER

Roland B. Bourne, John P. Tyskewicz, and Arthur E. Chase, assignors to The Maxim Silencer Company.
November 2, 1948, 3 Claims (Cl. 183-94).

This patent reports an improvement over earlier spark arresting mufflers. The backpressure of the centrifugal spark arrestor is reduced by a system of vanes which decrease the rotational velocity of the gas leaving the silencer.—CEN

2,453,240

7.7 ACOUSTICAL WAVE FILTER FOR PNEUMATIC HAND TOOLS

Gustav V. A. Malmros, assignor to International Business Machines Corporation.
November 9, 1948, 1 Claim (Cl. 181-48).

The exhaust of a pneumatic hand tool normally has an objectionable siren effect. In this patent a small collecting chamber in combination with its outlet slots form a wave filter or muffler. It is claimed that this construction produces only a high pitched hissing sound instead of the lower frequency siren-like sound.—CEN

2,455,965

7.7 WET-TYPE WATER-SEPARATING STEAM-INHIBITING EXHAUST MUFFLER

George Wohlberg.
December 14, 1948, 12 Claims (Cl. 181-52).

This patent describes a muffler in which water is mixed with the exhaust gas in order to quench sparks and reduce exhaust noise. The water is then separated from the gas by centrifugal action and the gas is heated so as to reduce the visibility of the steam in the exhaust. This exhaust system has been designed primarily for submarines in order to prevent their detection in wartime.—CEN

2,456,512

7.7 MUFFLER FOR INTERNAL-COMBUSTION ENGINES

George V. Johnson.
December 14, 1948, 3 Claims (Cl. 180-54).

For certain applications, it is essential that all sparks be eliminated from the exhaust of internal combustion engines. This patent describes a muffler in which the exhaust gas is directed at the surface of a tank of water so that the sparks are quenched in the water. It is claimed that muffling is accomplished by expanding the gas above the water and by "splashing the water about." It appears that the patent is limited to the construction in which the side panels of the muffler are an integral part of the chassis.—CEN

2,416,353

9.6 MEANS FOR VISUALLY COMPARING SOUND EFFECTS DURING THE PRODUCTION THEREOF

Barry Shipman and Robert H. Guhl.
February 25, 1947, 10 Claims (Cl. 35-1).

Though other applications are possible, this invention is essentially a training device, by means of which the instantaneous wave form of the sound produced by a pupil can be compared with that of a "teacher," either live or recorded. The signals derived from the two sources are amplified in separate channels and viewed on two oscillographs, placed close together, or on one oscillograph. In this latter case, the signals are fed to an electronic switch which samples first one and then the other at a sufficiently rapid rate to cause two separate traces to appear, one above the other, on the face of the CR tube. An additional feature is a means for varying the sweep rate of the oscillographs so as to get instantaneously a synchronization with the fundamental of the signal. This is useful in cases where the pupil is singing or playing a musical instrument, and it is accomplished by using a device similar to an organ keyboard. When a particular key is depressed it modifies the sweep rate either directly, by introducing different components into the sweep circuit, or indirectly, by actuating a relay which does the same thing. A monitoring loudspeaker is included in the apparatus.—WW-D

2,438,526

13.11n SYSTEM FOR DETERMINING THE DIRECTION OF A SOURCE OF SOUND

Charles H. Waterman, assignor to Submarine Signal Company.
March 30, 1948, 9 Claims (Cl. 177-352).

Many bearing deviation indicators used in sonar operate on the lobe comparison principle with a split hydrophone. This patent describes a circuit for intensifying the difference between the two voltages to be compared. The two signals from the split hydrophone are fed into similar amplifiers. A part of each output is rectified and used for differential control of the amplifier gains.—LB.

2,438,580

13.11n COMPENSATOR FOR DOPPLER EFFECT

O. Hugo Schuck, assignor to the United States of America.
March 30, 1948, 5 Claims (Cl. 177-386).

In sonar echo ranging equipment a device called an own Doppler nullifier eliminates the frequency shift caused by the motion of the searching ship. One such device is described in this patent. By either manual or automatic means a capacitance is varied in proportion to the product of the ship's speed by the cosine of the relative bearing of the sound beam. This capacitance controls the tuning of either the transmitter or the reciever so as to compensate for the Doppler effect of the ship's motion. Whatever frequency shift remains between the transmitted signal and an echo is then due entirely to the motion of the target.—LB

2,443,647

613.11n ELECTRICAL APPARATUS

Charles H. Waterman, assignor to Submarine Signal Company.
June 22, 1948, 1 Claim (Cl. 234-1.5).

In sonar echo ranging or depth sounding it is common to record the echoes on a chart of special paper. A stylus, traveling over the paper, produces a mark when an electric signal is applied. Available recording paper has a dynamic range between the faintest and the darkest mark corresponding to about 7 db in the electric signal. By a compressor circuit described in the patent this 7-db range at the stylus is made to represent a dynamic range of 40 db in the echo.—LB

2,444,069

13.11n SYSTEM FOR RECEIVING SOUNDS IN THE PRESENCE OF DISTURBING NOISES

Leon J. Sivian, assignor to Bell Telephone Laboratories, Incorporated.
June 29, 1948, 4 Claims (Cl. 177-386).

Interference from a local source, such as a listening ship's own propeller, can be reduced when two hydrophones are connected in opposition. One hydrophone is stiffness controlled and pressure operated. The other is mass controlled operated by pressure gradient. Paradoxically, they are located as close as possible to the disturbing source. The directionality of the pressure gradient hydrophone is helpful, but the important effect is an inherent property of sound propagation. A pressure gradient has two components. One varies inversely with the distance, and the other inversely with the square of the distance from the source. Close to the source, the inverse square component predominates, and produces the major portion of the response of the gradient hydrophone. If the two hydrophones have equal and opposite responses to the interference, the pressure operated one gives the predominant response to a signal from a remote source. Because the phase angle between the components of pressure gradient varies with distance, the interference can be exactly balanced out only at a single frequency. However, if the hydrophones are separated from the source by 1/20 of the wave-length of the highest frequency used, the interference is reduced at least 20 decibels. Obviously, the utility of the method is limited to very low frequencies. A mathematical analysis is given in the patent.—LB

2,437,088

13.11t MICROPHONE ASSEMBLY

Gabriel M. Giannini, assignor to Automatic Electric Laboratories, Incorporated.
March 2, 1948, 5 Claims (Cl. 179-116).

This patent describes a directional microphone which can be mounted on the periscope of a submarine. This microphone is constructed so no damage results when the submarine is submerged.—FHS

2,437,270

13.11t MAGNETOSTRICTIVE COMPRESSIONAL WAVE TRANSMITTER OR RECEIVER

Robert L. Peek, Jr., assignor to Bell Telephone Laboratories, Incorporated.
March 9, 1948, 8 Claims (Cl. 177–386).

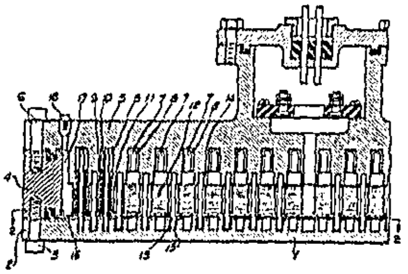
The magnetostrictive core of this transducer is in the form of a rectangular loop which can be readily assembled as a stack of laminations. Two opposite sides of the rectangle are the active legs which expand and contract in longitudinal vibration. The arms of the core which join these legs are effective only as connecting links in the system. The core is polarized by a permanent magnet having its poles near the ends of one arm. The flux from the magnet divides between two paths in the core. The useful flux flows through the two legs and the arm which connects their far ends. The useless flux takes the shorter path through the arm between the poles. A portion of this arm has its cross section reduced by a slot to provide a high reluctance for the unwanted polarizing flux. Since the reluctance of the core is relatively high at signal frequencies, the constriction at the slot does not significantly increase the reluctance to the alternating flux around the rectangular core.—LB

2,437,282

13.11t ELECTROACOUSTICAL TRANSDUCER

Edwin E. Turner, Jr., assignor to Submarine Signal Company.
March 9, 1948, 8 Claims (Cl. 177–386).

This patent describes a magnetostrictive transducer of a type commonly used for echo ranging or depth sounding. A large number of magnetostrictive tubes 7 are secured by forced fit joints to the back of the diaphragm 1. The diaphragm and tubes constitute a resonant half-wave-length vibrator, with a nodal plane in the tubes but close to the diaphragm. Each vibrating tube 7 is surrounded by a coil 12 which is supported by the cover 5. The transducer is polarized by Alnico rods 13 interspersed between the tubes. The magnet rods are also supported by the cover, and extend into clearance holes in the diaphragm.



For frequencies between 12 and 30 kilocycles per second, transducers of this type are made with diaphragms some 16 inches in diameter. Secondary lobes in the directivity pattern are reduced by distributing the driving force so that the diaphragm vibrates as a clamped edge disk. This shading is accomplished by annular grouping of the coils and by appropriate connections of the groups. Of course, the directivity pattern in receiving is the same as in transmitting.—LB

2,438,925

13.11t MAGNETOSTRICTIVE SUBMARINE SIGNAL TRANSMITTER OR RECEIVER

Hubert K. Krantz, assignor to Bell Telephone Laboratories, Incorporated.
April 6, 1948, 7 Claims, (Cl. 177–386).

Various magnetostrictive transducers are described in which the active element is a hollow cylinder in radial vibration. The cylindrical core may be a stack of annular laminations, or a spirally wound ribbon of magnetostrictive material. The core is linked with a toroidal coil. This assembly is protected by a surrounding housing and an internal piece of rubber. Only the inner surface of the vibrating ring is acoustically coupled to the water medium by the rubber. The rubber coupling member may be a solid plug, with one end in contact with the water, and the other end backed up by a metal resonator. Another embodiment has several cores arranged coaxially around a rubber tube, open to the medium at one or both ends. The directional characteristic is controlled in this case by suitable spacing of the cores and appropriate phasing of the coils. A broad frequency response can be obtained with several cores, of progressively different size, assembled on a conical rubber tube.—LB

2,438,926

13.11t MAGNETOSTRICTIVE SUPERSONIC TRANSDUCER

Edward E. Mott, assignor to Bell Telephone Laboratories, Incoporated.
April 6, 1948, 8 Claims (Cl. 177–386).

A companion to the Krantz Patent No. 2,438, 925 shows further variations of the hollow cylinder principle, with particular emphasis on the type M-5 transducer. In this design, fourteen cup-shaped elements are mounted in a plane array. The rings 14 are surrounded by pressure release material 11a and have negligible acoustic loading on their outer surfaces. Radiation occurs at the inner surfaces where the rings are in contact with the liquid medium. The cavities are closed at the back by lead plates 13 which rest against a Corprene sheet 11b. Each lead backing plate, together with the adjacent half of the liquid in the cavity, forms a quarter wave-length resonator at the operating frequency. Transducers of this type have been built for operation over a 6-kilocycle band with a mid-frequency of 25 kilocycles per second.

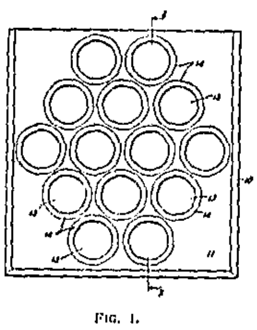


FIG. 1.

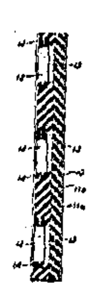


FIG. 2.

Each ring 14 contains a spirally wound core of magnetostrictive material, impregnated with phenolic material to prevent parasitic vibration. The core and its toroidal coil are molded in more phenolic material, in which copper dust is dispersed for better thermal conductivity. Design data for the core include heat treatment and magnetic properties of two magnetostrictive alloys. By suitable design, efficiencies as high as 88 percent have been obtained for individual ring elements.—LB

2,438,936

13.11t ELECTROMECHANICAL TRANSDUCER

Warren P. Mason, assignor to Bell Telephone Laboratories, Incorporated.
April 6, 1948, 1 Claim (Cl. 177-386).

The beam of a sonar transducer may be broadened by a lens which also serves the function of a sound window in the housing. The material suggested for the lens is a synthetic rubber which has a higher velocity of propagation than water. Accordingly a convex lens causes divergence of the sound. Excessive thickness of rubber may be avoided with a Fresnel lens shaped like the glass lens of a searchlight. A single rubber lens permits spreading the beam to a total width of 45°. This angle can be doubled by placing a second lens in front of the first.—LB

2,440,903

13.11t UNDERWATER TRANSDUCER

Frank Massa, assignor to The Brush Development Company.
May 4, 1948, 15 Claims (Cl. 177-386).

A flexible hose, with small transducers spaced at frequent intervals along its several hundred feet of length, is designed to be towed through the water. Requirements of water-tightness, tensile strength, and ability to withstand explosions at close range, are all satisfied by a rubber hose reinforced with stress cords. Because a hose of this kind is not acoustically transparent, windows of plain rubber must be inserted where the transducer elements are located. Each transducer, with its windows, is contained in a metal housing which also serves as a splicing sleeve to join adjacent lengths of hose. Either piezoelectric or magnetostrictive elements may be used. The details of the rather intricate construction, and some of the assembly processes, are described at considerable length.—LB

2,443,177

13.11t SUBMARINE SIGNALING APPARATUS

John T. Beechlyn, assignor to Submarine Signal Company.
June 15, 1948, 15 Claims (Cl. 177-386).

A long magnetostrictive tube is divided by flexible couplings into sections of suitable length. Each coupling is formed by expanding a portion of the tube itself into a bead which is compliant to longitudinal vibration. The section between tubes have longitudinal and circumferential resonances which may be separately chosen in any desired relationship. A variety of designs are illustrated, some with arrays of several tubes. The tubes are watertight and may be immersed directly in the sound medium. Internal electric windings may be arranged for either circumferential or longitudinal magnetization of the tubes.—LB

2,443,178

13.11t PIEZOELECTRIC VIBRATOR

Hugo Benioff, assignor to Submarine Signal Company.
June 15, 1948, 5 Claims (Cl. 177-386).

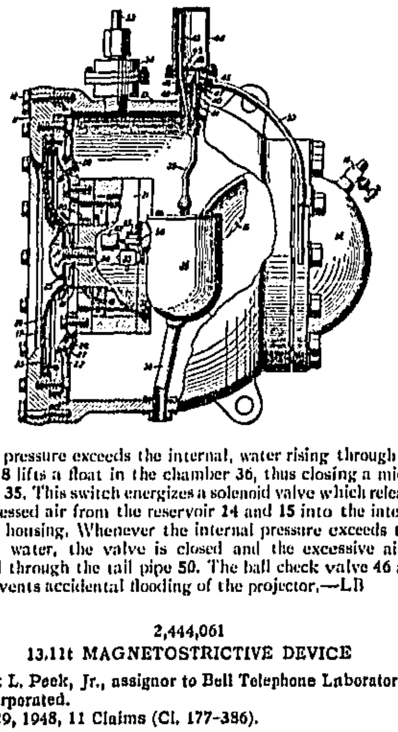
A resonant transducer is described which is very similar to one covered by Patent No. 2,406,792 [Reviewed J. Acous. Soc. Am. 20, 84 (1948)]. In both versions, the crystals are cemented in recesses in resonant metal blocks. The blocks are attached to the back of a radiating plate.—LB

2,444,049

13.11t PRESSURE COMPENSATED SUBMARINE SOUND TRANSMITTER OR RECEIVER

John H. King, assignor to Bell Telephone Laboratories, Incorporated.
June 29, 1948, 5 Claims (Cl. 177-386).

In calibrating hydrophones, a commonly used source of low frequency sound is the type 4B projector. Essentially, it is the well-known Bostwick speaker adapted for underwater use. Since the thin compliant diaphragm of this transducer cannot withstand appreciable difference between external and internal static pressures, a mechanism is provided to balance them within 0.02 pound per square inch. Whenever the ex-



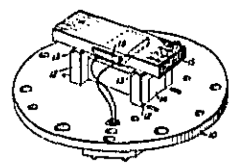
ternal pressure exceeds the internal, water rising through the tube 38 lifts a float in the chamber 36, thus closing a micro-switch 35. This switch energizes a solenoid valve which releases compressed air from the reservoir 14 and 15 into the interior of the housing. Whenever the internal pressure exceeds that of the water, the valve is closed and the excessive air is vented through the tail pipe 50. The ball check valve 46 and 49 prevents accidental flooding of the projector.—LB

2,444,061

13.11t MAGNETOSTRICTIVE DEVICE

Robert L. Peek, Jr., assignor to Bell Telephone Laboratories, Incorporated.
June 29, 1948, 11 Claims (Cl. 177-386).

A block of laminations 15 is supported at the nodes of flexural vibration for its fundamental mode. The supports 13 are the pole pieces of a permanent magnet 14. The signal coil



19 passes through a slot in the laminated block. Alternating flux flowing around the slot adds to the polarizing flux on one side and opposes it on the other. If the laminations have

sufficient remanence, the permanent magnet may be eliminated. The supports 13 may then be of resilient rubber. The base 10 is only the back cover of a complete sonar transducer, in which the block 15 is coupled to a diaphragm by a pad of rubber. The device has advantage at low frequencies because the flexural mode requires relatively small dimensions. A block 2½ inches long resonates at 5 kilocycles per second.—LB

2,444,911
13.11t ACOUSTIC STRUCTURE

Hugo Benioff, assignor to Submarine Signal Company. July 13, 1948, 2 Claims (Cl. 181-0.5).

To avoid turbulence, sonar transducers are commonly streamlined by a surrounding housing, or dome, of thin steel. The thin steel shell is reasonably transparent to sound at normal incidence. At oblique incidence, however, reflection or reradiation occurs as a result of transverse waves in the shell. The dome described in this patent is designed to damp out the transverse vibration. Inside the steel shell is a second shell of aluminum, and the thin space between the two is filled with a viscous substance such as asphalt or pitch.—LB

2,444,967
13.11t OSCILLATOR

Edwin E. Turner, Jr., assignor to Submarine Signal Comapny. July 13, 1948, 6 Claims (Cl. 177-386).

This patent describes a sonar transducer in which concentric magnetostrictive tubes are secured to the back of a radiating plate. The tubes distribute the driving force over the entire radiating area, which may be two feet or more in diameter. Exciting coils are located in the annular spaces between the concentric tubes.—LB

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Program of the Thirty-Seventh Meeting of the Acoustical Society of America

HOTEL STATLER, NEW YORK CITY, NEW YORK

May 5, 6, and 7, 1949

Acoustics in Communications

1. Invited paper BY HARRY F. OLSON, *RCA, Princeton, New Jersey.*

Contributed Papers

2. The Acoustic Impedance of Closed, Rectangular, Loudspeaker Housings. WILLARD F. MEEKER, FRANK H. SLAYMAKER, AND LYNN L. MERRILL, *Stromberg-Carlson Company, Rochester, New York* (15 min.).—Direct-radiator loudspeakers are often mounted with the back of the diaphragm working into a completely enclosed space. Conventional theory states that when the maximum linear dimension of such an enclosure is small compared with the wave-length, the acoustic impedance which it presents to the loudspeaker is capacitive and is given by the expression $Z=-j/\omega(V/\rho c^2)$, where V is the enclosed volume. Since it has not been established how small an enclosure must be before it is "small compared with the wave-length," the foregoing expression is frequently used, at low audiofrequencies, to calculate the acoustic impedance of closed loudspeaker housings, and in many cases its use results in considerable error. It is shown here that while the impedance of a closed, rectangular housing is capacitive at very low frequencies, it passes through zero as the frequency increases and becomes that of an inertance as the frequency of the first normal mode is approached. For a typical housing, 11″×22″×22″, the point at which the impedance presented to a very small speaker passes through zero occurs in the vicinity of 70 c.p.s.; at this frequency the maximum linear dimension of the enclosure is less than one seventh of the wave-length at this frequency. These results are obtained by following methods given by Morse for determining the pressure distribution throughout a room. Assuming a point-source loudspeaker, the pressure at the source is calculated as the summation of the pressures due to the normal modes of the enclosure. Measurement of the pressure at the back of the loudspeaker diaphragm support this analysis. From measurements of the pressure distribution over the surface of the loudspeaker diaphragm, we may deduce that the magnitude of the acoustic impedance which the enclosure presents to the loudspeaker diaphragm and the frequency at which the reactance becomes zero depend upon the dimensions of the loudspeaker diaphragm as well as the dimensions of the enclosure.

3. Non-Linear Distortion in Dynamic Loudspeakers Due to Magnetic Effects. W. J. CUNNINGHAM, *Yale University, New Haven, Connecticut* (15 min.).—Two sources of non-linear distortion in a dynamic loudspeaker are considered in this discussion; both are related to the magnetic characteristics of the driving mechanism. The first type of distortion arises due to a force of attraction between the voice coil, carrying a current, and the iron of the field structure. This force varies as the square of the current and produces second harmonic distortion. The force may be related to the space rate of change of self-inductance of the voice coil as it moves in the air gap. The magnitude of the distortion produced in this way may be several tenths of one percent, and is greater for low frequencies and large currents. This distortion may be reduced by proper proportioning of the voice coil and field structure, and by using a short-circuited winding on the field structure. The second type of distortion arises due to non-uniformity of the magnetic field in which the voice coil moves. This effect is well known qualitatively, but equations are given here for its quantitative evaluation. These equations indicate that the distortion characteristically is less than one percent, and is greater for large amplitudes of motion. If the voice coil is centered in a symmetrical field, only odd-order distortion is produced. If the field is not symmetrical about the voice coil, even-order distortion is also present. This distortion may be reduced by proportioning the voice coil and field structure so that the mean field in which the coil moves remains as constant as possible.

4. A Continuously Adjustable Filter for Audiofrequencies. GLENN E. TISDALE, *Yale University, New Haven, Connecticut* (15 min.).—An electronic instrument has been developed to operate as a filter with cut-off frequencies continuously adjustable over the audio spectrum. The circuit consists of a low-pass and a high-pass network in the form of four-terminal impedances. These may be used singly or connected in tandem to give band pass action. A single control selects the cut-off frequency of each network over a range of 10 to 1, while a decade switch extends the over-all range to 100 to 1, or more. The low pass section provides a pass band flat within 1 db below the cut-off frequency. Attenuation past cut-off is at the rate of 18 db per octave down to the noise level of the circuit. The high-pass section provides a pass band flat within 2 db above the cut-off frequency. Suppression is more than 32 db for the first octave below cut-off, and the attenuation continues to increase down to the noise level. Noise is at least 60 db below a signal of one volt. The voltage attenuation in the pass band of the complete filter is about 6 db. The principle of operation of the instrument is based on active feedback networks using only resistance and capacitance. The variable elements are ganged resistors, whose values may be in error by several percent without affecting the performance described. Because there are no high gain stages in the circuit, distortion and noise problems are reduced to a minimum.

5. On the Propagation of Sound in Narrow Conduits.* OSMAN K. MAWARDI, *Harvard University, Cambridge, Massachusetts* (15 min.).—The analysis of the propagation of sound in narrow tubes has usually been restricted to shapes yielding tractable mathematical expressions. A great number of practical applications do not fall within these categories and await a solution. An approximate solution of sufficient accuracy for narrow tubes of arbitrary shape has been derived. The derivation has been applied to a multiple capillary tube formed by filling a conduit with circular wires, a device often used as a high acoustical impedance. The theoretical predictions check satisfactorily with the experimental results. It is believed that the results of the analysis will be useful to other similar applications.

* This work was supported in part by the ONR under Project Order X of Contract N5ori-76.

6. Simplified Acoustic Impedance Measurements. R. W. LEONARD, *University of California at Los Angeles* (15 min.).—This paper describes an impedance measuring assembly consisting of a piston and pressure microphone arranged in such a manner that the ratio of the pressure to the particle velocity may be measured in both magnitude and phase at the surface of the piston. The assembly measures the driving point impedance at the surface of the piston. The sample to be measured is confined in a tube of diameter equal to that of the piston and is placed almost in contact with the vibrating surface of the piston. Thus, the pressure and particle velocity at the surface of the sample are prescribed by the velocity of the piston and the specific acoustic impedance of the sample. Constructional details and performance characteristics of the assembly are given.

7. The Least Discriminable Intensity for Random Noise. J. DONALD HARRIS, *U. S. Naval Medical Research Laboratory, U. S. Naval Submarine Base, New London, Connecticut* (15 min.).—When a continuous white noise is increased in intensity for one second every fifth second, a subject can be asked to judge when such increases occur. A device using moving tape to record a subject's responses largely eliminates invalid judgments. Under these conditions, the just noticeable increase was measured at each of several sensation levels from 3 to 50 db. A surprisingly slight loss of discriminability is found as the over-all sensation level decreases from moderately loud to very weak. Differences between the present data and previous experiments can, however, probably be attributed to differences in the manner of computing zero sensation level. The just noticeable increase which is the largest theoretically possible is inferred to be something less than 3 db, reasoning from the psychophysical curve for the noise *vs.* no-noise judgment. The just noticeable difference is found to be somewhat smaller than by the above method when the subject is forced to judge whether the second of two short noises is louder or softer than the first. It is possible that this second method drives the subject more nearly to his physiological limit. If so, it should perhaps receive special attention.

8. Uniform* Speech-Peak Clipping in a Uniform* Signal-to-Noise Spectrum Ratio. DANIEL W. MARTIN, *RCA Victor Division, Camden, New Jersey* (15 min.).—A graphical function $W(r, c)$ has been determined experimentally, in which W is word articulation, r is the relative level of unclipped speech and the noise, and c is the amount of uniform, symmetrical, speech-peak clipping. Preemphasis of the speech signal gave an approximately uniform speech spectrum prior to clipping. Uniform, random noise was mixed with the clipped speech before post-equalization, making the final noise spectrum similar in shape to the speech spectrum. The real-ear response of the earphones was compensated electrically to yield a uniform orthotelephonic response for the communication system, in the frequency range contributing significantly to articulation index. For constant clipping c_n the function $W(r, c_n)$ approaches $W(r, 0)$ as a limit for sufficiently large values of r. For $c<r+5$, $W \doteq W(r, 0)$. For the case of no clipping $W(r)$ when transformed to $W(A)$, A being articulation index, resembles closely the curve by Pollack.

* Uniformity here signifies uniformity with respect to frequency.

Acoustics in the Arts

9. **Invited paper** BY WILMER T. BARTHOLOMEW, *Harvard University.*

Contributed Papers

10. **The Acoustics Department of the Juilliard School of Music.** HARRY L. ROBIN, *Juilliard School of Music, New York* (15 min.).—In recognition of the ever increasing dependence by musicians upon acoustical engineering techniques, an Acoustics Department was established in the Juilliard School of Music in 1947. This Department has the following functions: 1. To conduct courses in musical acoustics for degree and diploma students. 2. To record student and faculty performances and all School concerts. 3. To supervise the technical aspects and production of the broadcasts of School concerts. Some observations on the work of the Department are made and discussed.

11. **Analysis and Synthesis of Speech-Like Sounds.*** FRANKLIN S. COOPER, JOHN M. BORST, AND ALVIN M. LIBERMAN,** *Haskins Laboratories, New York* (15 min.).—The study of the perception of speech is aided by instruments capable of representing physical patterns which are complex in frequency, time, and intensity. The sound spectrograph developed by Potter and co-workers provides visual patterns which are highly suggestive for the isolation of the distinctive aspects of auditory patterns. However, comparison and judgment by ear are desirable, and this requires additional instrumentation to play back modified spectrograms, or completely synthetic spectrographic patterns which differ only in the particular characteristic under examination. The spectrograph developed for these studies records on film, portraying a dynamic range of *ca.* 40 db by a linear density variation of 2.0. The use of a Photoformer to control the recording light permits compensation for non-linearities of other components, or the intentional introduction of high contrast (compression), black-to-white reversal, or intensity-contour characteristics into the spectrograms. The pattern playback uses these spectrograms directly, or after retouching or printing, to control the modulation at syllabic rates of an optical scanning beam which is then converted into sound. Synthetic spectrograms, hand-drawn on a transparent medium, are used in a similar manner. Spectrograph and playback were designed for specific application to speech-like sounds and to studies of the perception of such sounds. Design considerations and performance data are discussed.

* This research was made possible by funds granted by Carnegie Corporation of New York.
** Also, Wesleyan University.

12. **The Speaking Machine of Wolfgang von Kempelen.** T. H. TARNÓCZY, *Budapest; (Abstract and Presentation by Homer Dudley, Bell Telephone Laboratories, Inc.)* (15 min.).—Wolfgang von Kempelen made significant contributions to the investigation of the human mechanism of speech production around 1770–90. During the preceding century there had been considerable speculation on various aspects of speech by linguists, teachers, physiologists and others. Kempelen became interested from the standpoint of the problem of the deaf-and-dumb. About this time also Professor Kratzenstein produced five vowel sounds synthetically with some degree of satisfaction. Kempelen who made a hobby of building intriguing mechanisms proceeded experimentally to set up mechanical equivalents of the parts of the human vocal system on a cut-and-try basis. He used a bellows for the lungs, a slit membrane for the glottis, a box with two variable cavities for the mouth and a set of controls for the various openings—lips, nostrils, and tongue-palate. With some use of one hand for resonance effects he fingered the keys to produce all the consonant and vowel sounds or approximations thereto. The quality was evidentally good enough for visiting observers to recognize a number of words, particularly Italian and French. Kempelen was thus the first to produce a complete synthetic speech mechanism which he described in detail in a book titled *Mechanismus der Menschlichen Sprache nebst der Beschreibung seiner sprechenden Maschine* published in 1791. For general interest the oral presentation will include other early speech-producing machines.

13. **The Effect of Room Characteristics upon Vocal Intensity and Rate.** JOHN W. BLACK, *Kenyon College, Gambier, Ohio* (15 min.).—Groups of 23 males read 12 test phrases in each of eight rooms. The rooms represented two sizes, shapes, and reverberation times. Microphones led to two meters that registered vocal intensity and, in one instance, duration of the phrases. Each set of measurements was treated by analysis of variance. Both rate and intensity of reading were affected by the size and reverberation time of the room and not by the shape. Rate was significantly slower in the larger and the less reverberant rooms. Apparently vocal intensity was greater in the smaller and less reverberant rooms; and readers consistently increased their intensity as they read the 12 phrases in the less reverberant rooms. Differences in this regard, occasioned by the sound treatment of the rooms, were highly significant.

14. **Musical Scales and Their Classification.** J. MURRAY BARBOUR, *Michigan State College, East Lansing, Michigan* (10 min.).—A musical scale is a sequence of musical intervals in a certain range, such as an octave; a mode is a cyclic permutation of a scale; a key is a mode at a certain pitch-level; a raga is a melodic pattern of a key. Most writers, confusing scales and modes, have listed fewer than $\frac{1}{4}$ the possible scales. The older writers, such as Delezenne, Gandillot, Ellis, and Hatherly, were further limited by harmonic considerations. Slonimsky's 1330 scales and melodic patterns (1947) are mostly ragas, in which as with Schillinger (1946), symmetry is paramount; Slonimsky's pentatonic and heptatonic scales are actually modes. The inverse of a scale contains the same intervals in reverse order. The complement of a scale contains all the notes of an octave not in the scale itself. A scale may be measured by the total mean-square deviation of all its intervals, the index for complementary pairs of scales differing by a constant. The notation of a heptatonic scale as a harp scale (with seven different letter names) agrees with the deviation index. However, the function of notes in the scale may often be expressed by fewer than seven letter names or by two names for the same note.

15. **Influence of Humidity on the Tuning of a Piano.** ROBERT W. YOUNG, *San Diego, California* (15 min.).—A six-foot grand piano has been studied for the change of its tuning with relative humidity. During more than a year of observations in the living room in which the piano is located, the relative humidity varied between 20 and 70 percent. The change of tuning lagged the rise and fall of humidity as if the structure were characterized by a "time constant" of the order of 15 days. The relative humidity weighted according to this time constant varied only between 35 and 62 percent. Within the three central octaves, the tuning of this 39-year old piano rose on the average 5 cents (0.3 percent in frequency) for each increase of 10 percent in weighted relative humidity. This rise would result if the sound board would swell enough to lift the bridge at the end of the A_4 (440 c.p.s.) string 0.5 mm.

16. Twenty Years of Research in Phonological Biophysics. C. H. Voelker, *Washington College, Chestertown, Maryland* (10 min.).—This interim report will emphasize the great number of books published which were of prime interest to the research worker in this speciality at the time of the founding of the Acoustical Society of America. It will continue by outlining the development of the broader issues with reference to the glottal source, labial coupling and give particular attention to the recently suggested hypothesis that the glottal-labial tube may be an acoustic filter.

17. Comparison of Performances of the Same Melody Played in Solo and in Ensemble with Reference to Equal Tempered, Just, and Pythagorean Intonations. James F. Nickerson, *University of Kansas* (introduced by Arnold M. Small, Navy Electronics Laboratory, San Diego).—A study was made of solo and ensemble performance of the same musical material as related to systems of intonation postulated by certain acoustical, musical, and psychological theories. In particular, it was desired to check earlier findings that unaccompanied performance and listener preferences approximate Pythagorean intonation[1,2] and to extend a similar line of investigation to ensemble performance. Solo and ensemble performances by 24 well-trained string quartet players were recorded from which stratified random samples of tones were obtained for frequency analysis. This analysis was made through the use of 16-mm sound-on-film loops with a chromatic stroboscope (Stroboconn). The results confirm earlier findings for unaccompanied melodies and indicate that Pythagorean intonation is also most typical of ensemble performance. This tendency appears to dominate any "cultural conditioning" which may exist for equal-tempered intonation.

[1] Paul C. Green, "Violin intonation," J. Acous. Soc. Am. 9, 43–44 (1937).
[2] A. M. Small and Barrett Stout, "Present-day preferences for certain melodic intervals in the natural, equal-tempered, and Pythagorean scales," J. Acous. Soc. Am. 10, 256 (A) (1939).

Invited Papers

18. Acoustics in Communication. Dr. Ralph Bown, *Bell Telephone Laboratories.*
19. Acoustics in Comfort and Safety. Dr. Vern O. Knudsen, *University of California at Los Angeles.*
20. Acoustics and Modern Physics. Dr. Philip M. Morse, *Massachusetts Institute of Technology.*
21. Acoustics in the Arts. Dr. Harvey Fletcher, *Bell Telephone Laboratories.*

Papers Presented at Bell Telephone Laboratories, Murray Hill, New Jersey

22. Welcoming Address. Dr. Ralph Bown, *Director of Research, Bell Telephone Laboratories.*
23. Demonstration Lectures (Arnold Auditorium).
24. A. Recent Research on Barium Titanate Used as a Transducer Material. W. P. Mason.
25. B. Recent Studies of Transistors in Transducer Applications. R. L. Wallace, Jr.
26. C. The Ring Armature Receiver—An Improved Transducer for Telephone Use. W. C. Jones.
27. D. Action Pictures of Sound—A Motion Picture Portrayal of Dynamic Spectra. R. C. Mathes.
28. E. Methods for Focusing, Guiding, and Refracting Sound Waves. Winston E. Kock.

Acoustics in Comfort and Safety

29. Invited paper by Leo L. Beranek, *Massachusetts Institute of Technology.*

Contributed Papers

30. San Diego County Fair Hearing Survey. H. W. Himes and J. C. Webster, *Psychology Division, U. S. Navy Electronics Laboratory, San Diego, California* (15 min.).—A recorded hearing test similar in part to the Bell Telephone Laboratory's World's Fair Test was given at the San Diego County Fair. In addition to the absolute pure tone thresholds for the five-octave frequencies 440 c.p.s. through 7040 c.p.s. a white noise masking threshold was found for 880 c.p.s. and 3520 c.p.s. A report was made by each of the approximately 3700 participants as to age (within 10 year groupings), sex, musical training, noise environment, and known hearing difficulty. Statistical breakdown of hearing losses as a function of sex and age agreed in general with the earlier study.[1] Functional relationships indicate that (a) musical training was negatively related to hearing loss especially with the older age groups, (b) noise environment was positively related to hearing loss at 3520 c.p.s. for males and (c) declared (known) hearing difficulties correlate with actual hearing losses. Masked thresholds, in general, show trends similar to those of the absolute thresholds in relationship to the above factors. However, these trends are not as pronounced as those shown by the absolute thresholds. Subsequent analyses and work on masked and absolute thresholds on naval recruits will supplement these results.

[1] J. C. Steinberg, H. C. Montgomery, and M. B. Gardner, J. Acous. Soc. Am. 12, 291–301 (1940).

31. The Sounds of Disease-Carrying Mosquitoes.* W. H. Offenhauser, Jr. and Morton C. Kahn,** *Department of Public Health and Preventive Medicine, Cornell University Medical College, New York* (15 min.).—In the summer of 1947, the authors made recordings of a number of species of native mosquitoes in West Africa. The equipment used to make the recordings is described, and some of the records obtained are listed. In the summer of 1948, the authors went to Cuba where they recorded the sounds of the female Anopheles albimanus mosquito. The recordings were played back in the Husillo Swamp there for the purpose of calling male mosquitoes of the same species. Relatively large numbers of mosquitoes were killed in the sound-baited trap. It is believed that this was the first time that a sound-baited trap has been used successfully for catching mosquitoes. Mosquito sounds are quite distinctive; gross differences occur and are readily detected both by listening and by wave analysis. In general, male sounds seem higher pitched than female; examination of sound spectrograms suggests that the difference is due in considerable degree to the differences in harmonic emphasis. All fundamental sounds seem to occur in the center of the sonic range (from 300–1000 c.p.s.); all mosquito sounds are rich in harmonics. All mosquito sounds are warble-modulated; some at a single vibrato rate (in the order of 5 c.p.s.), others at a double rate (with the higher rate in the order of some 5 times the lower). In some tones, some harmonics are rather completely interrupted or pulsed, while the fundamental remains quite undisturbed. Warble amplitudes are usually quite large. The fundamental pitch often drifts; in one case, for example, there was a 25 percent increase in fundamental pitch in as little as 0.05 sec. Generally speaking, mosquitoes do not respond to sine-wave tones; this may explain why the tones they generate are complex. Mosquito sounds are low in energy level. A rough measurement has been made of the total sound power output of the insect whose sound was used for sound baiting the trap used in Cuba. The power was in the order of 10^{-12} watt. With such a low power level, it is difficult to obtain high signal-to-noise ratios as we know them in conventional high quality sound recording. Despite this, it has been possible to turn out recordings frequently with as much as 50-db signal-to-noise ratio. The usual operating nuisances of microphonics, hum induction, noise, and the like are encountered in aggravated form.

* Aided by a grant from the Tropical Diseases Study Section, United States Public Health Service.
** Associate Professor of Public Health and Preventive Medicine, Director of Parasitology.

32. The Acoustic Gallstone Detector. E. G. THURSTON AND ERIC A. WALKER, *Ordnance Research Laboratory, The Pennsylvania State College, State College, Pennsylvania* (15 min.).—A serious problem during a gallstone operation is to determine if all the stones have been removed, and, if not, where they are located. They may be in the bile ducts, in the gall bladder, or even in the liver itself. An electro-acoustic instrument to assist in this determination has been devised. It consists of a very small transducer mounted on the end of a slim metal rod. This is connected through a cable to a standard amplifier-loudspeaker system. The characteristics of the system are such that when the dilator touches healthy tissue no sound is emitted; but on striking a stone, a ringing sound is emitted. Another variation of the probe uses a Bakes dilator mounted in a handle containing the crystal. Still another variation which is being prepared is to be used for locating kidney stones.

33. Universal Phonograph Styli. JOHN D. REID, *Crosley Division, Avco Manufacturing Corporation.* The advent of "slow speed" records with small grooves has created mechanical problems in phonograph reproducers in that the needle size and in some cases the tone arm weight has to be adjusted as well as the speed of rotation of the record. This leads to undue complexity of operation. Several solutions to the problem in the form of universal styli which will fit both standard and small grooves will be presented and the relative merits of each compared.

34. Levels and Spectra of Noise in Industrial and Residential Areas. G. L. BONVALLET, *Armour Research Foundation of Illinois Institute of Technology, Chicago, Illinois* (15 min.).—At the Cleveland meeting of the Acoustical Society, in November, 1948, some data were presented on the noise in and near transportation vehicles. These were taken as part of the work on the Chicago Noise Survey. The survey includes investigation of noise conditions in industrial zones and in residential areas, and some data on these now are available. These consist of over-all noise taken with a sound level meter, and spectra taken by the use of an octave band filter unit. The noise conditions vary with the season, and the data on industrial noise, up to the present, were taken when factory doors, windows, and other openings were not open to the extent they are in warm weather. Further investigation will be made in this respect. It is difficult to interpret residential area noise levels since the noise sources may be traffic conditions rather than industrial noise. However, data have been analyzed and the attempt has been made to obtain meaningful information. Slides of characteristic over-all and octave band levels will be presented. Study of the data indicate that the average spectrum is peaked below 300 c.p.s., and the sound energy per cycle drops about 9 db per octave toward the higher frequencies.

35. Sound Transmission of Walls With Known Receiving Room Conditions. F. G. TYZZER, L. G. RAMER, AND J. ANCELL, *Armour Research Foundation of Illinois Institute of Technology, Chicago, Illinois* (15 min.).—Previous study of an improved sound transmission measuring technique at the Riverbank Acoustical Laboratories was reported at the November, 1948 meeting. This technique was shown to have promise in measuring properties of test walls instead of properties of the test walls plus the adjoining rooms. Further study has shown that practical measurements can be made which are directly related to the effective "impedance" of a test wall or panel with random sound on the incident side. The random incident sound is provided by the sound field of the reverberation chamber. An exploration has been made of the transmitted sound field in a narrow room with hard walls except for a very absorbent wall opposite the test panel. Results are given which indicate a fairly uniform energy flow from the test panel to the absorbing wall. The effect of high Q cross modes, which do not greatly affect this energy flow, can be minimized by the use of velocity microphones directed toward the test panel. Since the area of the test panel is less than the cross-sectional area of the room at which measurements are made, a correction must be made. This is a function of frequency since considerable beaming is found at the higher frequencies. By the use of pressure microphones to obtain an average sound pressure on the incident side of the test wall and velocity microphones in the transmitted sound field, chart recordings can be made, giving incident pressure and transmitted velocity as a function of frequency. Examples are shown for several test walls, and also curves of effective wall impedance *versus* frequency. An analysis of the probable error in these measurements is given and compared with some of the errors in the older technique.

36. Transient Sounds in Rooms. DAVID MINTZER, *Acoustics Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts* (15 min.).—Previous work on acoustical transients, where the result is obtained by a Fourier analysis of the steady-state response of the system, has been considered by Morse[1,2] and Bolt.[2] Their results are in the form of a summation over resonant modes, and do not give a simple picture of the effect of each reflection. The case of transients in one dimension (plane-waves) is solved here by applying the Laplace transform to the wave equation for the velocity potential, and to the boundary conditions. The boundary conditions assumed are a given particle-displacement at $x=0$ and a series resistance-inertiance-compliance termination at $x=l$. The transformed velocity potential is then expanded as a series, and the inverse transform is obtained in the form of a (finite) series of terms, the nth term representing the effect of the nth reflection from the termination. The same method is also applied to the case of a rectangular room with a point source arbitrarily located in it and each wall having an arbitrary impedance. By using plane-wave expansions around image points we may solve the transformed wave equation in the form of a series of integrals, which are then approximated. The inverse transform is then taken, and the velocity potential is obtained as a summation over the reflections between the walls of the room. The oral presentation of this paper will be confined to a brief description of the method and to graphical presentation of the results.

[1] P. M. Morse, *Vibration and Sound*, Second Ed. (McGraw-Hill Book Company, Inc., New York, 1948).
[2] P. M. Morse and R. H. Bolt, "Sound waves in rooms," Rev. Mod. Phys. 16, 69 (1944).

37. Some Practical Problems Involved in a Study of the Industrial Noise Problem. RALPH MARTIN MCGRATH.—Medical authorities all agree that the noise problem is serious. How serious it is for a given industry can only be ascertained by a properly conducted noise survey which will give data on the noise levels to which its industrial workers are subjected and an audiometric survey which will give data on the hearing acuity of workers subjected to such environments. Noise level meters, filter sets and analyzers, and audiometers are available with which to conduct these surveys. There is need for standardization in the methods by which the two surveys are conducted and interpreted. There is need, also, for data on how the adverse effects of noise upon personnel in turn affect the quality of the product, labor turnover, absenteeism, accidents, and compensation claims. Industry, confronted with a serious problem, cannot wait for elaborate refinement of data but must act with the data available. Fortunately, industries all over the country are acting and the steps being taken are reviewed. More before and after studies are needed to help others tackle their problems. The steps taken at Hawthorne are summarized with a brief review of the fundamental theoretical problems involved in compensation claims. The role that the Acoustical Society of America can play in coordinating the efforts of all organizations interested in assisting industry to solve the noise problem is outlined briefly.

38. Method of Calculating Hearing Loss for Speech from an Audiogram. Harvey Fletcher, *Bell Telephone Laboratories Murray Hill, New Jersey.*

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39. Invited paper by Charles Kittel, *Bell Telephone Laboratories.*

Contributed Papers

40. Proposed Acoustic System for Ordnance Research Laboratory Water Tunnel. Paul M. Kendig, *Ordnance Research Laboratory, The Pennsylvania State College, State College, Pennsylvania* (15 min.).—A description is given of the acoustic methods and equipment for detecting and locating sources of cavitation on underwater bodies and propellers (under test) in the high speed water tunnel now under construction at the Ordnance Research Laboratory at the Pennsylvania State College. Sound arising inside the tunnel will pass through an acoustically transparent window to an external tank in which is placed a small piezoelectric crystal hydrophone at the focus of an ellipsoidal reflector. The source of sound can thus be located because the hydrophone is most sensitive to sounds originating at the conjugate focus of the ellipsoid which is inside the tunnel. The reflector and hydrophone assembly has three degrees of freedom.

41. Acoustic Filter for Water Filled Pipes. R. M. Hoover, D. Laird, and L. N. Miller, *Ordnance Research Laboratory, The Pennsylvania State College, State College, Pennsylvania* (15 min.).—It is anticipated that some of the auxiliary equipment to be used in conjunction with the new water tunnel under construction at the Pennsylvania State College will be particularly noisy in the ultrasonic frequency region where certain low level acoustic measurements are to be made. This potentially noisy equipment includes such items as a deaerator, an energy dissipator, and a pressure control system which are located in a water loop, external to the main tunnel section. To effectively isolate this loop acoustically from the remainder of the tunnel where the acoustic measurements will be made, it has been considered desirable to design an acoustic filter which can be inserted at the junctions of the loop with the tunnel. This filter should serve the multiple function of reducing the sound transmission through the pipe walls and through the water in the pipes while permitting the passage of relatively large volumes of water at small loss of pressure. The problem of isolation in the pipe path is a fairly common one and essentially requires the use of sound attenuating gasket material. However, the isolation of the water path is more complex in that water must be allowed to flow while sound is attenuated. In this case a honey-combed structure of a pressure release material is used. This paper discusses the hydraulic consideration of the problem and presents the results of acoustic transmission measurements on the filter components. The measurements include: 1. The frequency characteristics of the acoustic filter under free field conditions, 2. The transmission properties of the isolating gaskets mounted in the pipe flanges, and 3. The effective response of the filter and isolation gaskets combined.

42. The Properties of Gaseous Solutions as Revealed by Acoustic Cavitation Measurements.* F. G. Blake, Jr., *Harvard University, Cambridge, Massachusetts* (15 min.).—That very small gas bubbles can serve as nuclei for the formation of cavities in liquids is well established. That experimental observations on the rupture of liquids can be interpreted only on this basis is perhaps not so obvious but none the less true. In the present series of experiments on acoustic cavitation in water, two distinct types of bubble formation occur. One, the violent formation and collapse of vapor-filled cavities, results from mechanical instability of gas nuclei. The other, relatively quiet gas bubble formation, occurs as a consequence of the slow growth of nuclei by "rectified" diffusion of dissolved gas from the surrounding liquid. Each process has a threshold of excitation by a sound field; which has the lower threshold in any given case depends largely upon the concentration of dissolved gas in the liquid. Measurements of the acoustic cavitation threshold in conventionally "deaerated" water, as a function of temperature and ambient hydrostatic pressure, reveal how the equilibrium size of gas nuclei depends upon these variables. Observations on sonically induced effervescence in saturated solutions provide at least a qualitative explanation for the pulse length and viscosity effects observed elsewhere. Cavitation at the surface of a sound projector apparently is profoundly affected by adsorbed gases. The conclusion that gaseous nuclei exist more or less in equilibrium with solutions not supersaturated with gas is contrary to the conventional theory of gaseous solutions. Stabilization of nuclei in the surface cracks of suspended solid particles is a very plausible but not entirely satisfactory explanation. Revision of the theory is a tempting subject for speculation.

* This work was supported in part by the ONR under Project Order X of Contract N5ori-76.

43. The Ripple Tank as a Device for Studying Wave Propagation.* H. D. Rix, *Physics Department, The Pennsylvania State College, State College, Pennsylvania* (15 min.).—The ripple tank has proven to be a useful instrument for studying wave propagation, particularly wave patterns arising from inhomogeneities in the medium. We have built a tank 3 ft. by 2 ft. in which we produce ripples in tap water with a mechanically driven vibrator consisting of a strip of plate glass. The wave field is observed directly or photographed by means of stroboscopic illumination, the wave crests acting as lenses to focus the transmitted light in a plane at a convenient height above the water surface. Refraction, interference, and diffraction effects are produced with the help of obstacles made of glass or plastic. Both phase and amplitude measurements are possible to a fair degree of accuracy. Good agreement has been found with the calculated acoustic field resulting from transmission of plane waves through a vertical cylinder of warm air. We have been able to simulate rather closely in the ripple tank the radiation field experimentally observed in air above a high frequency piston source a few wave-lengths in diameter. The ease with which certain types of acoustic field, for which no specific theory exists or which would be difficult to measure directly, can be simulated and observed makes the tank an important research tool in the acoustics laboratory.

* Part of this work was done under Contract No. W 36-039 sc-32001, U. S. Army Signal Corps Engineering Laboratories, Bradley Beach, New Jersey.

44. A New High Speed Inkless Recorder. A. W. NIEMANN AND L. P. REITZ, *Sound Apparatus Company, Stirling, New Jersey* (15 min.).—A portable instrument is described for recording level changes or low frequency wave forms on a 2-inch wide strip chart. A novel servo system, employing a power driven stylus, is utilized, giving a greater force-to-mass ratio than has heretofore been possible. Recording speeds in excess of 1000 db per second on a logarithmic db scale are attainable, with a continuously variable stylus-speed control. A novel feed-back loop used in conjunction with antihunt circuits makes possible very rapid stylus movement insuring maximum accuracy and stability. Eight synchronous chart speeds from 1 to 200 mm/sec. are push-button selected for accurate time axis recording. A take-up chart spindle is provided for continuous recording. Especially designed for acoustic reverberation studies, it provides accurate means for investigating low reverberant chambers of 0.1 second, and less, reverberation time.

45. The Present Status of Piezoelectric Transducer Crystals. HANS JAFFE, *The Brush Development Company, Cleveland, Ohio* (15 min.).—The piezoelectric characteristics of transducer crystals may be expressed in terms of electromechanical coupling coefficient, mechanical compliance coefficient, and dielectric constant, all referring to the mode to be applied, and the density. The modes considered are face shear (utilized in the torque "Bimorph"), lateral expansion, thickness expansion, and thickness shear. Properties not readily expressed in circuit terms but of the highest practical importance are dielectric and mechanical strength and permissible temperature rise. Rochelle salt offers a coupling coefficient of 0.5 or more, combined with fairly high mechanical compliance and high dielectric constant for face shear and lateral expansion. Quartz combines a rather low coupling of about 0.1 with low electric compliance; these factors are disadvantageous for most transducer applications but may be outweighed by excellent stability. The newer crystals, ammonium dihydrogen phosphate (ADP) and lithium sulfate monohydrate (LH), combine fair stability against atmospheric conditions with intermediate dielectric constant and couplings about 0.3, the former for face shear and lateral expansion, the latter for thickness and volume expansion. Neutral potassium tartrate (DKT) in face shear or lateral expansion is indicated for transducer uses requiring fairly high coupling, up to 0.25, with a low temperature coefficient of resonant frequency. Polarized barium titanate ceramic takes its place besides the single crystals. It combines low mechanical compliance and very high dielectric constant with coupling of 0.20 for the lateral expansion and 0.45 for the thickness expansion. It may be shaped into curved transducer elements. A comparative table for these and some other substances will be given.

46. Scattering of Ultrasonic Waves in Water by Cylindrical Liquid Filled Obstacles.* PAUL TAMARKIN, *Brown University, Providence, Rhode Island* (Introduced by R. B. Lindsay) (15 min.).—The scattering of an underwater ultrasonic beam from effectively infinitely long cylindrical liquid filled obstacles is studied. The wave-length of the radiation used is 1.3 mm and the obstacle diameter is 13 mm, thus placing this type of scattering between the extremes of scattering from obstacles large compared with the wave-length, and scattering from very small obstacles. In a previous paper** the obstacles studied were air and steel, affording the two extremes of ρc mismatch, and the scattering was found to be a diffraction phenomenon. The present communication describes the scattering patterns produced by a cylinder of methyl alcohol, the first of a series of liquids used to determine the type of scattering (i.e., diffraction, refraction, or combinations) produced, as the ρc of the obstacle is varied slowly from values smaller, to values greater than that of water.

* Work supported by Navy Contract N6 ori-215, Task Order 3.
** L. Bauer, P. Tamarkin, and R. B. Lindsay, J. Acous. Soc. Am. 20, 858 (1948).

47. Absorption Measurements in Magnesium Sulfate.* R. T. BEYER, M. C. SMITH, AND R. BARRETT, *Brown University* (15 min.).—Ultrasonic absorption measurements are reported for dilute water solutions of $MgSO_4$. For frequencies in the range 3 to 10 megacycles, the measurements are made by measuring the rectified output of a crystal detector. Measurements proportional to the acoustic intensity are made along the axis of the radiated beam, and the absorption is calculated from formulas which take into account the spreading of the beam and the size of the microphone.[1] In this connection, the necessary measurement of the wave-length of the radiation can be made by use of the spatial modulation of the acoustic signal by the electromagnetic pick-up.[2] Measurements at frequencies in the range 10–30 megacycles are made with a conventional radiation pressure balance. Measurements are reported for various concentrations at the different frequencies. The values obtained indicate an increase in the absorption coefficient which is approximately linear with concentration.

* Work supported by the ONR under contract N6 ori-215.
[1] L. W. Labaw and A. O. Williams, Jr., J. Acous. Soc. Am. 19, 30 (1947).
[2] L. W. Labaw, J. Acous. Soc. Am. 17, 19 (1945).

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Contributed Papers

48. Techniques of Research Used in Quieting Machinery and Appliances. H. C. HARDY, *Armour Research Foundation of Illinois Institute of Technology, Chicago, Illinois* (15 min.).—As our age becomes more industrialized, the problems of quieting machinery and appliances—whether they occur in industry, office or home—become more numerous and more difficult. The economy of using improved research methods and competent scientific personnel is emphasized. Determination of qualitative data (source of energy, source of radiation, relative spectral distribution, relative intensity of frequency peaks, etc.) is usually more important than spending excess time obtaining quantitative data (precise sound intensity, exact frequency spectrum, percent harmonic content, etc.). The importance of analysis of sound energy sources, sound radiation sources, and coupling factors between them will be emphasized. A system of analyzing noise problems, by schematic diagrams giving the energy flow between sound energy sources and the surrounding propagating medium, will be outlined and illustrated by actual data from typical research problems.

49. Method for Quieting Ram Jet Motor Test Stations. W. B. SNOW AND C. J. T. YOUNG, *Kellex Corporation, 233 Broadway, New York, New York* (15 min.).—The Applied Physics Laboratory of Johns Hopkins University, operating under contract with the Bureau of Ordnance of the U. S. Navy, set up during the war a testing laboratory for ram jet motors at Forest Grove, Maryland, a short distance outside of Washington, D. C., in a location surrounded by open country. Postwar building in the neighborhood made it necessary to quiet this installation if operation were to continue at the same location and the Kellex Corporation undertook design of the revisions. Since very large volumes of air and hot gases had to enter and leave the test cells, it was necessary to design a duct system which offered extremely low resistance to gas flow, but high attenuation to sound. This paper gives a brief description of the resulting construction which has allowed the laboratory to continue operation in the midst of a residential community.

50. Acoustic Absorption Coefficients at High Frequencies. WILLIAM S. CRAMER, *Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland* (15 min.).—The measurement of the acoustic absorption coefficient by a steady state method was carried out at frequencies of 9, 20, and 30 kc for seven different materials. This involved the construction of a sound chamber with facilities for creating a diffuse sound field and a sample area where materials could be mounted. The average intensity in the chamber was measured with the sample area covered with the material under test and the results compared with similar measurements when the area was covered successively with a material of negligible absorption and when it was open to the air outside. The expression giving the absorption coefficient in terms of these three relative intensity readings is derived.

51. Transmission of Reverberant Sound through Double Walls. ALBERT LONDON, *National Bureau of Standards, Washington, D. C.* (15 min.).—In a previous communication[1] the transmission of reverberant sound through homogeneous single walls was investigated theoretically and experimentally. The attenuation of an obliquely incident plane sound wave upon transmission through a single wall was computed and using the customary reverberant sound field statistics the attenuation was integrated over all angles of incidence to give the average transmission loss. A similar technique is employed in this paper in studying the transmission of sound through a double wall consisting of two identical single walls. The materials comprising the double walls are the same as was used in the single walls, i.e., aluminum, plywood, and plasterboard. From the single wall experimental results, an expression for the wall impedance, Z_w, for each material, was determined, this expression containing terms which include the effects of the mass, dissipation or resistance, and flexural motion of the wall. This value of Z_w is used in the double wall theory to compute the transmission loss for a double wall. Good agreement was obtained between theory and experiment.

[1] A. London, "Transmission of sound through homogeneous walls," J. Acous. Soc. Am. 20, 595 (Abstract 69) (1948).

52. The Sound Absorption of Perforated Rigid Facings Backed by Porous Materials. L. W. SEPMEYER, *U. S. Naval Ordnance Test Station, Pasadena, California* AND R. W. LEONARD, *University of California at Los Angeles, Los Angeles, California* (15 min.).—A chart for designing specialized absorption characteristics utilizing perforated rigid facings backed by porous materials has been presented by R. H. Bolt in [J. Acous. Soc. Am. 19, 917 (1947)]. In order to check the validity of these charts, reverberation chamber measurements have been made on six different combinations comprised of three different perforated facings and two different porous backing materials. The perforated facings were selected to yield maximal absorption in the range 300 to 2000 cycles per sec. and the flow resistance of the backing materials differed by approximately 10 to 1. The results of these measurements in comparison to the values predicted by the charts and the measured surface impedance of the backing material will be presented.

53. The First Symmetrical Mode of Vibration of a Conical Shell. H. C. HARDY, *Armour Research Foundation* AND B. S. RAMAKRISHNA, *Illinois Institute of Technology, Chicago, Illinois* (15 min.).—An aid in the design of loudspeakers would be a solution of the vibration of a conical shell. The general solution is exceedingly complex, but a partial solution can be obtained for the lowest symmetrical modes, the ones which appear to be most important in loudspeaker performance. The conical shell can be considered to be divided into identical radial lamina. The vibration is thus found to correspond to a beam whose width varies linearly with the distance from the apex, and whose stiffness varies inversely as the square of the distance from the apex. The problem, therefore, reduces to a fourth-order differential equation. Power series solutions of this equation are given in this paper. However, for the truncated cone the necessity of using four boundary conditions leads to very complicated calculations to obtain the eigentones. A more convenient method for obtaining the first symmetrical mode is the Rayleigh-Ritz method. For the free-free vibration, a simple function is found to fit the boundary conditions. This leads easily to the calculation of the resonant frequency. McLachlan* has measured the fundamental frequency of such a metallic cone obtaining 4000 c.p.s. for its first symmetrical mode. The calculated frequency, using the method outlined above, is 3700 c.p.s. The first symmetrical mode in loudspeakers occurs at 500 to 900 c.p.s. and is controlled greatly by the boundary conditions at the skiver, spider, and dust cap.

* N. W. McLachlan, *Loudspeakers (Theory, Performance, Testing, Design)*, (Oxford University Press, New York, 1934).

54. The Acoustic Impedance of a Bubbly Mixture and the Determination of its Bubble Size Distribution Function. NORMAN DAVIDS AND E. G. THURSTON, *Ordnance Research Laboratory, The Pennsylvania State College, State College, Pennsylvania.*—A statistical method is developed which enables an analytical bubble size distribution to be formulated from the rough size grouping usually obtained with practical methods of experimental observation. This method is applied to data taken from photographs of bubbly, turbulent water. The resulting function is used with certain known formulae for evaluating the acoustic impedance of such a mixture. A brief physical interpretation of the effects predicted by the theory is given.

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Contributed Papers

55. Kinetic Theory Equations for Sound in Gases. Henry Harrison (10 min.).—By applying Enskog's first-order solution of Boltzmann's equation one can find kinetic theory analogs of Rayleigh's equations of continuity, momentum, and energy. By specializing these equations to one dimension, one obtains equations describing the propagation of plane waves of arbitrary amplitude. By further restricting these equations to small amplitude perturbations on a uniform medium one obtains a first-order wave equation, containing all loss effects. This kinetic theory equation is almost identical with the equation which Rayleigh uses to discuss viscous losses alone. The heat flow losses which Rayleigh finds appear to be the result of the rather artificial concept of heat conductivity used in the theory of continuous media. A change to the kinetic theory equations would, apparently, increase the gap between theoretical and measured values of sound attenuation in gases.

56. The Absorption and Scattering of Sound Power by a Microphone. Richard K. Cook, *National Bureau of Standards, Washington, D. C.*—Some years ago Lamb determined the maximum power which can be scattered by small non-rigid objects (e.g., resonators, microphones, etc.) located in a plane wave. Recently Foldy has found that an omni-directional microphone located in a plane wave of wave-length λ transmits maximum power equal to $(\lambda^2/4\pi)\times$ the incident plane wave intensity. This result is identical with that for an antenna picking up electromagnetic waves. This paper presents results showing the relation between the sound scattered and absorbed by several types of microphones which are small in comparison with the wave-length. Microphones located in plane waves near reflecting boundaries are included. The design of a microphone small in comparison with the wave-length for transmitting maximum power will be discussed.

57. An Experimental Study of the Velocities of Rayleigh and Lamb Waves.* J. R. Frederick and A. E. Martin, Jr., *Brown University, Providence, Rhode Island.*—Over the surface of a solid medium of infinite thickness Rayleigh waves may be propagated. As the thickness of the medium approaches the order of magnitude of the wave-length of the vibrations Lamb waves[1] are produced. These may be either of a symmetric or antisymmetric type across the thickness of the material, and they travel with their own characteristic velocities depending on the ratio of wave-length to material thickness. Measurements of velocities were made with an ultrasonic reflectoscope by either using a single Y cut quartz crystal to generate and receive pulses of the waves, or by using a separate receiving crystal. The frequencies used ranged from one to 15 megacycles. Measurements are reported on a variety of solid materials and the results are compared with the theory.

* The work reported in this paper has been supported in part by the ONR under Contract N6 ori-715/3.

[1] H. Lamb, Proc. Roy. Soc. A93, 114 (1917).

58. Ultrasonic Radiation from an Ideal Piston Source. G. S. Heller, *Brown University.*—The Rayleigh formula for acoustic radiation from an ideal piston source reduces, under suitable approximation given below, to the ordinary Fresnel integral for diffraction through a circular opening. The Fresnel integral is usually expressed in a series of Bessel functions (Lommel function of two arguments) and reduces in the region of validity, to the correct solution along the axis, to the Fraunhofer solution at large distances, and agrees with R. B. Lindsay's solution on a cylinder based on the piston and coaxial with it. In this expansion, the first term represents the Fraunhofer solution but all the terms of the series must be included to give the solution along the axis. An alternative expansion can be given in which the first term represents both the Fraunhofer solution at large distances and the solution along the axis. This expansion is much easier to handle for regions near the axis than the first. For a piston of radius a, these expansions are valid for points at a distance d from the center of the piston within a cone of half-angle θ coaxial with the piston such that $(a/d)\sin^2\theta \ll \lambda/a$, where λ is the wave-length, and where $(a/z)^2 \ll 1$, z being the distance along the axis from the center of the piston.

59. The Threshold of Hearing for Continuous and Interrupted Tones. Walter A. Rosenblith and George A. Miller, *Psycho-Acoustic Laboratory, Harvard University* (15 min.).—The quiet threshold of hearing for tones, as measured with earphones, shows large variations depending upon the method of presentation of the tones. When the tone is continuous the threshold may be much higher than when the tone is interrupted. This difference is especially marked at high frequencies. For both continuous and interrupted tones the thresholds were determined (1) by starting above threshold, and progressively decreasing the intensity, and (2) by starting below threshold and progressively increasing the intensity. With tones interrupted at slow rates the threshold for the descending series lies below the threshold for the ascending series. This is the usual result obtained with this method. However, when the tones are continuous the descending threshold may lie far above the ascending threshold depending upon the frequency, and the starting point for the descending series.

60. The Sonalator, A 29 Channel Visible Speech Translator. Harry R. Foster and Elmo E. Crump, *Kay Electric Company, Pine Brook, New Jersey.*—A heterodyne type of visible speech translator has been developed with 29 channels. This unit employs 29 separate crystal filters and a high speed rotary beam commutation tube. The heterodyne feature makes it possible to explore any desired 4000-cycle band, from 100 cycles up to several hundred kilocycles, by changing the local oscillator crystal. The Sonalator also employs a fast-acting avc system and selective high frequency boost of the type found desirable by R. K. Potter and associates at Bell Telephone Laboratories.

61. Extraction and Portrayal of Pitch in Speech Sounds. O. O. Gruenz and L. O. Schott, *Bell Telephone Laboratories, Inc., Murray Hill, New Jersey* (15 min.).—An improved method for automatically extracting the pitch information of speech sounds has been devised. It employs a combination of gain control, double detection, voiced sound selection, unvoiced sound exclusion, and a means for counting the fundamental vibrations in the voiced sound intervals. Reliable indications of pitch have been obtained over a range corresponding to frequencies from 100 to 600 cycles for a wide variety of voices. Several visual portrayal means that have been used to show pitch changes are described. One means involves a display of colored light which changes from purple through amber to blue as the pitch increases. Another is in graphical form employing an array of staff-like lines whose spacings widen or narrow in contour fashion to show how the pitch varies with time, permitting a detailed study of the changes.

62. Electrical Stimulation of the Skin at Audio Frequencies. A. B. Anderson and W. A. Munson, *Bell Telephone Laboratories, Inc., Murray Hill, New Jersey* (15 min.).—This paper reports the subjective responses of observers to alternating potentials applied directly to the surface of the skin. The curve of threshold of sensitivity was measured for frequencies from 100 to 10,000 cycles. The intensity range that could be used without extreme discomfort was also determined for this same band of frequencies. Intensity and frequency difference limen tests indicated that frequency discrimination is poor in this frequency range but intensity discrimination compares favorably with auditory results.

63. X-Ray Study of Vibrating Crystal Plates. J. E. White, *Acoustics Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts* (15 min.).—The ability of a perfect crystal to reflect x-rays is markedly enhanced by vibration, as has been reported by many writers. Measurements on statically bent quartz are given which establish a quantitative relationship between increased reflecting power and radius of bending. Several pictures show quartz plates in various modes of vibration, light nodal lines separating the dark zones of maximum bending. Other types of vibration are discussed.

64. Dispersion of Compressional Waves in Rods of Rectangular Cross Section.* R. W. Morse, *Brown University.* (Introduced by R. B. Lindsay) (15 min.).—An earlier paper [J. Acous. Soc. Am. 20, 833 (1948)] reported measurements of the phase velocity of compressional waves as a function of frequency in rectangular rods where the cross-sectional dimensions are of comparable magnitude. Two modes of propagation were observed each being determined by one of the lateral dimensions, there being a discontinuity in the dispersion curve determined by the larger side. The present paper is an attempt to explain the behavior of these curves using the general methods of the theory of elasticity applied to the symmetrical vibrations of an infinitely long isotropic rod. Harmonic wave solutions of the displacement equations of motion are constructed which satisfy the free-surface boundary conditions for certain wave-length regions in the general case and for all wave-lengths in certain limiting configurations. Dispersion curves are calculated and compared with the observed ones. The frequencies at which the experimental discontinuities were found to occur are closely predicted and the theoretical curves are in good general agreement with the measured values.

* Work supported by the ONR under Contract N6onr-215.

65. Oblique Reflection and Refraction of Plane Shear Waves in Viscoelastic Media. H. T. O'Neil, *Bell Telephone Laboratories, Murray Hill, New Jersey* (15 min.).—At sufficiently high frequencies, liquids exhibit both viscous and elastic effects in shear. A recently developed method of measuring the shear elasticity and viscosity of liquids involves ultrasonic plane shear waves, generated in a fused quartz rod and reflected obliquely from a plane interface of the quartz and the liquid. Comparing the effects observed in the presence and absence of the liquid, the change of amplitude and phase of the reflected wave can be correlated with the shear wave parameters of the liquid. The method has been described in two recently published papers.[1,2] This discussion will be concerned with some of the characteristics of the waves in the two media, which affect the theoretical relations involved in the reduction of the data. The waves in the quartz and in the liquid are of different types; the refracted wave in the liquid has nonuniform amplitudes over the wave fronts.

[1] Mason, Baker, McSkimin, and Heiss, "Measurement of shear elasticity and viscosity of liquids at ultrasonic frequencies," Phys. Rev. 75, 936 (1949).
[2] H. T. O'Neil, "Reflection and refraction of plane shear waves in viscoelastic media," Phys. Rev. 75, 928 (1949).

66. Improved Devices for the Concentration of Ultrasonic Energy. Paul J. Ernst, *Department of Physics, Villanova College, Villanova, Pennsylvania* (10 min.).—The efficiency of all existing devices for the concentration of ultrasonic energy is considerably lowered by the losses cause by reflection, absorption, and diffraction. These losses, though generally unavoidable, can be minimized in various ways. The choice of suitable materials, use of "matching coatings," design of "stepped lenses," and development of "acceleration plates" will be discussed, samples of the improved devices shown and numerical and experimental data given.

67. A Precedence Effect in Sound Localization. Hans Wallach, *Swarthmore College* and E. B. Newman and M. R. Rosenzweig, *Harvard University.*—The fact that sounds are localized in reverberant surroundings points up a critical problem which has not been explored sufficiently. A brief description will be given of experiments we have done which demonstrate that there is a *precedence effect*, whereby the first in line of a series of closely spaced sounds is the one which determines the place where the sound is heard. This demonstration of the importance of first arrival makes clear how we are able to discount the ambiguous clues from the reflected sounds of an ordinary hard-walled room. More extended measurements of the *precedence effect* have been made by synthesizing a sound out of four clicks arranged to give first one pair to the two ears representing one location, then a second pair to the ears representing a different location. Two parameters have been studied systematically, the interval between first pair and second pair, and the temporal disparity of the second pair. All measurements were made by varying the disparity of the first pair until the fused sound appeared to be in the middle of the head. Results of these experiments will be discussed.

68. Some Determinants of Interaural Phase Effects. I. J. Hirsh and F. A. Webster, *Psycho-Acoustic Laboratory, Harvard University.*—Recently the binaural masked threshold has been shown to depend upon the phase angle between the two ears for both the masked signal and the masking signal. These interaural phase effects are particularly clear for pure tones of fairly low frequency (100–800 c.p.s.) that are masked by white noise. It has been shown that a pure tone, in-phase at the two ears, that is presented against a background of white noise is more easily heard when the noise is out-of-phase than when the noise is in-phase at the ears. The converse is also true, namely, that if the tone is out-of-phase it is more easily heard when the noise is in-phase. The masked threshold of a 250-cycle tone presented against a background of noise (100 db SPL in a 7000-cycle low pass band) is approximately 15 db lower under antiphasic (tone-in, noise-out or tone-out, noise-in) than under homophasic (tone-in, noise-in or tone-out, noise-out) conditions. When the same tone is masked by another tone whose frequency is fairly close but not close enough to produce beats, no such differences appear. These two masking signals represent the extremes of a continuum of complexity along which the necessary characteristics of the adequate stimulus for these differences should appear. The present experiment constitutes an attempt to find these characteristics. A pure tone at 250 c.p.s. was used as the signal to be masked throughout the experiment. Four different kinds of masking signals were used; pure tones, 'regular' pulses (125 p.p.s.), 'random' pulses (average 125 p.p.s.) and random noise. The signals were presented in frequency-bands which were varied in respect of band-width and center frequency. The results indicate that a regular, periodic masking signal will not produce these interaural phase effects. A necessary condition is randomness or irregularity with respect to time but not necessarily with respect to amplitude. Frequency spectrum does not enter on any such all-or-none basis but rather contributes to the magnitude of the response. The nearer a frequency-band is to the frequency of the masked tone, the greater are the differences between the homophasic and antiphasic conditions. There is no substantial difference between the interaural phase effects produced by a narrow *versus* a wide band, provided that both bands contain the frequency of the masked tone.

69. Distortion of Acoustic Beam Patterns by Echoes and Electric Pick-Up.* A. O. WILLIAMS, JR., W. KECK, AND M. C. SMITH, *Brown University.*—When the acoustic intensity is measured, in water, along the axis of a diverging ultrasonic beam, with a very small piezoelectric microphone as a receiver, the results plotted *vs.* distance from the source follow only approximately the beam pattern predicted by theory. On the steadily falling plot is superimposed a spatial variation repeating at half-wave-length intervals relatively near the source. Another variation shows full wave-length intervals farther away, and there is a transition region between. These variations are found with either CW or pulsed beams, if an averaging detector is used. The full wave-length variation is due to electric pick-up and has previously been used to locate acoustic wave fronts. The other variation cannot be a standing wave between source and microphone, because successively reflected waves are too much weakened by divergence, nor can it be due to standing waves in the whole tank. It seems to be the effect of the first echo returning to the microphone, compounding with the main acoustic and electric signals, and falling off much more rapidly than the electric pickup effect. Measurements and calculations in the 1–3-mc region agree with this explanation, and suggest how to identify the correct beam pattern from the data. Without such corrections the determination of wave fronts at intermediate distances might be markedly affected.

* Work supported in part by the ONR under Contract N6 ori-215 Task 3.

70. Intensity Distribution in Ultrasonic Beams.* W. KECK, G. S. HELLER, AND J. D. NIXON, *Brown University.*—A pulse method has been used to investigate the intensity distribution in the ultrasonic beam produced in water by vibrating quartz crystals having disk- and ring-shaped electrodes. The plot of intensity *vs.* angular position was observed directly on an oscilloscope screen. Measurements were made at a frequency of about 1 mc sec.$^{-1}$. The intensity distribution found for the disk agrees closely with theory and with results which have been obtained by other methods in this laboratory. Most of the energy is confined to a narrow cone with a small fraction of the energy appearing in side lobes which become practically negligible about 8 diameters from the source. For ring sources the intensity distribution is similar to that for a disk. However, the cone of the main beam becomes narrower as the ring is made thinner, as predicted by Williams, Heller and Hellens (J. Acous. Soc. Am. 20, 583 (A) (1948)). In addition a larger fraction of the energy appears in the side lobes, which are discernible at greater distances from the source than in the case of the disk.

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